TEMPERATURE EFFECT ON NUCLEAR STATISTICAL QUANTITIES IN THE CASE OF PURE ISOSCALAR NEUTRON-PROTON PAIRING

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Abstract. Gap equations at finite temperature are deduced in the pure isoscalar pairing case (T=0). Expressions of the nuclear statistical quantities, *i.e.* the energy, the entropy and the heat capacity are deduced. A numerical study is then performed using the schematic one-level model.

It is found that the behavior of the isoscalar neutron-proton gap parameter $\Delta_{np}^{T=0}$ as a function of the temperature is similar to that of Δ_{pp} and Δ_{nn} in the conventional Finite Temperature BCS (FTBCS) approach. The presence of a critical temperature value beyond which $\Delta_{np}^{T=0}$ is null is noted.

Dealing with the nuclear statistical quantities, their behavior as a function of the temperature is similar to that obtained using the conventional FTBCS theory in the pairing between like-particles case. An increase in the energy value is noted compared with other types of pairing.

Key words: Pure isoscalar neutron-proton pairing, Temperature, Statistical quantities.

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1. INTRODUCTION

Nuclear pairing correlations are an essential ingredient for describing the properties of atomic nuclei. These correlations are usually studied with the classification of the isospin quantum number T: T=1 represents the isovector one and corresponds to proton-proton (p-p), neutron-neutron (n-n) and neutron-proton (n-p) pairing, and T=0 represents the isoscalar one and only corresponds to neutron-proton pairing. Most of the studies of pairing correlations have been focused on the isovector pairing interaction [1–6]. On the other hand, it is shown from both theoretical and experimental studies that, besides the isovector pairing, isoscalar pairing may also be of importance in $N \approx Z$ nuclei [7–10]. The attraction between protons and neutrons, which is the basic ingredient for the pairing correlations, is even stronger in the isoscalar pairing [11, 12], which gives rise to the deuteron bound state. The importance of T = 0 pairing was already pointed out in the 1960s. Indeed, Goswami and Kisslinger [13] worked out the Hartree-Fock-Bogoliubov (HFB) equations in the case of a pure isoscalar pair field. From a survey of experimental level spectrum data they pointed out that the

T=0 force is stronger than the T=l force in light nuclei, especially in nuclei of $A < 30$. Since that, there have been many discussions and works on possible signatures of the isoscalar pairing correlations. Among others, Langanke *et al.* [14] have studied pairing correlations in the ground states of $N = Z$ pf-shell nuclei using Shell Model Monte Carlo (SMMC) calculations and investigated the thermal dependence of selected observables for the odd-odd nucleus ^{54}Co and the even-even nuclei ^{60}Zn and ^{60}Ni. They found that when comparing the results to those obtained with the realistic Kuo-Brown interaction KB3 a transition with increasing temperature from a phase of isovector pairing dominance to one where isoscalar pairing correlations dominate. Bertsch and Luo [15] considered all the shell-model interaction matrix elements with total spin and isospin coupling (J,T)=(0,1),(1,0) and made a least-squares fit to each set using the contact interaction and harmonic oscillator orbitals. The interactions they used were the USDB Hamiltonian fitted to sd-shell nuclei and the GX1A Hamiltonian fitted to fp-shell nuclei. They performed HFB calculations using that contact interaction as the pairing interaction and the Woods-Saxon and the spin-orbit potentials. It was shown that in the case of ^{48}Cr with the full fp space, the correlation energy with (0,1) is larger than that with (1,0). They also showed that the isoscalar pairing can be dominant in much heavier nuclei of $A \approx 130 - 140$, although such exotic nuclei are currently not experimentally accessible. Chazono *et al.* [16] suppose that an isoscalar n-p pair is expected to emerge in nuclei that have similar proton and neutron numbers and it may be candidate for a deuteron "cluster". Isacker *et al.* [17] have shown that when the isoscalar component is dominant, the ground state is not paired to $J^\pi = 0^+$ but, rather behaves as a state of aligned 1^+ quasideutrons.

