SEARCHING FOR MULTIWAVE INTERACTION SOLUTIONS FOR A SPATIAL SYMMETRIC GENERALIZED KP MODEL IN (2+1)-DIMENSIONS

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Abstract. A spatial symmetric generalized Kadomtsev-Petviashvili (KP) model in (2+1)-dimensions is taken into consideration in the present study. Advanced computer algebra system software is employed for symbolic calculations, enabling the construction of diverse multiwave interaction solutions for this model. Moreover, by appropriately setting the associated constants, the profiles of some acquired solutions are plotted. This graphical representation effectively clarifies the structural characteristics of waves, providing valuable insights into the underlying physical phenomena.

Key words: KP model, Symbolic calculation, Multiwave interaction solutions.

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1. INTRODUCTION

In the exploration of a wide range of natural phenomena, multiwave interactions are commonly observed. For example, within the realm of fluid mechanics, when studying the propagation of shallow water waves, such interactions often emerge. Multiwave interactions involve different types of waves such as solitary waves, rogue waves, and breathers [1–3]. When these different types of waves interact and superimpose, they will present complex and diverse characteristics. When it comes to describing numerous physical phenomena, multiwave interaction solutions can more accurately mirror the actual circumstances. For instance, in plasma physics, the existence and interactions of multiwaves have a significant effect on energy transmission and matter distribution [4]. The ways of studying multiwave interaction solutions are diverse, including analytical methods, numerical simulations, and experimental observations [5–16]. Overall, the research on multiwave interaction solutions is of great worth for deeply comprehending complex physical systems and mathematical models.

It is a known fact that the KP equation is the natural generalization of the KdV equation from one dimension to two dimensions in space [17]. In the research of shallow water waves, when the weakly nonlinear restoring force and the wave length of frequency dispersion are taken into account, the KP equation serves as a highly useful model for exploring the behavior of water waves. For instance, during the propagation of water waves, when the surface tension is less than gravity, the

KP equation can be used to characterize the behavior of water waves under such a circumstance. With this equation, the propagation, evolution and diverse phenomena of water waves can be explored, which is of great significance for understanding the water wave dynamics in shallow water areas [18–20]. The KP equation can also serve to simulate the propagation of waves in magnetic media [21]. Studies on the KP equation enable an in-depth exploration of the propagation and evolution mechanisms of waves under the interaction of nonlinear effects and dispersion effects, helping to gain a more profound understanding of the complex phenomena in fields such as plasma physics and quantum field theory [22]. Previously, scholars have executed probes into KP equations from different perspectives [23–26].

In 2024, a nonlinear spatial symmetric generalized KP model

$$\begin{aligned}&\alpha(6u_xv_x+6u_{xx}v+u_{xxxx}+6u_yw_y+6u_{yy}w+u_{yyyy})\\&+\beta(4uu_{xy}+4v_yw_x+u_xu_y+u_{yy}v+u_{xx}w+v_xw_y+u_{xxyy})\\&+\gamma_1(u_{tx}+u_{ty})+\gamma_2(u_{xx}+u_{yy})+\gamma_3u_{xy}=0,\end{aligned}\tag{1}$$

where $v_y=u_x, w_x=u_y, p_x=v, q_y=w$, was introduced [27]. Lump waves of model (1) were worked out, and their dynamical characteristic properties were elaborately examined, including critical points and optimal values [27].

What we are interested in is to look for some multiwave interaction solutions for model (1) with the help of computer algebra systems and graphically manifest the localized structures of some acquired solutions.

2. MULTIWAVE INTERACTION SOLUTIONS FOR MODEL (1)

We can achieve the desired bilinear form

$$\begin{aligned}&(\alpha(D_x^4+D_y^4)+\beta D_x^2D_y^2+\gamma_1(D_tD_x+D_tD_y)\\&\qquad+\gamma_2(D_x^2+D_y^2)+\gamma_3D_xD_y)\Psi\cdot\Psi=0\end{aligned}\tag{2}$$

from model (1) *via* the logarithmic derivative transformations

$$u=2(\ln\Psi)_{xy},\ v=2(\ln\Psi)_{xx},\ w=2(\ln\Psi)_{yy},\ p=2(\ln\Psi)_x,\ q=2(\ln\Psi)_y,\tag{3}$$

where D is the Hirota D-operator [28], and Ψ is a test function that depends on x, y, and t.

From equation (2), it is straightforward to derive that

$$\begin{aligned}&2\alpha(\Psi_{xxxx}\Psi-4\Psi_{xxx}\Psi_x+3\Psi_{xx}^2+\Psi_{yyyy}\Psi-4\Psi_{yyy}\Psi_y+3\Psi_{yy}^2)\\&+2\beta(\Psi_{xxyy}\Psi-2\Psi_{xxy}\Psi_y-2\Psi_{xyy}\Psi_x+\Psi_{xx}\Psi_{yy}+2\Psi_{xy}^2)\\&+2\gamma_1(\Psi_{xt}\Psi-\Psi_t\Psi_x+\Psi_{yt}\Psi-\Psi_t\Psi_y)+2\gamma_2(\Psi_{xx}\Psi-\Psi_x^2+\Psi_{yy}\Psi-\Psi_y^2)\\&+2\gamma_3(\Psi_{xy}\Psi-\Psi_x\Psi_y)=0.\end{aligned}\tag{4}$$

Evidently, once a solution of equation (4) is found, a solution of model (1) can be constructed *via* transformations (3). In the subsequent subsections, we shall calculate solutions for the examined equation relying on three assumptions [29, 30].

