GENERALIZED METHOD FOR ANALYSIS OF COMPOUND EPICYCLIC GEAR TRAINS

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Abstract. The paper is devoted to the presentation of a method able to calculate kinematic ratios, torque and power flow distribution of epicyclic gear trains having more than 2DOF (two Degrees of Freedom). The method is based on the transformation of the compound epicyclic gear train into a structure consisting of an epicyclic gear train with 2DOF, preserving the DOF (number of Degrees of Freedom) of the overall structure as for the compound epicyclic gear trains. Consequently, it become possible to use analytical tools, including the computer. The method is exemplified on the most frequently used compound epicyclic gear trains such as Ravigneaux.

Key words: compound epicyclic gear train, epicyclic gear box.

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1. INTRODUCTION

The compound epicyclic gear trains (hereinafter referred to as EGT) are frequently used in cars epicyclic gear boxes due to their main advantages: compact design, robustness, and space saving as well as versatility in obtaining a large number of gear ratios. The complete analysis of an epicyclic gear box consists of the calculation of the following: ratios, torques and power flows and overall efficiency for every gear box stage.

The structure of certain compound epicyclic gear trains (Ravigneaux and Lepelletier) are presented in paper [1] focused on their systematization and presentation of gear boxes with such mechanisms, without any analysis.

The graphical methods are used in [2] in order to calculate the speed and forces; the overall efficiency is calculated for low speed ratio and for reverse ratio starting with the specific power flow existing in the above mentioned stages.

A splitting of the Ravigneaux mechanism into basic EGT according to specific status of elements in each stage is presented in [3] as part of the analysis of a seven ratio epicyclic gear box. The performed analysis does not include torques, power flows and overall efficiencies of the gear box, being focused only on calculating the ratios and jump ratios.

In order to apply the bond graph theory for analysis of the 6HP26 automatic transmission, the authors of the paper [4] have performed a reduction of a Ravigneaux-type 3DOF compound EGT to a 2DOF basic EGT considering the spurs effectively used for power transmission in each stage; thus, it was possible to use analytical methods for calculation of gear box characteristics (speed ratios, torques, overall efficiency). The papers [5, 6] detail a comprehensive method for calculation of speed ratios, torque and power flows for epicyclic gear boxes which includes only 2DOF basic EGTs, bringing the advantage of the matrix formulation providing efficiency in usage of commercial calculation software.

The recirculating power flows are analysed in [7] in the case of a Simpson compound EGT using a graphical – analytical method. The computer aided analysis of a gear box including Lepelletier [8] compound EGT is presented in [9] taking advantage of obvious splitting of Lepelletier into two inter-connected 2DOF EGTs.

The above referenced literature provides specific analysing methods which can be applied to particular solutions of compound epicyclic gear trains. The methods used for analysis are not appropriate for computer use and, consequently, are time consuming.

The hereby paper has the objective of taking advantage of the method presented in [5, 6] by extending its applicability to gear boxes which include compound EGTs too. In order to achieve this objective, the method of transformation of the compound EGT into a structure consisting of several interconnected 2DOF basic EGT was elaborated.

The paper presents the basics of a 2DOF epicyclic gear train in order to familiarize the reader with the adopted notations followed by a short presentation of the proposed method.

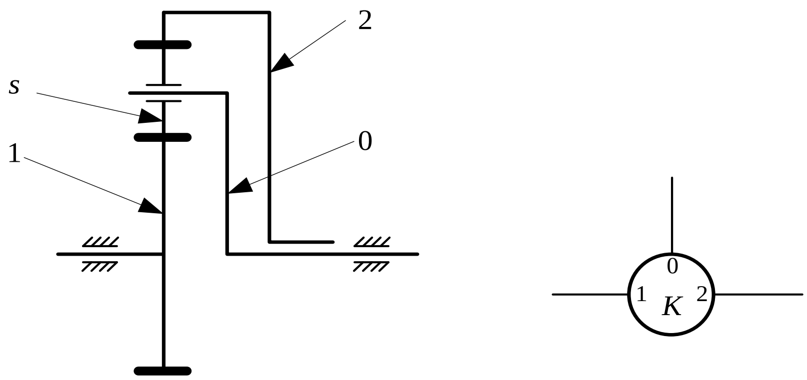
The following sections of the paper illustrate the proposed method for two widely used gear boxes using the Ravigneaux compound EGT.