For their part, Bai *et al.* [11] studied the Gamow Teller (GT) states in $N = Z$ nuclei with the mass number A from 48 to 64 using Hartree-Fock-Bogoliubov plus quasi-particle random phase approximation (HFB+QRPA) with Skyrme interactions. The isoscalar pairing interaction is taken into account in QRPA calculations. It is found in the context of SU(4) symmetry in the spin-isospin space that the GT strength of lower energy excitations is largely enhanced by the T=0 pairing interaction which works cooperatively with the T=1 pairing interaction in the ground state. A two-peaked structure observed recently in (p,n) reaction on ^{58}Ni can be considered as a manifestation of the role of T=0 pairing in the GT collective states.

On the other side, the properties of nuclei at finite temperature have attracted recent experimental [18] and theoretical [19–21] interest. In the case of pairing between like-particles, it may be found in the literature a wealth of theoretical studies which have been performed in order to illustrate the thermodynamic quantities dependence on both temperature and pairing gap using different approaches and models such as, the Finite Temperature Bardeen Cooper Shrieffer (FTBCS) method [22–25], Finite Temperature Hartree Fock Bogoliubov (FT-HFB) [26–28] method, Shell Model Monte Carlo (SMMC) approaches [29, 30], the temperature dependent random phase

approximation (RPA) [31–33] and the Feyman path integral technique [34–40] which is based on the Hubbard-Stratonovitch transformation [41, 42].

When only isovector pairing is considered, the effect of the temperature has also been analyzed in the framework of various approaches [43–46]. In the isoscalar plus isovector pairing case, only few studies have been performed to illustrate the temperature effect on pairing correlation [47–52].

In a previous paper [52] we have established explicit gap equations at finite temperature using a path integral approach. Expressions of the energy, the entropy and the heat capacity were deduced. The formalism has been numerically studied using the schematic one-level model.

The aim of this paper is to study the influence of the pure n-p isoscalar pairing on the nuclear statistical quantities using the method described in Ref. [52] in which the effect of isoscalar plus isovector n-p pairing on the nuclear statistical quantities has been studied. The formalism for the present work is deduced from that of Ref. [52] by neglecting the like-particles interaction, as well as the n-p isovector one. The Hamiltonian and the gap equations are presented in Sec. 2. The various statistical quantities are derived in Sec. 3. Numerical results are presented and discussed in Sec. 4. Main conclusions are summarized in last section.

2. HAMILTONIAN-GAP EQUATIONS

The Hamiltonian of a system constituted by N neutrons and Z protons, in the second quantization and isospin formalism and in the case of isovector and isoscalar neutron-proton pairing is given by: [52, 53]

$$\mathcal{H}= \sum_{\substack{\nu>0\\ t=n,p}} \varepsilon_{\nu t}\,(a^{+}_{\nu t}\,a_{\nu t}+a^{+}_{\tilde{\nu}t}a_{\tilde{\nu}t}) - \sum_{t=n,p} G_{tt} \sum_{\nu\mu>0} b^{+}_{\nu tt} b_{\mu tt}$$
$$-G^{T=1}_{np} \sum_{\nu\mu>0} \left(b^{+}_{\nu np}\right)^{T=1} \left(b_{\mu np}\right)^{T=1} - G^{T=0}_{np} \sum_{\nu\mu>0} \left(b^{+}_{\nu np}\right)^{T=0} \left(b_{\mu np}\right)^{T=0}, \qquad (1)$$

where t corresponds to the isospin component ($t = n, p$), $a^{\dagger}_{\nu t}$ and $a_{\nu t}$ represent respectively the creation and annihilation operators of the particle (neutron or proton) in the state $|\nu t\rangle$, of energy $\varepsilon_{\nu t}$, $|\tilde{\nu t}\rangle$ is the time reversed state of $|\nu t\rangle$, G_{tt} is the pairing-strength between like-particles, $G^{T=1}_{tt'}$ and $G^{T=0}_{tt'}$ characterize respectively the isovector and isoscalar pairing strengths. The pair operator $b^{+}_{\nu tt}$ creates a pair of particles in the time reversal states. It is defined by:

$$b^{+}_{\nu tt} = a^{+}_{\nu t} a^{+}_{\tilde{\nu}t}\ ,\ t = n, p \quad \text{and} \qquad \left(b^{+}_{\nu np}\right)^{T} = \left(a^{+}_{\nu p} a^{+}_{\tilde{\nu}n} + (-1)^{T+1} a^{+}_{\nu n} a^{+}_{\tilde{\nu}p}\right). \quad (2)$$