2.1. ASSUMPTION I

Firstly, we adopt the test function in the form of

$$\Psi = \kappa_1 \cos\chi_1 + \kappa_2 \cosh\chi_2 + m_1 e^{\chi_3} + \theta, \tag{5}$$

where

$$\chi_1 = a_1 x + a_2 y + a_3 t + a_4,\ \chi_2 = b_1 x + b_2 y + b_3 t + b_4,\ \chi_3 = c_1 x + c_2 y + c_3 t + c_4,$$

and $\kappa_1, \kappa_2, m_1, a_j, b_j, c_j, 1 \leq j \leq 4$ are constants to be worked out. Incorporating assumption (5) into equation (4) and combining like terms results in a group of algebraic equations. Upon executing some calculations on this group of algebraic equations, the relationships serving to restrict the constants are ascertained:

Family 1.

$$a_2 = \epsilon\Upsilon_1 a_1, b_2 = \epsilon\Upsilon_1 b_1, c_2 = \epsilon\Upsilon_1 c_1, a_3 = -\frac{a_1(\epsilon\Upsilon_1\gamma_3 + \gamma_2 + \Upsilon_2)}{\gamma_1(1+\epsilon\Upsilon_1)},$$
$$b_3 = -\frac{b_1(\epsilon\Upsilon_1\gamma_3 + \gamma_2 + \Upsilon_2)}{\gamma_1(1+\epsilon\Upsilon_1)}, c_3 = -\frac{c_1(\epsilon\Upsilon_1\gamma_3 + \gamma_2 + \Upsilon_2)}{\gamma_1(1+\epsilon\Upsilon_1)},$$
$$\Upsilon_1 = \frac{\sqrt{2}\sqrt{\alpha(-\beta + \sqrt{\beta^2 - 4\alpha^2})}}{2\alpha}, \Upsilon_2 = \frac{\gamma_2(-\beta + \sqrt{\beta^2 - 4\alpha^2})}{2\alpha}, \epsilon = \pm 1.$$

Taking these relationships into (5), followed by utilizing transformations (3), gives multiwave interaction solutions

$$u = \frac{2\epsilon(-\Upsilon_1 a_1^2\kappa_1\cos\chi_1 + \Upsilon_1 b_1^2\kappa_2\cosh\chi_2 + \Upsilon_1 c_1^2 m_1 e^{\chi_3})}{\Psi} - \frac{2(-a_1\kappa_1\sin\chi_1 + b_1\kappa_2\sinh\chi_2 + c_1 m_1 e^{\chi_3})\Omega_1}{\Psi^2}, \tag{6}$$

$$v = \frac{2(-a_1^2\kappa_1\cos\chi_1 + b_1^2\kappa_2\cosh\chi_2 + c_1^2 m_1 e^{\chi_3})}{\Psi} - \frac{2(-a_1\kappa_1\sin\chi_1 + b_1\kappa_2\sinh\chi_2 + c_1 m_1 e^{\chi_3})^2}{\Psi^2}, \tag{7}$$

$$w = \frac{2(-\Upsilon_1^2 a_1^2\kappa_1\cos\chi_1 + \Upsilon_1^2 b_1^2\kappa_2\cosh\chi_2 + \Upsilon_1^2 c_1^2 m_1 e^{\chi_3})}{\Psi} - \frac{2(-\Upsilon_1 a_1\kappa_1\sin\chi_1 + \Upsilon_1 b_1\kappa_2\sinh\chi_2 + \Upsilon_1 c_1 m_1 e^{\chi_3})^2}{\Psi^2}, \tag{8}$$

$$p = \frac{2(-a_1\kappa_1\sin\chi_1 + b_1\kappa_2\sinh\chi_2 + c_1 m_1 e^{\chi_3})}{\Psi}, \tag{9}$$

$$q=\frac{2\epsilon(-\Upsilon_1 a_1\kappa_1\sin\chi_1+\Upsilon_1 b_1\kappa_2\sinh\chi_2+\Upsilon_1 c_1 m_1 e^{\chi_3})}{\Psi}, \tag{10}$$

where

$$\Omega_1=-\epsilon\Upsilon_1 a_1\kappa_1\sin\chi_1+\epsilon\Upsilon_1 b_1\kappa_2\sinh\chi_2+\epsilon\Upsilon_1 c_1 m_1 e^{\chi_3},$$

Ψ is defined by (5), and

$$\chi_1=a_1x+\epsilon\Upsilon_1 a_1 y-\frac{a_1(\epsilon\Upsilon_1\gamma_3+\gamma_2+\Upsilon_2)}{\gamma_1(1+\epsilon\Upsilon_1)}t+a_4,$$
$$\chi_2=b_1x+\epsilon\Upsilon_1 b_1 y-\frac{b_1(\epsilon\Upsilon_1\gamma_3+\gamma_2+\Upsilon_2)}{\gamma_1(1+\epsilon\Upsilon_1)}t+b_4,$$
$$\chi_3=c_1x+\epsilon\Upsilon_1 c_1 y-\frac{c_1(\epsilon\Upsilon_1\gamma_3+\gamma_2+\Upsilon_2)}{\gamma_1(1+\epsilon\Upsilon_1)}t+c_4.$$

Family 2.