2. 2DOF EPICYCLIC GEAR TRAIN

Many cars epicyclic gear boxes includes epicyclic gear train having internal and external mesh (see Fig. 1a) [10], are characterized by existence of 2DOF and consists of three external elements: the sun gear (indexed 1), the ring (annular) gear (indexed 2) and the carrier (indexed 0) hosting the revolute axis of planetary gears.

The characteristic parameter of a 2DOF EGT is defined, in general, as:

$$K = \frac{\prod z_{driven\ gears}}{\prod z_{driving\ gears}}. \tag{1}$$



a) Kinematic diagram b) Nodal representation

Fig. 1 – Structure and representations of a 2DOF epicyclic gear train.

The kinematics of a 2DOF EGT is fully described by Willis's equation:

$$\omega_1 - i_{1,2}^0 \omega_2 - \left(1 - i_{1,2}^0\right)\omega_0 = 0, \tag{2}$$

where $i_{1,2}^0$ represents the ratio of EGB from input shaft 1 to output shaft 2 and having element 0 blocked:

$$i_{1,2}^0 = \left.\frac{\omega_1}{\omega_2}\right|_{\omega_0}; \qquad i_{1,2}^0 = (-1)^{n_e} \cdot K, \tag{3}$$

in which n_e is the notation for the number of external mesh of the EGT; for the EGT presented in Fig. 1, $n_e = 1$, and, consequently $i_{1,2}^0 = -K$.

It is noted that it is necessary to know two angular speeds in order to calculate the angular speed of the third element.

The torques acting on the external elements of a 2DOF EGT are calculated using the following equations:

$$T_1 + T_2 + T_0 = 0; \qquad T_1 \cdot i_{1,2}^0 + T_2 = 0. \tag{4}$$

We can observe that it is enough to know the value of the torque acting on a single element in order to calculate the torque acting on the remaining two elements.

The power flow can be calculated using the definition formula:

$$P_j = T_j \cdot \omega_j, \quad j = 1, 2, 0. \tag{5}$$

Consequently, a structure consisting of inter-connected 2DOF EGTs could be analysed using Eqs. (2), (4) and (5); the method is detailed in [5, 6]. Thus, converting an EGT having more than 3DOF into a structure including only 2DOF EGTs becomes possible, in order to efficiently analyse it.

For a compound epicyclic gear train having *N* DOF, it results that there are *N* external links. Assuming that the equivalent structure consisting of *m* 2DOF EGTs (basic EGTs) having three external links each, it results a total of $3\cdot m$ external links. In order to keep the same *N* DOF for the equivalent structure, it is necessary to identify $3\cdot m-N$ links connecting the *m* basic EGTs. Thus, in order to build the equivalent structure of N DOF it is necessary to identify *m* basic EGTs, and to connect their external elements according to the design of the compound EGT. The method is illustrated below considering a relevant example.

3. THE RAVIGNEAUX COMPOUND EGT

The Ravigneaux compound EGT is widely used in the design of planetary gear boxes, almost all the manufacturers adopting its usage due to compactness and robustness as well as versatility in obtaining a greater number of stages [10].

The typical structure of the Ravigneaux EGT is presented in Fig. 2 A.

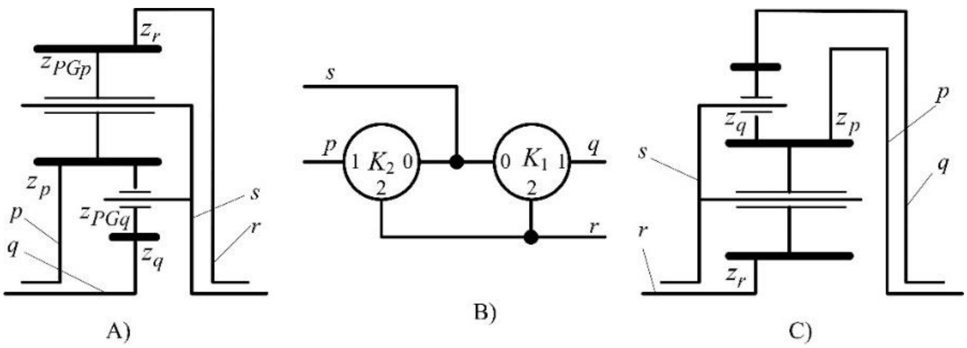


Fig. 2 – The Ravigneaux Compound EGT.