The Hamiltonian corresponding to the pure neutron-proton isoscalar pairing can be obtained from Eq. (1) by neglecting the interaction between like-particles as well

as the isovector one, *i.e.* $G_{tt} = G_{tt'}^{T=1} = 0$. It's given by [54]:

$$\mathcal{H}= \sum_{\substack{\nu>0 \\ t=n,p}} \varepsilon_{\nu t} \left(a_{\nu t}^{+}\, a_{\nu t} + a_{\tilde{\nu} t}^{+} a_{\tilde{\nu} t}\right) - G_{np}^{T=0} \sum_{\nu\mu>0} b_{\nu np}^{+} b_{\mu np}, \tag{3}$$

In order to conserve on average neutron and proton numbers, let us define the auxiliary Hamiltonian

$$H = \mathcal{H} - \sum_{t} \lambda_t N_t, \tag{4}$$

where λ_t (t=n,p) are the Fermi energies and N_t (t=n,p) are the particle-number operators

$$N_t = \sum_{\nu>0} (a_{\nu t}^{+}\, a_{\nu t} + a_{\tilde{\nu} t}^{+} a_{\tilde{\nu} t}) \qquad\qquad t = n, p. \tag{5}$$

The grand partition function of a thermal equilibrium system is defined by

$$Z = Tr\ \exp(-\beta H) \tag{6}$$

where $\beta = \frac{1}{kTemp}$ is the inverse of the temperature, noted $Temp$, of the system, and Tr means the trace.

H may be approximately diagonalized using the results of Ref. [52] where the Feyman path integral technique has been used. The latter uses the Hubbard-Stratonovich transformation [41] and the static path approximation [55]. H then reads

$$H = \sum_{\substack{\nu>0 \\ \tau=1,2}} E_{\nu\tau} \left(\alpha_{\nu\tau}^{+} \alpha_{\nu\tau} - \alpha_{\tilde{\nu}\tau} \alpha_{\tilde{\nu}\tau}^{+}\right) + \sum_{\substack{\nu>0 \\ t=n,p}} \widetilde{\varepsilon}_{\nu t}. \tag{7}$$

where we set $\widetilde{\varepsilon}_{\nu t} = \varepsilon_{\nu t} - \lambda_t$, $\alpha_{\nu\tau}^{+}$ and $\alpha_{\nu\tau}$ are the creation and annihilation operators of one quasi-particle given by the generalized Bogoliubov-Valatin transformation

$$\begin{cases} \alpha_{\nu\tau}^{+} = \sum\limits_{t} \left(u_{\nu\tau t} a_{\nu t}^{+} - v_{\nu\tau t} a_{\tilde{\nu} t}\right) \\ \alpha_{\tilde{\nu}\tau} = \sum\limits_{t} \left(\bar{v}_{\nu\tau t} a_{\nu t}^{+} + u_{\nu\tau t} a_{\tilde{\nu} t}\right) \end{cases}, \tag{8}$$

$\tau = 1, 2$ is the quasi-particle type. The quasiparticle energy is given by

$$E_{\nu\tau}^{2} = \frac{1}{2}\left[\tilde{\varepsilon}_{\nu n}^{2} + \tilde{\varepsilon}_{\nu p}^{2} + 2\left(\Delta_{np}^{T=0}\right)^{2} + (-)^{\tau+1} R_{\nu}\right] \qquad , \qquad \tau = 1, 2, \tag{9}$$

with

$$R_{\nu} = \sqrt{\left(\tilde{\varepsilon}_{\nu n}^{2} - \tilde{\varepsilon}_{\nu p}^{2}\right)^{2} + 4\left(\Delta_{np}^{T=0}\right)^{2} (\tilde{\varepsilon}_{\nu n} - \tilde{\varepsilon}_{\nu p})^{2}} \tag{10}$$

$\Delta_{np}^{T=0}$ being the pairing gap parameter.

2.1. GAP EQUATIONS

The gap equations are deduced from those of Ref. [52] by canceling the pairing gap parameters in the isovector case as well as in the like-particles case, *i.e.* $\Delta_{pp} = \Delta_{nn} = \Delta_{np}^{T=1} = 0$.