$$a_2=\epsilon\Upsilon_3 a_1, b_2=\epsilon\Upsilon_3 b_1, c_2=\epsilon\Upsilon_3 c_1, a_3=-\frac{a_1(\epsilon\Upsilon_3\gamma_3+\gamma_2-\Upsilon_4)}{\gamma_1(1+\epsilon\Upsilon_3)},$$
$$b_3=-\frac{b_1(\epsilon\Upsilon_3\gamma_3+\gamma_2-\Upsilon_4)}{\gamma_1(1+\epsilon\Upsilon_3)}, c_3=-\frac{c_1(\epsilon\Upsilon_3\gamma_3+\gamma_2-\Upsilon_4)}{\gamma_1(1+\epsilon\Upsilon_3)},$$
$$\Upsilon_3=\frac{\sqrt{2}\sqrt{-\alpha(\beta+\sqrt{\beta^2-4\alpha^2})}}{2\alpha}, \Upsilon_4=\frac{\gamma_2(\beta+\sqrt{\beta^2-4\alpha^2})}{2\alpha}, \epsilon=\pm1.$$

Making use of these relationships along with transformations (3), we get multi-wave interaction solutions

$$u=\frac{2\epsilon(-\Upsilon_3 a_1^2\kappa_1\cos\chi_1+\Upsilon_3 b_1^2\kappa_2\cosh\chi_2+\Upsilon_3 c_1^2 m_1 e^{\chi_3})}{\Psi} - \frac{2(-a_1\kappa_1\sin\chi_1+b_1\kappa_2\sinh\chi_2+c_1 m_1 e^{\chi_3})\Omega_2}{\Psi^2}, \tag{11}$$

$$v=\frac{2(-a_1^2\kappa_1\cos\chi_1+b_1^2\kappa_2\cosh\chi_2+c_1^2 m_1 e^{\chi_3})}{\Psi} - \frac{2(-a_1\kappa_1\sin\chi_1+b_1\kappa_2\sinh\chi_2+c_1 m_1 e^{\chi_3})^2}{\Psi^2}, \tag{12}$$

$$w=\frac{2(-\Upsilon_3^2 a_1^2\kappa_1\cos\chi_1+\Upsilon_3^2 b_1^2\kappa_2\cosh\chi_2+\Upsilon_3^2 c_1^2 m_1 e^{\chi_3})}{\Psi} - \frac{2(-\Upsilon_3 a_1\kappa_1\sin\chi_1+\Upsilon_3 b_1\kappa_2\sinh\chi_2+\Upsilon_3 c_1 m_1 e^{\chi_3})^2}{\Psi^2}, \tag{13}$$

$$p=\frac{2(-a_1\kappa_1\sin\chi_1+b_1\kappa_2\sinh\chi_2+c_1 m_1 e^{\chi_3})}{\Psi}, \tag{14}$$

$$q=\frac{2\epsilon(-\Upsilon_3 a_1\kappa_1\sin\chi_1+\Upsilon_3 b_1\kappa_2\sinh\chi_2+\Upsilon_3 c_1 m_1 e^{\chi_3})}{\Psi}, \tag{15}$$

where

$$\Omega_2 = -\epsilon\Upsilon_3 a_1\kappa_1 \sin\chi_1 + \epsilon\Upsilon_3 b_1\kappa_2 \sinh\chi_2 + \epsilon\Upsilon_3 c_1 m_1 e^{\chi_3},$$

Ψ is defined by (5), and

$$\begin{aligned}
\chi_1 &= a_1 x + \epsilon\Upsilon_3 a_1 y - \frac{a_1(\epsilon\Upsilon_3\gamma_3 + \gamma_2 - \Upsilon_4)}{\gamma_1(1+\epsilon\Upsilon_3)} t + a_4,\\
\chi_2 &= b_1 x + \epsilon\Upsilon_3 b_1 y - \frac{b_1(\epsilon\Upsilon_3\gamma_3 + \gamma_2 - \Upsilon_4)}{\gamma_1(1+\epsilon\Upsilon_3)} t + b_4,\\
\chi_3 &= c_1 x + \epsilon\Upsilon_3 c_1 y - \frac{c_1(\epsilon\Upsilon_3\gamma_3 + \gamma_2 - \Upsilon_4)}{\gamma_1(1+\epsilon\Upsilon_3)} t + c_4.
\end{aligned}$$

2.2. ASSUMPTION II

Secondly, we consider the test function Ψ to be

$$\Psi = \kappa_1 \cos\zeta_1 + \kappa_2 \cosh\zeta_2 + m_1 e^{\zeta_3} + m_2 e^{\zeta_4} + \theta, \tag{16}$$

in which

$$\begin{aligned}
&\zeta_1 = a_1 x + a_2 y + a_3 t + a_4,\ \zeta_2 = b_1 x + b_2 y + b_3 t + b_4,\\
&\zeta_3 = c_1 x + c_2 y + c_3 t + c_4,\ \ \zeta_4 = l_1 x + l_2 y + l_3 t + l_4,
\end{aligned}$$

and the constants $\kappa_i, m_i, a_j, b_j, c_j, l_j, i = 1,2; 1 \le j \le 4$ are all to be solved for. Next, incorporating assumption (16) into equation (4) and combining like terms, a collection of sixteen algebraic equations regarding the unknowns can be produced. By computations, we build the following relationships among the constants:

Family 1.