Following the same method as described above, the resulting equivalent structure is presented as a nodal diagram in Fig. 2 B and has the characteristics detailed in Table 1.

Table 1

Assigned functions for external elements of the compound EGT

Basic EGT	Sun gear	Ring gear	Carrier	Planetary gears	n_e	K	$i^0_{1,2}$
1	z_q	z_r	s	z_{PGp} z_{PGq}	2	$K_1=\frac{z_r}{z_q}$	$+K_1$
2	z_p	z_r	s	z_{PGp}	1	$K_2=\frac{z_r}{z_p}$	$-K_2$

A particular solution is used by the Mercedes-Daimler 7 speed automatic transmission which includes an “inversed” Ravigneaux EGT consisting of a unique sun gear and two ring gears (see Fig. 2 C). This solution is analysed below, assuming the “inversed” roles for gears as it results from Fig. 2 C by noticing the usage of the gear notations. Consequently, the EGT characteristic parameters became smaller than 1 ($K_1 < 1$, $K_2 < 1$).

The W7A 700 type gear box is used on various models of Mercedes cars, such as: S340, S/CL/E/SL500, CLS 350, E350, M Class (W164), R Class (W251) and G Class (X164). The kinematic diagram presented in [11] was transformed according to the method described above, the results being presented in Fig. 3, and consists of: a Ravigneaux type EGT, two basic EGTs and seven friction elements (four brakes and three clutches); the gear box has 4DOF imposing to act on 3 friction elements in order to obtain the desired stage, as detailed in Table 2.

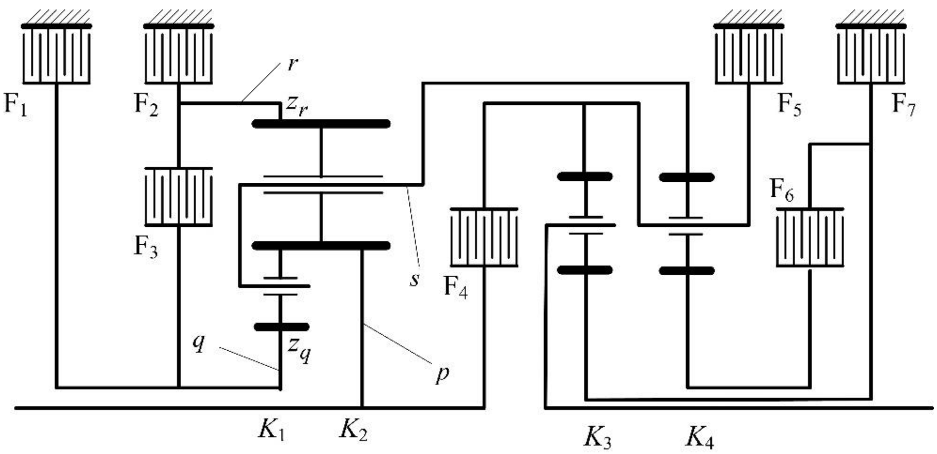


Fig. 3 – Kinematic diagram of the W7A 700 planetary gear box.

Table 2

Friction elements application

Stage	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**R1**	**R2**
Operated brakes, clutch	F_1, F_6, F_7	F_2, F_6, F_7	F_3, F_6, F_7	F_3, F_4, F_7	F_3, F_4, F_6	F_2, F_4, F_6	F_1, F_4, F_6	F_1, F_5, F_6	F_2, F_5, F_6
Ratio	4.377	2.859	1.921	1.368	1	0.820	0.728	–3.416	–2.231

The equivalent nodal diagram is presented in Fig. 4. In order to apply the analysis method detailed in [5] and [6], each branch was indexed accordingly.

The ratio calculation was performed symbolically and has as results functions having as variables the EGTs characteristic parameters. Because only numerical values of the gear box are known (denoted i_j), the values of characteristic parameters were calculated by solving the equations $F_j(K_1, K_2, K_3, K_4) = i_j$; the results are detailed in Table 3.