They are given by

$$\frac{2}{G_{np}^{T=0}} = \sum_{\nu>0,\tau} \frac{1}{E_{\nu\tau}} \tanh \frac{\beta E_{\nu\tau}}{2} \left\{ 1 + (-1)^{\tau+1} \frac{\left(\tilde{\varepsilon}_{\nu n} - \tilde{\varepsilon}_{\nu p}\right)^2}{R_\nu} \right\} \tag{11}$$

The expectation values of the particle-number operators become

$$\left\langle N_{n(p)} \right\rangle = \sum_{\nu>0} \left\{ 1 - \sum_{\tau} \frac{\tanh\left(\frac{\beta E_{\nu\tau}}{2}\right)}{2E_{\nu\tau}} \right. \tag{12}$$
$$\left. \times \left[\tilde{\varepsilon}_{\nu n(p)} + (-1)^{\tau+1} \frac{\widetilde{\varepsilon}_{\nu n(p)} \left(\tilde{\varepsilon}^2_{\nu p(n)} - \tilde{\varepsilon}^2_{\nu n(p)} \right) + 2 \left(\Delta_{np}^{T=0} \right)^2 \left(\widetilde{\varepsilon}_{\nu n(p)} - \widetilde{\varepsilon}_{\nu p(n)} \right)}{R_\nu} \right] \right\}.$$

3. STATISTICAL QUANTITIES

Dealing with the nuclear statistical quantities, they can be deduced from the grand potential Ω which is given by

$$\Omega = -\beta \sum_{\nu>0,t} \widetilde{\varepsilon}_{\nu t} - \beta \frac{\left(\Delta_{np}^{T=0}\right)^2}{G_{np}^{T=0}} + \sum_{\nu>0,\tau} \ln 4 \cosh^2 \left(\frac{\beta E_{\nu\tau}}{2} \right). \tag{13}$$

The total energy of the system is defined by

$$E = -\left(\frac{\partial \Omega}{\partial \beta} \right)_{\gamma_t = cte} \quad , \quad \gamma_t = \beta \lambda_t \quad , \quad t = n, p. \tag{14}$$

After all calculations made one obtains

$$\begin{aligned} E \;=\; & -\frac{\left(\Delta_{np}^{T=0}\right)^2}{G_{np}^{T=0}} + \sum_{\nu>0} \left(\varepsilon_{\nu n} + \varepsilon_{\nu p} \right) - \sum_{\nu>0,\tau} \frac{\tanh\left(\frac{\beta E_{\nu\tau}}{2}\right)}{2E_{\nu\tau}} \left\{ \left(\varepsilon_{\nu p} \widetilde{\varepsilon}_{\nu p} + \varepsilon_{\nu n} \widetilde{\varepsilon}_{\nu n} \right) \right. \\ & \left. + \frac{(-)^{\tau+1}}{R_\nu} \left\{ \left(\tilde{\varepsilon}^2_{\nu n} - \tilde{\varepsilon}^2_{\nu p} \right) \left(\varepsilon_{\nu n} \widetilde{\varepsilon}_{\nu n} - \varepsilon_{\nu p} \widetilde{\varepsilon}_{\nu p} \right) + 2 \left(\Delta_{np}^{T=0} \right)^2 \left(\widetilde{\varepsilon}_{\nu n} - \widetilde{\varepsilon}_{\nu p} \right) \left(\varepsilon_{\nu n} - \varepsilon_{\nu p} \right) \right\} \right\}. \end{aligned} \tag{15}$$

The entropy of the system is defined by

$$S = \Omega + \beta E - \beta \sum_t \lambda_t N_t. \tag{16}$$

Its final expression is, after some algebra

$$S = \sum_{\nu>0,\tau} \ln 4\cosh^2\left(\frac{\beta E_{\nu\tau}}{2}\right) - \beta \sum_{\nu>0,\tau} E_{\nu\tau} \tanh\left(\frac{\beta E_{\nu\tau}}{2}\right). \tag{17}$$