$$\begin{aligned}
&a_2 = \epsilon Y_1 a_1, b_2 = \epsilon Y_1 b_1, c_2 = \epsilon Y_1 c_1, l_2 = \epsilon Y_1 l_1,\\
&a_3 = -\frac{a_1(\gamma_2 + Y_2 + \epsilon Y_1\gamma_3)}{\gamma_1(1+\epsilon Y_1)}, b_3 = -\frac{b_1(\gamma_2 + Y_2 + \epsilon Y_1\gamma_3)}{\gamma_1(1+\epsilon Y_1)},\\
&c_3 = -\frac{c_1(\gamma_2 + Y_2 + \epsilon Y_1\gamma_3)}{\gamma_1(1+\epsilon Y_1)}, l_3 = -\frac{l_1(\gamma_2 + Y_2 + \epsilon Y_1\gamma_3)}{\gamma_1(1+\epsilon Y_1)},\\
&Y_1 = \frac{\sqrt{2}\sqrt{\alpha(-\beta + \sqrt{\beta^2 - 4\alpha^2})}}{2\alpha}, Y_2 = \frac{\gamma_2(-\beta + \sqrt{\beta^2 - 4\alpha^2})}{2\alpha}, \epsilon = \pm 1.
\end{aligned}$$

Through the utilization of these relationships along with transformations (3), we obtain multiwave interaction solutions

$$\begin{aligned}
u = &\frac{2\epsilon(-Y_1 a_1^2\kappa_1 \cos\zeta_1 + Y_1 b_1^2\kappa_2 \cosh\zeta_2 + Y_1 c_1^2 m_1 e^{\zeta_3} + Y_1 l_1^2 m_2 e^{\zeta_4})}{\Psi}\\
&- \frac{2(-a_1\kappa_1 \sin\zeta_1 + b_1\kappa_2 \sinh\zeta_2 + c_1 m_1 e^{\zeta_3} + l_1 m_2 e^{\zeta_4})\Omega_3}{\Psi^2},
\end{aligned} \tag{17}$$

$$v = \frac{2(-a_1^2\kappa_1\cos\zeta_1 + b_1^2\kappa_2\cosh\zeta_2 + c_1^2m_1e^{\zeta_3} + l_1^2m_2e^{\zeta_4})}{\Psi} - \frac{2(-a_1\kappa_1\sin\zeta_1 + b_1\kappa_2\sinh\zeta_2 + c_1m_1e^{\zeta_3} + l_1m_2e^{\zeta_4})^2}{\Psi^2}, \tag{18}$$

$$w = \frac{2(-Y_1^2a_1^2\kappa_1\cos\zeta_1 + Y_1^2b_1^2\kappa_2\cosh\zeta_2 + Y_1^2c_1^2m_1e^{\zeta_3} + Y_1^2l_1^2m_2e^{\zeta_4})}{\Psi} - \frac{2(-Y_1a_1\kappa_1\sin\zeta_1 + Y_1b_1\kappa_2\sinh\zeta_2 + Y_1c_1m_1e^{\zeta_3} + Y_1l_1m_2e^{\zeta_4})^2}{\Psi^2}, \tag{19}$$

$$p = \frac{2(-a_1\kappa_1\sin\zeta_1 + b_1\kappa_2\sinh\zeta_2 + c_1m_1e^{\zeta_3} + l_1m_2e^{\zeta_4})}{\Psi}, \tag{20}$$

$$q = \frac{2\epsilon(-Y_1a_1\kappa_1\sin\zeta_1 + Y_1b_1\kappa_2\sinh\zeta_2 + Y_1c_1m_1e^{\zeta_3} + Y_1l_1m_2e^{\zeta_4})}{\Psi}, \tag{21}$$

where

$$\Omega_3 = -\epsilon Y_1a_1\kappa_1\sin\zeta_1 + \epsilon Y_1b_1\kappa_2\sinh\zeta_2 + \epsilon Y_1c_1m_1e^{\zeta_3} + \epsilon Y_1l_1m_2e^{\zeta_4},$$

Ψ is defined by (16), and

$$\begin{aligned}
\zeta_1 &= a_1x + \epsilon Y_1a_1y - \frac{a_1(\gamma_2 + Y_2 + \epsilon Y_1\gamma_3)}{\gamma_1(1+\epsilon Y_1)}t + a_4,\\
\zeta_2 &= b_1x + \epsilon Y_1b_1y - \frac{b_1(\gamma_2 + Y_2 + \epsilon Y_1\gamma_3)}{\gamma_1(1+\epsilon Y_1)}t + b_4,\\
\zeta_3 &= c_1x + \epsilon Y_1c_1y - \frac{c_1(\gamma_2 + Y_2 + \epsilon Y_1\gamma_3)}{\gamma_1(1+\epsilon Y_1)}t + c_4,\\
\zeta_4 &= l_1x + \epsilon Y_1l_1y - \frac{l_1(\gamma_2 + Y_2 + \epsilon Y_1\gamma_3)}{\gamma_1(1+\epsilon Y_1)}t + l_4.
\end{aligned}$$

Family 2.

$$\begin{aligned}
&a_2 = \epsilon Y_3a_1, b_2 = \epsilon Y_3b_1, c_2 = \epsilon Y_3c_1, l_2 = \epsilon Y_3l_1,\\
&a_3 = -\frac{a_1(\gamma_2 - Y_4 + \epsilon Y_3\gamma_3)}{\gamma_1(1+\epsilon Y_3)}, b_3 = -\frac{b_1(\gamma_2 - Y_4 + \epsilon Y_3\gamma_3)}{\gamma_1(1+\epsilon Y_3)},\\
&c_3 = -\frac{c_1(\gamma_2 - Y_4 + \epsilon Y_3\gamma_3)}{\gamma_1(1+\epsilon Y_3)}, l_3 = -\frac{l_1(\gamma_2 - Y_4 + \epsilon Y_3\gamma_3)}{\gamma_1(1+\epsilon Y_3)},\\
&Y_3 = \frac{\sqrt{2}\sqrt{-\alpha(\beta + \sqrt{\beta^2 - 4\alpha^2})}}{2\alpha}, Y_4 = \frac{\gamma_2(\beta + \sqrt{\beta^2 - 4\alpha^2})}{2\alpha}, \epsilon = \pm 1.
\end{aligned}$$