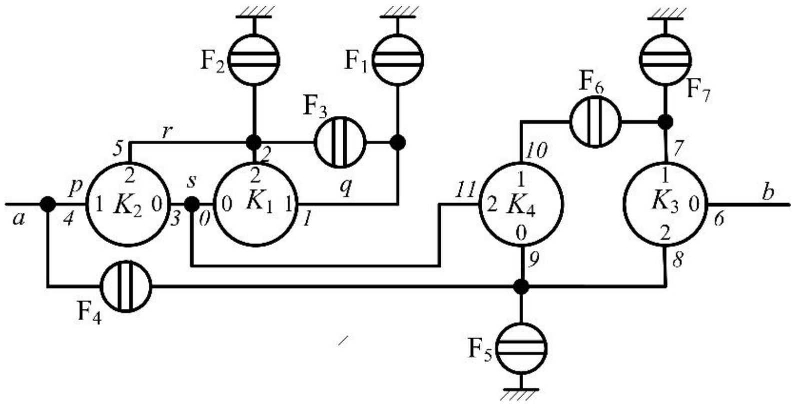


Fig. 4 – Nodal diagram of the W7A 700 gear box.

The values of the characteristic parameters calculated by using four equations are the following $K_1 = 0.3820$, $K_2 = 0.4882$, $K_3 = 2.7173$, $K_4 = 2.4737$. In Table 3, after the numerical value of ratios, between brackets, the errors from values indicated in [11] are mentioned; we can observe that the errors are very small.

Table 3

Results of the analysis – gear box ratios

Function $F_j(K_1, K_2, K_3, K_4)$	
1st Stage (j = 1) $\frac{(K_3+1)\cdot(K_4+1)\cdot(K_1+K_2)}{K_1\cdot K_3\cdot K_4}$ 4.3765 (0.01%)	2nd Stage (j = 2) $\frac{(K_2+1)\cdot(K_3+1)\cdot(K_4+1)}{K_3\cdot K_4}$ 2.8590 (0%)
3rd Stage (j = 3) $\frac{(K_3+1)\cdot(K_4+1)}{K_3\cdot K_4}$ 1.921 (0%)	4th Stage (j = 4) $\frac{(K_3+1)}{K_3}$ 1.3680 (0%)
6th Stage (j = 6) $\frac{(K_2+1)\cdot(K_3+1)}{(K_2+1)\cdot(K_3+1)+K_2\cdot K_4}$ 0.8208 (0.10%)	7th Stage (j = 7) $\frac{(K_1+K_2)\cdot(K_3+1)}{K_1+K_2+K_1\cdot K_3+K_2\cdot K_3+K_2\cdot K_4}$ 0.7281 (0.01%)
R1(S Mode) (j = 8) $-\frac{(K_1+K_2)\cdot(K_3+1)}{K_1\cdot K_4}$ -3.4236 (0.22%)	R2 (C Mode) (j = 9) $-\frac{(K_2+1)\cdot(K_3+1)}{K_4}$ -2.2365 (0.25%)

Following the matrix method detailed in [6], the speed of all 12 external elements were calculated using an application written in PTC Mathcad.

In order to calculate the torques acting on the external elements of the EGTs, additionally to Eq. (4), the following equations are considered for the simple nodes (nodes that distribute the torque keeping the same speed of the branches) which connect N_b branches indexed 1 to N_b:

$$\sum_{j=1}^{N_b} T_j = 0, \tag{6}$$

representing the law of balance of torques.

The constraints imposed for each stage have the following formulation:

- for non-operated brake: $T = 0$;
- for non-operated clutch placed between branches denoted i and j respectively: $T_i = T_j = 0$;
- for operation between branches denoted i and j: $T_i + T_j = 0$.

The calculations were performed numerically and had as results the torque acting on every external elements of the EGTs; it becomes possible to calculate the power flows for each stage, using Eq. (5). The results are presented in the following figures.