Dealing with the heat capacity of the system, it is defined by

$$C = -\beta \frac{\partial S}{\partial \beta}. \tag{18}$$

Using Eq. (17), one finds

$$C = \frac{\beta^2}{2} \sum_{\nu>0,\tau} \frac{E_{\nu\tau}}{\cosh^2\left(\frac{\beta E_{\nu\tau}}{2}\right)} \left(E_{\nu\tau} + \beta \frac{\partial E_{\nu\tau}}{\partial \beta}\right), \tag{19}$$

with

$$\frac{\partial E_{\nu\tau}}{\partial \beta} = \frac{\partial E_{\nu\tau}}{\partial \Delta_{np}^{T=0}} \frac{\partial \Delta_{np}^{T=0}}{\partial \beta}, \tag{20}$$

where

$$\frac{\partial E_{\nu\tau}}{\partial \Delta_{np}^{T=0}} = \frac{1}{4E_{\nu\tau}} \left[4\Delta_{np}^{T=0} + (-1)^{\tau+1} \frac{4\Delta_{np}^{T=0}(\widetilde{\varepsilon}_{\nu n} - \widetilde{\varepsilon}_{\nu p})^2}{R_\nu}\right]$$

and

$$\frac{\partial \Delta_{np}^{T=0}}{\partial \beta} = \frac{1}{2} \sum_{\nu>0,\tau} \frac{E_{\nu\tau} \frac{\partial E_{\nu\tau}}{\partial \Delta_{np}^{T=0}}}{\cosh^2\left(\frac{\beta E_{\nu\tau}}{2}\right)} \left[\frac{2}{G_{np}^{T=0}} - \frac{\beta}{2} \sum_{\nu>0,\tau} \frac{\left(\frac{\partial E_{\nu\tau}}{\partial \Delta_{np}^{T=0}}\right)^2}{\cosh^2\left(\frac{\beta E_{\nu\tau}}{2}\right)} - \sum_{\nu>0,\tau} \tanh\frac{\beta E_{\nu\tau}}{2} \frac{\partial^2 E_{\nu\tau}}{\partial \left(\Delta_{np}^{T=0}\right)^2}\right]^{-1}.$$

At zero temperature, the energy becomes

$$E = -\frac{\left(\Delta_{np}^{T=0}\right)^2}{G_{np}^{T=0}} + \sum_{\substack{\nu>0,\\ t=n,p}} \varepsilon_{\nu t} \left(1 - \frac{\widetilde{\varepsilon}_{\nu t}}{|\varepsilon_{\nu t}|}\right). \tag{21}$$

For the entropy and the heat capacity, one has S=0 and C=0.

4. NUMERICAL RESULTS - DISCUSSION

In the present work and in order to perform numerical comparison with Refs. [44] and [52] we use the simple schematic one-level model. In the latter, $\varepsilon_\nu = 0$ for any ν. We also use the same parameters, *i.e.* $Z = 4$, $N = 6$ and $\Omega = 11$, Ω being the total degeneracy of pairs.

We considered as in Ref. [52], four different values of the pairing gap parameter at zero temperature denoted $\Delta_{np}^{T=0}(0)$, *i.e.* $\Delta_{np}^{T=0}(0) = 0.3, 0.4, 0.5, 0.6$ MeV. The isoscalar pairing-strength values $G_{np}^{T=0}$ have been calculated such as to reproduce the latter values of $\Delta_{np}^{T=0}(0)$.

4.1. GAP PARAMETER

The variations of the gap parameter $\Delta_{np}^{T=0}$ have been studied as a function of the temperature. The results are presented in Fig. 1. It is clear from this figure that

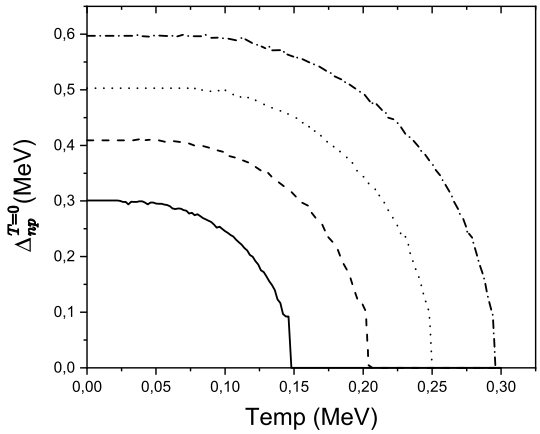

Fig. 1 – Variation of $\Delta_{np}^{T=0}$ as a function of the temperature for different values of $\Delta_{np}^{T=0}(0)$.