By means of these relationships and transformations (3), we can present multi-

wave interaction solutions as

$$u = \frac{2\epsilon(-Y_3a_1^2\kappa_1\cos\zeta_1 + Y_3b_1^2\kappa_2\cosh\zeta_2 + Y_3c_1^2m_1e^{\zeta_3} + Y_3l_1^2m_2e^{\zeta_4})}{\Psi} - \frac{2(-a_1\kappa_1\sin\zeta_1 + b_1\kappa_2\sinh\zeta_2 + c_1m_1e^{\zeta_3} + l_1m_2e^{\zeta_4})\Omega_4}{\Psi^2}, \tag{22}$$

$$v = \frac{2(-a_1^2\kappa_1\cos\zeta_1 + b_1^2\kappa_2\cosh\zeta_2 + c_1^2m_1e^{\zeta_3} + l_1^2m_2e^{\zeta_4})}{\Psi} - \frac{2(-a_1\kappa_1\sin\zeta_1 + b_1\kappa_2\sinh\zeta_2 + c_1m_1e^{\zeta_3} + l_1m_2e^{\zeta_4})^2}{\Psi^2}, \tag{23}$$

$$w = \frac{2(-Y_3^2a_1^2\kappa_1\cos\zeta_1 + Y_3^2b_1^2\kappa_2\cosh\zeta_2 + Y_3^2c_1^2m_1e^{\zeta_3} + Y_3^2l_1^2m_2e^{\zeta_4})}{\Psi} - \frac{2(-Y_3a_1\kappa_1\sin\zeta_1 + Y_3b_1\kappa_2\sinh\zeta_2 + Y_3c_1m_1e^{\zeta_3} + Y_3l_1m_2e^{\zeta_4})^2}{\Psi^2}, \tag{24}$$

$$p = \frac{2(-a_1\kappa_1\sin\zeta_1 + b_1\kappa_2\sinh\zeta_2 + c_1m_1e^{\zeta_3} + l_1m_2e^{\zeta_4})}{\Psi}, \tag{25}$$

$$q = \frac{2\epsilon(-Y_3a_1\kappa_1\sin\zeta_1 + Y_3b_1\kappa_2\sinh\zeta_2 + Y_3c_1m_1e^{\zeta_3} + Y_3l_1m_2e^{\zeta_4})}{\Psi}, \tag{26}$$

where

$$\Omega_4 = -\epsilon Y_3a_1\kappa_1\sin\zeta_1 + \epsilon Y_3b_1\kappa_2\sinh\zeta_2 + \epsilon Y_3c_1m_1e^{\zeta_3} + \epsilon Y_3l_1m_2e^{\zeta_4},$$

Ψ is defined by (16), and

$$\zeta_1 = a_1x + \epsilon Y_3a_1y - \frac{a_1(\gamma_2 - Y_4 + \epsilon Y_3\gamma_3)}{\gamma_1(1+\epsilon Y_1)}t + a_4,$$
$$\zeta_2 = b_1x + \epsilon Y_3b_1y - \frac{b_1(\gamma_2 - Y_4 + \epsilon Y_3\gamma_3)}{\gamma_1(1+\epsilon Y_3)}t + b_4,$$
$$\zeta_3 = c_1x + \epsilon Y_3c_1y - \frac{c_1(\gamma_2 - Y_4 + \epsilon Y_3\gamma_3)}{\gamma_1(1+\epsilon Y_3)}t + c_4,$$
$$\zeta_4 = l_1x + \epsilon Y_3l_1y - \frac{l_1(\gamma_2 - Y_4 + \epsilon Y_3\gamma_3)}{\gamma_1(1+\epsilon Y_3)}t + l_4.$$

2.3. ASSUMPTION III

Thirdly, we postulate the test function Ψ to be

$$\Psi = \kappa_1\cos\vartheta_1 + \kappa_2\cosh\vartheta_2 + \kappa_3\sin\vartheta_3 + \kappa_4\cosh\vartheta_4 + \theta, \tag{27}$$

with

$$\vartheta_1 = a_1x + a_2y + a_3t + a_4,\ \vartheta_2 = b_1x + b_2y + b_3t + b_4,$$
$$\vartheta_3 = c_1x + c_2y + c_3t + c_4,\quad \vartheta_4 = l_1x + l_2y + l_3t + l_4,$$

and the constants $\kappa_j, a_j, b_j, c_j, l_j, 1 \leq j \leq 4$ are to be computed. Conducting operations that are similar to those previously done gives a group of algebraic equations. After a sequence of further calculations on this group of algebraic equations, the constraints among the constants are found as follows:

Family 1.

$$a_2 = \epsilon\delta_1 a_1, b_2 = \epsilon\delta_1 b_1, c_2 = \epsilon\delta_1 c_1, l_2 = \epsilon\delta_1 l_1,$$
$$a_3 = -\frac{a_1(\delta_2 + \epsilon\delta_1\gamma_3 + \gamma_2)}{\gamma_1(\epsilon\delta_1 + 1)}, b_3 = -\frac{b_1(\delta_2 + \epsilon\delta_1\gamma_3 + \gamma_2)}{\gamma_1(\epsilon\delta_1 + 1)},$$
$$c_3 = -\frac{c_1(\delta_2 + \epsilon\delta_1\gamma_3 + \gamma_2)}{\gamma_1(\epsilon\delta_1 + 1)}, l_3 = -\frac{l_1(\delta_2 + \epsilon\delta_1\gamma_3 + \gamma_2)}{\gamma_1(\epsilon\delta_1 + 1)},$$
$$\delta_1 = \frac{\sqrt{2}\sqrt{\alpha(-\beta + \sqrt{\beta^2 - 4\alpha^2})}}{2\alpha}, \delta_2 = \frac{\gamma_2(-\beta + \sqrt{\beta^2 - 4\alpha^2})}{2\alpha}, \epsilon = \pm 1.$$