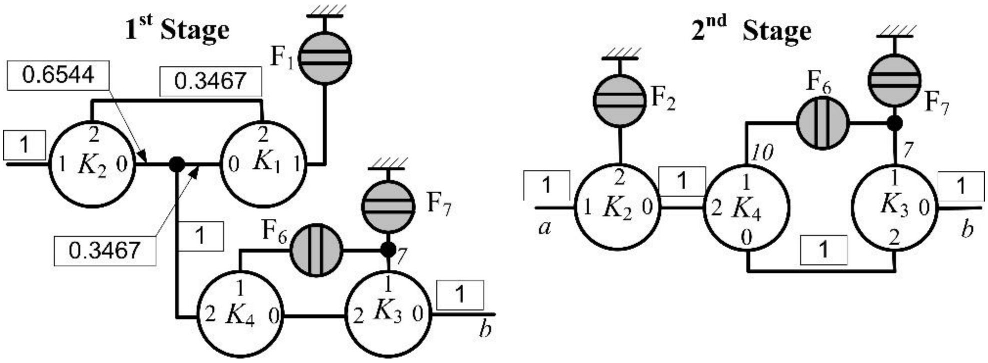


Fig. 5 – Equivalent nodal diagrams for the first two stages.

Figure 5 presents the equivalent nodal diagram for the first two stages (the framed numbers represents the power flows). The operated friction elements are filled in grey. The representations omit the non-operated friction elements as well as the EGTs which don't transmit power flow. For the first stage, the input power flow (assumed to be 1) is divided between the EGTs noted K_2 and K_1 and re-added in order to be applied to the EGTs noted K_4 and K_3 which work as ordinary gears having the sun gears blocked. The equivalent diagram for the second stage indicates that only the EGTs noted K_2, K_4 and K_3 successively transmit the same amount of power flow (serial transmission configuration).

In stage 3 (Fig. 6 left), only the EGTs noted K_4 and K_3 contribute to power flow transmission, keeping the same serial configuration as in the previous two stages.

The EGTs noted K_2 and K_1 are blocked due to operation of the clutch F_3. For the 4th stage, the power flow transmission is realized only by the EGT noted K_3.

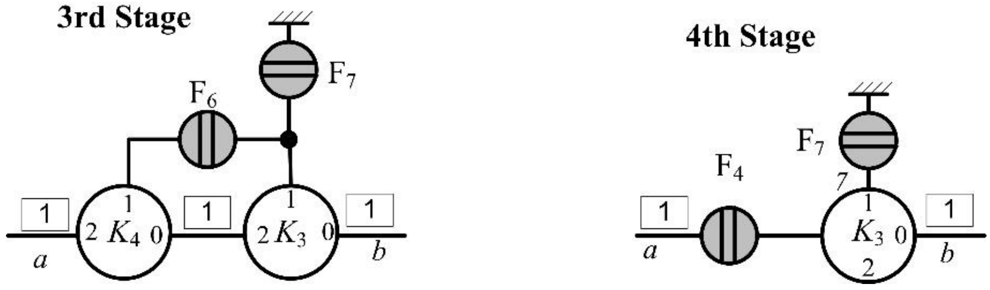


Fig. 6 – Equivalent nodal diagrams for stages 3 and 4.

The equivalent nodal diagrams for the last two forward stages are presented in Fig. 7; we can notice a ramification of power flows among EGTs, thus providing a better efficiency of the gear box.

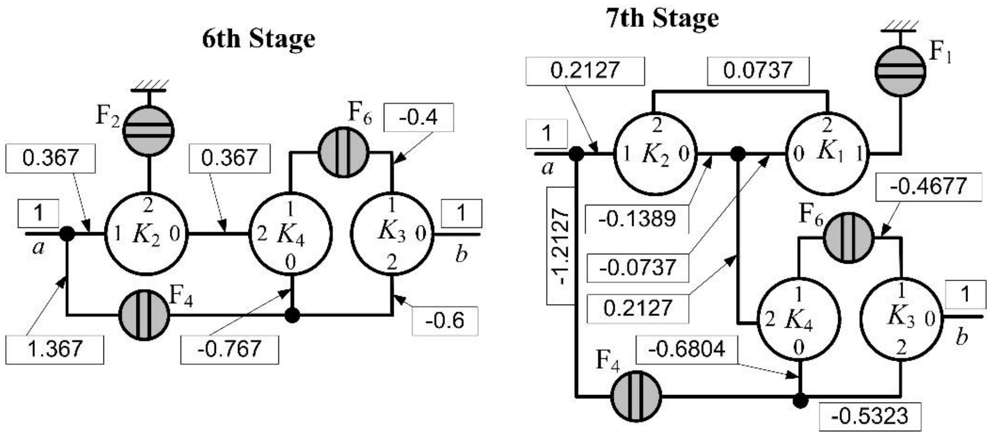


Fig. 7 – Equivalent nodal diagrams for stages 6 and 7.