the pairing gap parameter $\Delta_{np}^{T=0}$ is first constant at low temperatures values and then decreases with increasing temperature and vanish at a given temperature called critical temperature. In the following, it will be denoted $(Temp_C)_{np}^{T=0}$. The variation of the critical temperature is represented as a function of $\Delta_{np}^{T=0}(0)$ in Fig. 2. From this figure, it's found that the behavior of $(Temp_C)_{np}^{T=0}$ as a function of $\Delta_{np}^{T=0}(0)$ is linear, as in the case in like-particles pairing. In this latter and in the FTBCS theory, the critical temperature is related to the pairing gap at zero temperature by the coefficient 0.56. In the present study this coefficient is equal to 0.46, *i.e.* $(Temp_C)_{np}^{T=0} = 0.46\Delta_{np}^{T=0}(0)$.

In order to compare the behavior of $\Delta_{np}^{T=0}$ with the other pairing gap parameters, the variations of the four gap parameters Δ_{pp}, Δ_{nn}, $\Delta_{np}^{T=1}$ and $\Delta_{np}^{T=0}$ as a function of the temperature are represented in Fig. 3. The results corresponding to the value $\Delta_{np}^{T=0}(0) = 0.4$ MeV, chosen as an example, are shown in this figure (dotted lines). In the left-hand part of the figure, they are compared to the ones obtained in the pure isovector pairing case (dashed lines) and in the isovector plus isoscalar pairing case (solid lines) taken from Ref. [52]. In the right-hand part of the Fig. 3, the same

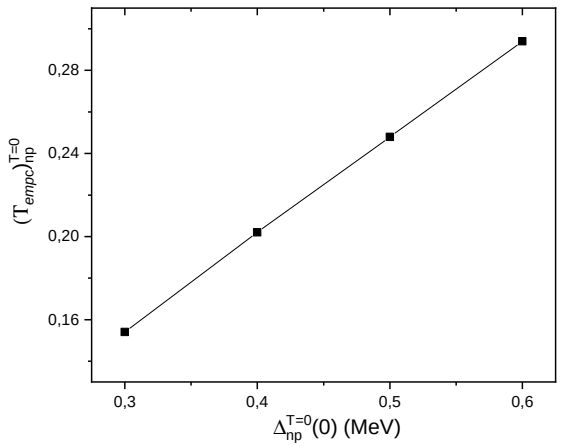

Fig. 2 – Variation of the critical temperature as a function of $\Delta_{np}^{T=0}(0)$.

results are compared to the conventional FTBCS theory, *i.e.* in the pairing between like-particles case (dash-dot lines).

From Fig. 3, it appears that the behavior of $\Delta_{np}^{T=0}$ as a function of the temperature is similar to that of Δ_{pp} and Δ_{nn} in the pairing between like-particles case and that of $\Delta_{np}^{T=0}$ in the isovector plus isoscalar pairing case as well as to that of $\Delta_{np}^{T=1}$ in the pure isovector pairing case.

There is a shift of the critical temperature value, when compared to the pairing gap parameter $\Delta_{np}^{T=1+T=0}$ in the isovector plus isoscalar pairing case.

4.2. STATISTICAL QUANTITIES

We have studied numerically the energy of the system, the entropy and the heat capacity defined by Eqs. (15), (17) and (19) respectively. These quantities have been calculated for the same values of isoscalar pairing gap parameter at zero temperature as in the previous section. The variations of E, S and C, which correspond to $\Delta_{np}^{T=0}(0) = 0.4$ MeV, chosen as an example, are shown as a function of the temperature in Fig. 4. The conventional FTBCS values [43] and those obtained when only the isovector pairing is considered [44] as well as those obtained when the isoscalar plus isovector pairing is taken into account [52] are also shown in the same figure.