By incorporating the obtained outcomes into (27) and employing transformations (3), we can write multiwave interaction solutions as

$$u = \frac{2\epsilon(-\delta_1 a_1^2\kappa_1\cos\vartheta_1 + \delta_1 b_1^2\kappa_2\cosh\vartheta_2 - \delta_1 c_1^2\kappa_3\sin\vartheta_3 + \delta_1 l_1^2\kappa_4\cosh\vartheta_4)}{\Psi} - \frac{2(-a_1\kappa_1\sin\vartheta_1 + b_1\kappa_2\sinh\vartheta_2 + c_1\kappa_3\cos\vartheta_3 + l_1\kappa_4\sinh\vartheta_4)\Delta_1}{\Psi^2}, \tag{28}$$

$$v = \frac{2(-a_1^2\kappa_1\cos\vartheta_1 + b_1^2\kappa_2\cosh\vartheta_2 - c_1^2\kappa_3\sin\vartheta_3 + l_1^2\kappa_4\cosh\vartheta_4)}{\Psi} - \frac{2(-a_1\kappa_1\sin\vartheta_1 + b_1\kappa_2\sinh\vartheta_2 + c_1\kappa_3\cos\vartheta_3 + l_1\kappa_4\sinh\vartheta_4)^2}{\Psi^2}, \tag{29}$$

$$w = \frac{2(-\delta_1^2 a_1^2\kappa_1\cos\vartheta_1 + \delta_1^2 b_1^2\kappa_2\cosh\vartheta_2 - \delta_1^2 c_1^2\kappa_3\sin\vartheta_3 + \delta_1^2 l_1^2\kappa_4\cosh\vartheta_4)}{\Psi} - \frac{2(-\delta_1 a_1\kappa_1\sin\vartheta_1 + \delta_1 b_1\kappa_2\sinh\vartheta_2 + \delta_1 c_1\kappa_3\cos\vartheta_3 + \delta_1 l_1\kappa_4\sinh\vartheta_4)^2}{\Psi^2}, \tag{30}$$

$$p = \frac{2(-a_1\kappa_1\sin\vartheta_1 + b_1\kappa_2\sinh\vartheta_2 + c_1\kappa_3\cos\vartheta_3 + l_1\kappa_4\sinh\vartheta_4)}{\Psi}, \tag{31}$$

$$q = \frac{2\epsilon(-\delta_1 a_1\kappa_1\sin\vartheta_1 + \delta_1 b_1\kappa_2\sinh\vartheta_2 + \delta_1 c_1\kappa_3\cos\vartheta_3 + \delta_1 l_1\kappa_4\sinh\vartheta_4)}{\Psi}, \tag{32}$$

where

$$\Delta_1 = -\epsilon\delta_1 a_1\kappa_1\sin\vartheta_1 + \epsilon\delta_1 b_1\kappa_2\sinh\vartheta_2 + \epsilon\delta_1 c_1\kappa_3\cos\vartheta_3 + \epsilon\delta_1 l_1\kappa_4\sinh\vartheta_4,$$

Ψ is defined by (27), and

$$\vartheta_1 = a_1x + \epsilon\delta_1 a_1 y - \frac{a_1(\delta_2 + \epsilon\delta_1\gamma_3 + \gamma_2)}{\gamma_1(\epsilon\delta_1 + 1)}t + a_4,$$
$$\vartheta_2 = b_1x + \epsilon\delta_1 b_1 y - \frac{b_1(\delta_2 + \epsilon\delta_1\gamma_3 + \gamma_2)}{\gamma_1(\epsilon\delta_1 + 1)}t + b_4,$$
$$\vartheta_3 = c_1x + \epsilon\delta_1 c_1 y - \frac{c_1(\delta_2 + \epsilon\delta_1\gamma_3 + \gamma_2)}{\gamma_1(\epsilon\delta_1 + 1)}t + c_4,$$
$$\vartheta_4 = l_1x + \epsilon\delta_1 l_1 y - \frac{l_1(\delta_2 + \epsilon\delta_1\gamma_3 + \gamma_2)}{\gamma_1(\epsilon\delta_1 + 1)}t + l_4.$$

Family 2.

$$a_2 = \epsilon\delta_3 a_1, b_2 = \epsilon\delta_3 b_1, c_2 = \epsilon\delta_3 c_1, l_2 = \epsilon\delta_3 l_1,$$
$$a_3 = -\frac{a_1(\delta_4 + \epsilon\delta_3\gamma_3 + \gamma_2)}{\gamma_1(\epsilon\delta_3 + 1)}, b_3 = -\frac{b_1(\delta_4 + \epsilon\delta_3\gamma_3 + \gamma_2)}{\gamma_1(\epsilon\delta_3 + 1)},$$
$$c_3 = -\frac{c_1(\delta_4 + \epsilon\delta_3\gamma_3 + \gamma_2)}{\gamma_1(\epsilon\delta_3 + 1)}, l_3 = -\frac{l_1(\delta_4 + \epsilon\delta_3\gamma_3 + \gamma_2)}{\gamma_1(\epsilon\delta_3 + 1)},$$
$$\delta_3 = \frac{\sqrt{2}\sqrt{-\alpha(\beta + \sqrt{\beta^2 - 4\alpha^2})}}{2\alpha}, \delta_4 = -\frac{\gamma_2(\beta + \sqrt{\beta^2 - 4\alpha^2})}{2\alpha}, \epsilon = \pm 1.$$