In Fig. 7, the equivalent nodal diagrams corresponding to the reverse stages are shown. For both stages, the EGTs noted K_4 and K_3 work as reduction gears without splitting the power flow. In the case of the 1st Reverse Stage (Fig. 8 left), the power flow is divided between the EGTs noted K_1 and K_2, while for the 2nd Reverse Stage, all EGTs perform as reduction gears, and the power flow is missing.

Considering the formulae for calculation of gear box ratios presented in Table 3, it results that the ratio (for seven forward stages as well as for two reverse stages) can be expressed as functions depending of the variables K_1, K_2, K_3 and K_4:

$$R_j = R_j(K_1, K_2, K_3, K_4), \qquad j \in \{1, \ldots, 9\}. \tag{7}$$

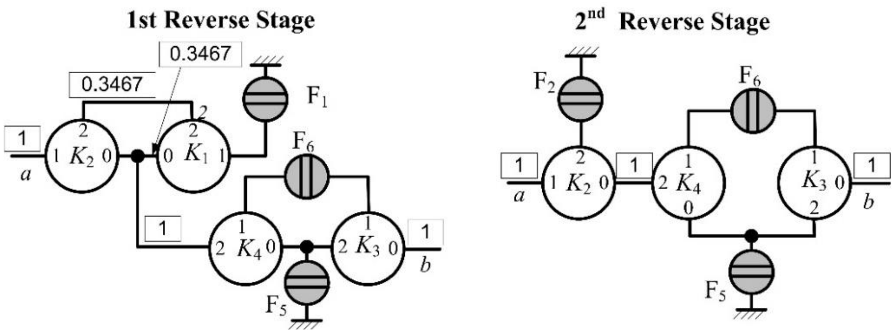


Fig. 8 – Equivalent nodal diagrams for reverse stages.

Using the methodology detailed in [5], the efficiency of the gear box is calculated by the following formulae:

$$\eta_j = \frac{R_j\left(\eta_0^{u_{1j}}\cdot K_1,\ \eta_0^{u_{2j}}\cdot K_2,\ \eta_0^{u_{3j}}\cdot K_3,\ \eta_0^{u_{4j}}\cdot K_4\right)}{R_j(K_1,K_2,K_3,K_4)}, \quad j \in \{1,\dots,9\}, \tag{8}$$

where: η_0 – the efficiency of the basic EGT; for the situation in which the input is on the sun gear, output on the ring gear and the carrier is blocked, the value 0.98 is assumed for η_0 [12]; u_{jk} – the exponent corresponding to stage *j* and the EGT noted K_k, calculated with the formulae:

$$u_{jk} = \operatorname{sign}\left(\frac{K_k}{R_j}\cdot\frac{\partial R_j}{\partial K_k}\right), \quad j \in \{1,\dots,9\},\ k \in \{1,\dots,4\}, \tag{9}$$

where R_j is defined by Eq. (7).

It is assumed that all basic EGTs have the same efficiency η_0; if a more accurate efficiency is required, the specific value of η_0 for each EGT can be calculated using the formulae detailed in [12, 13]. The results of the calculations are detailed in the Table 4.

Table 4

The efficiency of the gear box

Gear box stages								
1st	2nd	3rd	4th	5th	6th	7th	1st Rev.	2nd Rev.
0.9669	0.9824	0.9889	0.9946	1	0.9912	0.9856	0.9942	0.9593

4. CONCLUSIONS

The proposed method facilitates the analysis of gear boxes including compound epicyclic gears by its replacement with an equivalent structure which includes only basic 2DOF EGTs. The method has a general applicability, proved by this paper through two relevant examples, *i.e.* the Simpson mechanism and the Ravigneaux mechanism.

The proposed method for analysing compound EGT allows the usage of analytical methods for calculation of speed ratios, the speeds of all external elements as well as the torques acting on them, the power flows and overall gear box efficiencies. All calculations can be made symbolically and numerically by computer, using commercial software, resulting in significant time savings.

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