4.2.1. Energy of the system

From Fig. 4, it may be seen that in the case of pure isoscalar n-p pairing, the behavior of the energy, as a function of the temperature is different from that obtained

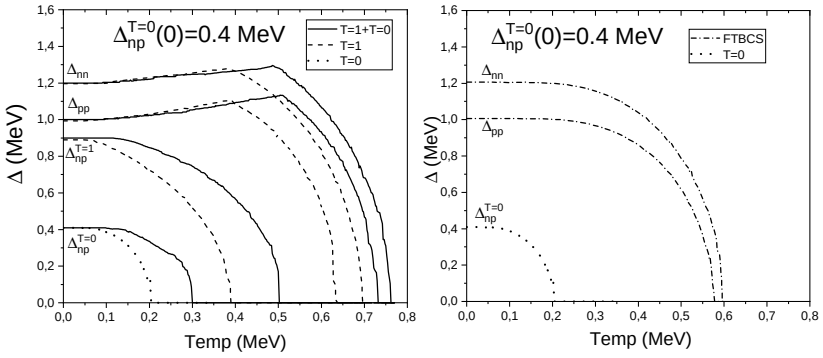

Fig. 3 – Variation of $\Delta_{np}^{T=0}$ as a function of the temperature in the pure isoscalar n-p pairing case compared to that of Δ_{pp}, Δ_{nn} and $\Delta_{np}^{T=1}$ in the pure isovector pairing case (dashed lines at the left hand part) and that of Δ_{pp}, Δ_{nn} , $\Delta_{np}^{T=1}$ and $\Delta_{np}^{T=0}$ in the case of isoscalar plus isovector n-p pairing case (solid lines at the left hand part) and compared to that of Δ_{pp} and Δ_{nn} in the pairing between like-particles case (dashed-dot lines at the right-hand part).

in the pure isovector pairing and in the isoscalar plus isovector pairing cases. There is no increase as it's observed for these two types of pairing. However, it's similar to that obtained in the conventional FTBCS approach. The energy is first constant when $\Delta_{np}^{T=0}$ is constant, then it increases smoothly until $(Temp_c)_{np}^{T=0}$ at low temperature and when $Temp > (Temp_c)_{np}^{T=0}$, the energy is null since there is no more pairing and $\varepsilon_\nu = 0$ for any ν. One notices an increasing of the energy when compared to the other pairing cases. The discrepancy between the curves decreases with the $\Delta_{np}^{T=0}(0)$ value. The relative discrepancy between T=1+T=0 n-p pairing and the isoscalar one at zero temperature is about 72.92% when $\Delta_{np}^{T=0}(0) = 0.3$ and 64.79% when $\Delta_{np}^{T=0}(0) = 0.4$. We also notice that at any temperature, the energy in the isovectorial plus isoscalar pairing case is not equal to the sum of the pure isoscalar energy and the pure isovectorial one.

We also considered the excitation energy E^* which is defined as the difference between the energy of the system at a given temperature and the one at zero temperature. The variation of E^* as a function of the temperature is represented in Fig. 5 for the different pairing types. From this figure, it appears that the behavior of the excitation energy is similar to that of FTBCS and different from those obtained in the pure isovector pairing and in the isoscalar plus isovector pairing. E^* is null when the pairing gap parameter is constant and then increases up to the critical temperature. Starting from this value, E^* is constant. It appears that the excitation energy in the pure isoscalar pairing is more important than in the other pairing cases when the

