EPR PARADOX AND QUANTUM FIELD THEORIES

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Abstract. We discuss the EPR paradox and study an example of this in a quantum field theoretical context. We show that there is always an alternative description of QFT in which particles that interact at a moment in time remain entangled and therefore in a form of contact at any spatial or temporal separation. Consequently the correlations of spin or other properties of the particles long time after they cease to interact is perfectly natural and justifiable.

Key words: EPR paradox, quantum field theory.

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In 1935 Einstein Podolsky and Rosen published the famous EPR paper [1] that deals with the inconsistencies between the quantum mechanics principles and the causality and locality principles that the physical world must respect. The authors argued that in order for quantum mechanics to be a complete theory it needs additional assumptions or completions. A possible attempt to reconcile this was proposed later in the form of hidden variables, the consequences of which were discussed in the famous Bell paper [2]. Bell showed that in local realistic hidden variables theories the quantum mechanics predicted correlations must satisfy some inequalities called "Bell inequalities". Later on was proved without any doubt that these inequalities were not respected by theoretical or experimental data.

Here we will briefly review the EPR argument and the substrate of Bell inequalities for illustration. Let us consider the decay of a scalar particle into two fermions. The first fermion will be assimilated to system I in the EPR paper whereas the second to system II. Initially there is a common wave function characterizing the system I+II. After the fermions get separated at moment t and any interaction between the two parts cease to exist one can measure an eigenvalue for some observable for system I. According to the EPR paper the wave function of the composed system can be represented as such:

$$\Psi(x_1, x_2) = \sum_n \Psi_n(x_2) u_n(x_1), \tag{1}$$

where $u_n(x_1)$ are eigenvectors of system I corresponding to some eigenvalues a_n of the observable A. This observable might commonly be position, momentum or spin. If the eigenvalue a_k is obtained for system I then the wave function of the system II

becomes $\Psi_k(x_2)$. Now assume that another observable B is measured for system I. The wave function of the composed system in this basis is:

$$\Psi(x_1,x_2)=\sum_n \Psi'_n(x_2)u'_n(x_1), \tag{2}$$

where u'_n are eigenvectors corresponding to eigenvalues b_n of the observable B. Then if a measurement b_r of the eigenvalue for B of system I is performed then the wave function of system II collapses to $\Psi'_r(x_2)$. The EPR paper concludes that two wave function characterize the system II. If the two variables A and B are position and momentum this would mean that one can determine them both thus contradicting the Heisenberg principle. This statement however was refuted easily and it is not the main contradiction outlined in the paper. The main argument is given by the fact that in principle it should not be possible to determine instantaneously the state of system II separated in space by system I by measuring an eigenvalue of system I. The authors conclude that the quantum theory must be incomplete. Future experimental data showed without any doubt that the correlation of system I or II interacting in the past exist even of the two systems are at the moment of measurement separated in space.

In 1964 Bell considered the main ideas in the EPR paper in the context of the presence of hidden variables that satisfy the principle of locality. He showed that the presence of the hidden variables in a theory with two particles with correlated spins would lead to inequalities that would violate the basic principles of quantum mechanics. He concludes that the theory of hidden variables might work only if these variables are nonlocal. In the subsequent years quantum entanglement stemmed from this early controversy evolved as a separate field of knowledge with both theoretical and experimental applications connecting several subfields of quantum physics, condensed matter physics, atomic physics or quantum field theories.

In this work we shall discussed the basic ideas in the EPR and Bell papers from the point of view of quantum field theories. Interesting discussions in this direction were also given in [3] and [4]. The system we will study is that of a scalar particles that decays into two fermion and the observable of interest will be the spin. This case was chosen for its clarity and simplicity but the arguments we shall give here are generally valid.

The Lagrangian of interest is:

$$\mathcal{L}=\bar{\Psi}(i\gamma^\mu\partial_\mu-m)\Psi+\bar{\chi}(i\gamma^\mu\partial_\mu-m)\chi+ \\ \frac{1}{2}\partial^\mu h\partial_\mu h+y\bar{\Psi}h\chi+y\bar{\chi}h\Psi, \tag{3}$$

where Ψ and χ are the two fermions of interest and h is a scalar which couples to the two fermions.