Incorporating the obtained outcomes into (27) and taking advantage of transformations (3) yields

$$u = \frac{2\epsilon(-\delta_3 a_1^2\kappa_1\cos\vartheta_1 + \delta_3 b_1^2\kappa_2\cosh\vartheta_2 - \delta_3 c_1^2\kappa_3\sin\vartheta_3 + \delta_3 l_1^2\kappa_4\cosh\vartheta_4)}{\Psi} - \frac{2(-a_1\kappa_1\sin\vartheta_1 + b_1\kappa_2\sinh\vartheta_2 + c_1\kappa_3\cos\vartheta_3 + l_1\kappa_4\sinh\vartheta_4)\Delta_2}{\Psi^2}, \tag{33}$$

$$v = \frac{2(-a_1^2\kappa_1\cos\vartheta_1 + b_1^2\kappa_2\cosh\vartheta_2 - c_1^2\kappa_3\sin\vartheta_3 + l_1^2\kappa_4\cosh\vartheta_4)}{\Psi} - \frac{2(-a_1\kappa_1\sin\vartheta_1 + b_1\kappa_2\sinh\vartheta_2 + c_1\kappa_3\cos\vartheta_3 + l_1\kappa_4\sinh\vartheta_4)^2}{\Psi^2}, \tag{34}$$

$$w = \frac{2(-\delta_3^2 a_1^2\kappa_1\cos\vartheta_1 + \delta_3^2 b_1^2\kappa_2\cosh\vartheta_2 - \delta_3^2 c_1^2\kappa_3\sin\vartheta_3 + \delta_3^2 l_1^2\kappa_4\cosh\vartheta_4)}{\Psi} - \frac{2(-\delta_3 a_1\kappa_1\sin\vartheta_1 + \delta_3 b_1\kappa_2\sinh\vartheta_2 + \delta_3 c_1\kappa_3\cos\vartheta_3 + \delta_3 l_1\kappa_4\sinh\vartheta_4)^2}{\Psi^2}, \tag{35}$$

$$p = \frac{2(-a_1\kappa_1\sin\vartheta_1 + b_1\kappa_2\sinh\vartheta_2 + c_1\kappa_3\cos\vartheta_3 + l_1\kappa_4\sinh\vartheta_4)}{\Psi}, \tag{36}$$

$$q = \frac{2\epsilon(-\delta_3 a_1\kappa_1\sin\vartheta_1 + \delta_3 b_1\kappa_2\sinh\vartheta_2 + \delta_3 c_1\kappa_3\cos\vartheta_3 + \delta_3 l_1\kappa_4\sinh\vartheta_4)}{\Psi}, \tag{37}$$

in which

$$\Delta_2 = -\epsilon\delta_3 a_1\kappa_1 \sin\vartheta_1 + \epsilon\delta_3 b_1\kappa_2 \sinh\vartheta_2 + \epsilon\delta_3 c_1\kappa_3 \cos\vartheta_3 + \epsilon\delta_3 l_1\kappa_4 \sinh\vartheta_4,$$

Ψ is defined by (27), and

$$\begin{aligned}
\vartheta_1 &= a_1 x + \epsilon\delta_3 a_1 y - \frac{a_1(\delta_4 + \epsilon\delta_3\gamma_3 + \gamma_2)}{\gamma_1(\epsilon\delta_3 + 1)} t + a_4,\\
\vartheta_2 &= b_1 x + \epsilon\delta_3 b_1 y - \frac{b_1(\delta_4 + \epsilon\delta_3\gamma_3 + \gamma_2)}{\gamma_1(\epsilon\delta_3 + 1)} t + b_4,\\
\vartheta_3 &= c_1 x + \epsilon\delta_3 c_1 y - \frac{c_1(\delta_4 + \epsilon\delta_3\gamma_3 + \gamma_2)}{\gamma_1(\epsilon\delta_3 + 1)} t + c_4,\\
\vartheta_4 &= l_1 x + \epsilon\delta_3 l_1 y - \frac{l_1(\delta_4 + \epsilon\delta_3\gamma_3 + \gamma_2)}{\gamma_1(\epsilon\delta_3 + 1)} t + l_4.
\end{aligned}$$

3. CONCLUDING REMARKS

For the visual demonstrations of the localized features of some acquired solutions, we utilize the drawing tools within Maple to create a few 3D plots. First of all, we assign the constants in solutions (6)–(10) as

$$\begin{aligned}
&\alpha = 1, \beta = -2.5, \kappa_1 = -2, \kappa_2 = 2, m_1 = 16, a_1 = 16, b_1 = 2,\\
&c_1 = -1, a_4 = 1, b_4 = 1, c_4 = 1, \gamma_1 = 1, \gamma_2 = 1, \gamma_3 = 1, \theta = 40, \epsilon = 1.
\end{aligned} \tag{38}$$

Relying on these values, we outline the 3D structures, as illustrated in Figure 1. Then, we take the constants in solutions (17)–(21) as

$$\begin{aligned}
&\alpha = 1, \beta = -2.5, \kappa_1 = 2, \kappa_2 = 2, m_1 = 16, m_2 = 16,\\
&a_1 = 16, b_1 = 2, c_1 = -1, l_1 = 1, a_4 = 1, b_4 = 1, c_4 = 1,\\
&l_4 = 1, \gamma_1 = 1, \gamma_2 = 1, \gamma_3 = 1, \theta = 40, \epsilon = 1.
\end{aligned} \tag{39}$$

Figure 2 signifies the 3D localized visualizations of (17)–(21) with (39). The periodicities exhibited by the waves in both figures are owing to the presence of trigonometric functions within the solutions. As seen in subfigures (d) and (e) of Figure 1, a breather-kink wave interacts with a kink wave. While in subfigures (d) and (e) of Figure 2, a breather-kink wave can be observed interacting with two kink waves.