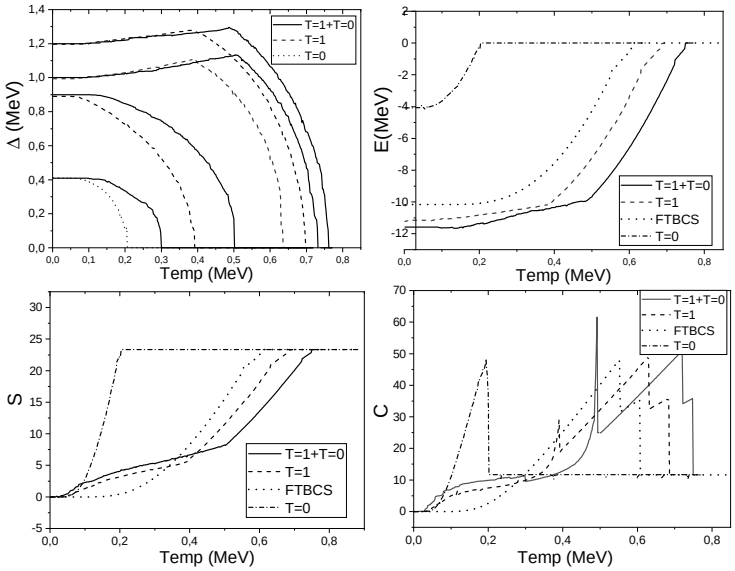

Fig. 4 – Variations of the various gap parameters, the energy, the entropy, and the heat capacity of the system as a function of the temperature, when $\Delta_{np}^{T=0}(0) = 0.4$ MeV. Solid lines refer to the isoscalar plus isovector pairing case. Dashed lines refer to the pure isovector pairing case, dotted lines to the pairing between like-particles and dashed-dot lines to the pure isoscalar pairing. The gap parameters in the pairing between like-particles case have not been represented for the clarity of the figure.

temperature value is lower than $(Temp_c)_{np}^{T=0}$.

4.2.2. Entropy

It appears from Fig. 4, that the behavior of the entropy as a function of the temperature is different from those obtained in pure isovector n-p pairing and isoscalar plus isovector n-p pairing cases. It's similar to that of the conventional FTBCS approach. Indeed, it's null when the pairing gap parameter is constant and increases rapidly until $(Temp_c)_{np}^{T=0}$ value. From this value of temperature there is no more pairing and the entropy becomes constant.

4.2.3. Heat capacity

It also appears from Fig. 4 that the pure isoscalar n-p pairing graph of C is similar to that of the conventional FTBCS when only one system of like-particles is considered, *i.e.* proton or neutron. The heat capacity C is null when the pairing gap parameter $\Delta_{np}^{T=0}$ is constant at low temperature. It increases with the values of temperature and decreases abruptly at $(Temp_c)_{np}^{T=0}$. As it's well known, this peak in the heat capacity curve, is the signature of the pairing phase transition in the system of particles from the superfluid phase to the normal one in heated nuclei. When

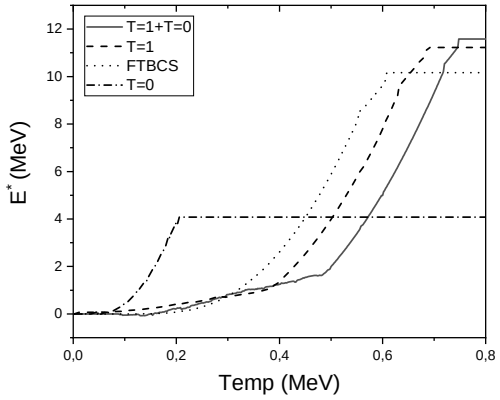

Fig. 5 – Variations of excitation energy as a function of the temperature. Solid lines refer to the isoscalar plus isovector pairing case. Dashed lines refer to the pure isovector pairing case, dotted lines to the pairing between like-particles and dashed-dot lines to the pure isoscalar pairing.

$Temp > (Temp_c)_{np}^{T=0}$ C becomes constant since there is no more pairing in this region.

5. CONCLUSION

Effect of temperature in the pure isoscalar n-p pairing case has been studied. Gap equations, expressions of the energy, the entropy and the heat capacity have been deduced from the results of Ref. [52].

A numerical study has been performed in the framework of the schematic one-level model. The variations of the pairing gap parameter $\Delta_{np}^{T=0}$ have been studied as a function of the temperature for several values of $\Delta_{np}^{T=0}(0)$. It was then found that the behavior of $\Delta_{np}^{T=0}$ is similar to that of Δ_{pp} and Δ_{nn} obtained within the conventional FTBCS theory in the pairing between like-particles case. The existence of a critical temperature, beyond which $\Delta_{np}^{T=0}$ is null has been noted. Moreover, the corresponding critical temperature value is shifted, relatively to the gap parameter $\Delta_{np}^{T=1+T=0}$ in the isovector plus isoscalar pairing case. Dealing with the nuclear statistical quantities, *i.e.* the energy, the entropy and the heat capacity, their behavior as a function of the temperature is similar to those obtained using the conventional FTBCS theory in the pairing between like-particles case. An increase in the value of the energy is noted compared with the other types of pairing. Let us note that the present work results deals only with the schematic case, the realistic one is the focus of work in progress.

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