In this study, after conducting computations with the computer algebra systems, diverse multiwave interaction solutions formulated as combinations of trigonometric, hyperbolic, and exponential functions to a spatial symmetric generalized KP model in (2+1)-dimensions were found. The acquired solutions provide valuable insights into the nature of the investigated model. Moreover, it is clear that multiwave interaction solutions of nonlinear models help in interpreting wave-wave interactions in fluid dynamics, and predicting wave-related phenomena, guiding practical applications in oceanography and engineering.

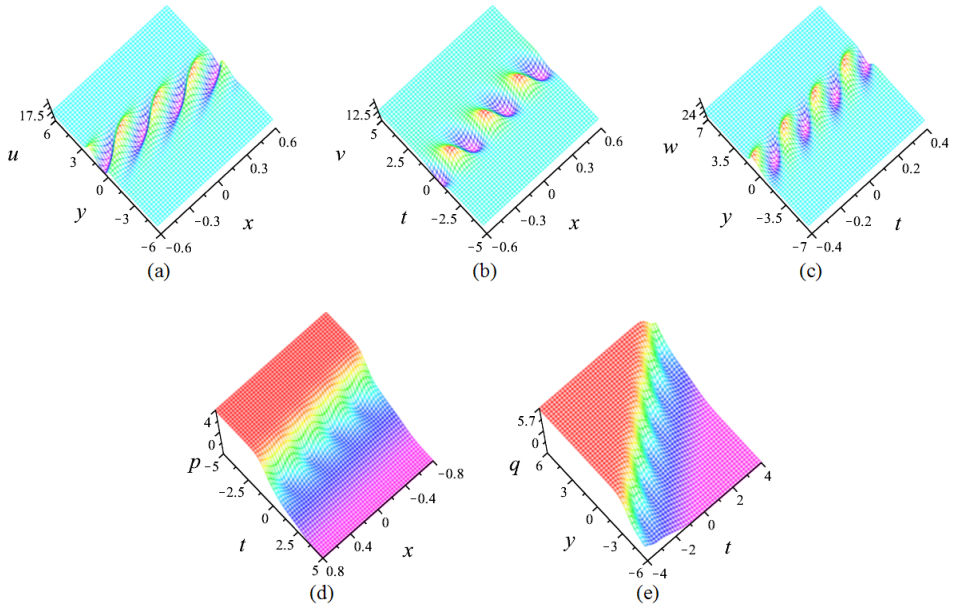

Fig. 1 – Profiles of (6)–(10) with (38); (a) u when $t = 0$; (b) v when $y = 0$; (c) w when $x = 0$; (d) p when $y = 0$; (e) q when $x = 0$.

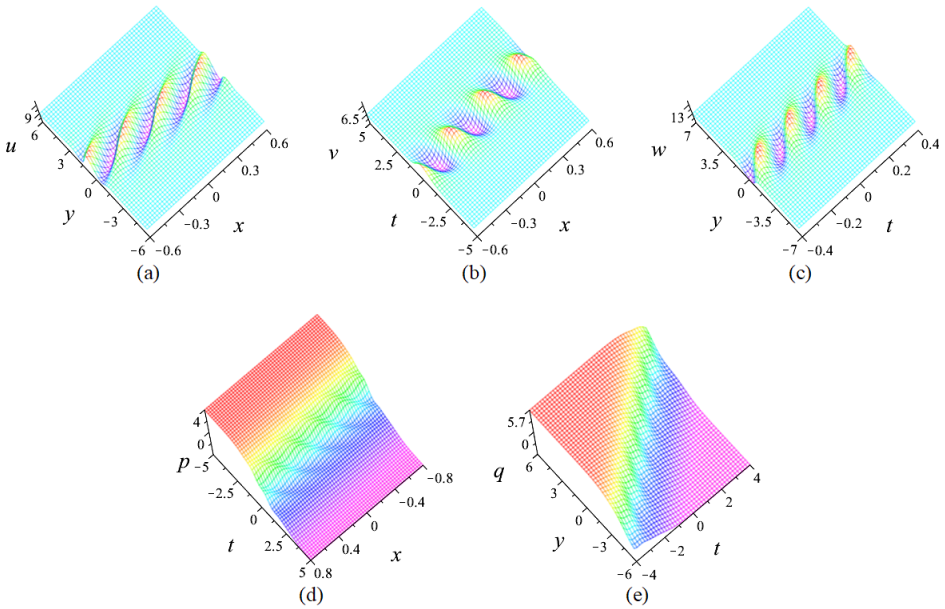

Fig. 2 – Profiles of (17)–(21) with (39); (a) u when $t = 0$; (b) v when $y = 0$; (c) w when $x = 0$; (d) p when $y = 0$; (e) q when $x = 0$.

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