Assume we introduce a source $J(x)$ for the scalar $h(x)$. Then the kinetic term for the scalar field will become:

$$\int d^4x[\frac{1}{2}\partial^\mu h\partial_\mu h + J(x)h(x)] =$$
$$-\frac{1}{2}\int d^4x \int d^4y h'(x)\delta(x-y)\partial^2 h'(y) +$$
$$\frac{1}{2}\int d^4x \int d^4y \times$$
$$J(x)\int \frac{d^4k}{(2\pi)^4}\exp[ik(x-y)]\frac{1}{k^2}J(y), \tag{4}$$

where

$$h'(x) = h(x) - \frac{1}{\partial^2}J(x). \tag{5}$$

This is the standard procedure for extracting correlation functions in the presence of a source. The whole approach can be also regarded in the reversed order. One may start with the last of equations in Eq. (4) and retrieve the first equation in (4).

We first rewrite the last term in Eq. (4):

$$\frac{1}{2}\int d^4x \int d^4y J(x)\int \frac{d^4k}{(2\pi)^4}\exp[ik(x-y)]\frac{1}{k^2}J(y) =$$
$$\frac{1}{2}\int \frac{d^4k}{(2\pi)^4}J(k)\frac{1}{k^2}J(-k). \tag{6}$$

We consider $J(k)$ of the form:

$$J(k) = a\sin[k_0 z_0]\frac{1}{|\vec{k}|^{3/2}}, \tag{7}$$

and introduce it in Eq. (6) to get:

$$\frac{1}{2}\int \frac{d^4k}{(2\pi)^4}J(k)\frac{1}{k^2}J(-k) =$$
$$a^2\frac{1}{2}\frac{4\pi}{16\pi^4}i\int dk_0\int dk_r \sin^2(k_0 z_0)\frac{1}{k_r}\frac{1}{-k_0^2-k_r^2} =$$
$$\frac{a^2}{2}\frac{4\pi}{16\pi)^4}\pi i(-i)\int dk_r \sin^2(k_r x_0)\frac{1}{k_r^2} =$$
$$\frac{a^2}{8\pi}|x_0|. \tag{8}$$

Here the integral is done in the Euclidean space and the last line corresponds to $z_0 = ix_0$. Moreover a is a constant with mass dimension $\frac{1}{2}$.

Now we can proceed to our EPR interpretation and method. We may consider the low energy limit where the spin operator does not interfere with the angular momentum to write the projection of the spin operator on the direction z for the fermion Ψ in the quantum field theory as [4]:

$$S_z = \int d^3x \langle \Psi(x)^\dagger \frac{1}{2}\Sigma_z \Psi\rangle = \int d^3x \int \frac{d^4p}{(2\pi)^4}\int \frac{d^4q}{(2\pi)^4}\times \langle \Psi^\dagger(p)\frac{1}{2}\Sigma_z\Psi(q)\rangle \exp[-i(p-q)x] = \int \frac{d^3p}{(2\pi)^3}\int \frac{dp_0}{2\pi}\int \frac{dq_0}{2\pi}\exp[-i(p_0-q_0)x_0]\times \langle \Psi^\dagger(p)\frac{1}{2}\Sigma_z\Psi(p)\rangle, \tag{9}$$

where in the last line $\vec{p}=\vec{q}$.

The vacuum expectation value of the operator means that it is placed in the path integral approach that contains the exponential of the action $\exp[iS]$. We introduce the exponential $\exp[-i(p_0-q_0)x_0]$ in the action to write the full exponential term in the path integral approach as:

$$\exp[iS - i(p_0-q_0)x_0], \tag{10}$$

and denote using Eq. (8):

$$q_0 - p_0 = \frac{a^2}{8\pi}. \tag{11}$$

Retracing the steps which lead to Eq. (8) we obtain that the scalar field should be replaced by:

$$h'(x) = h(x) + \frac{1}{\partial^2}J(x), \tag{12}$$

to get an action with a kinetic term for $h'(x)$ and with a source term $J(x)h'(x)$. But then one needs to make also the substitution in the interaction term in the action. This leads to:

$$\mathcal{L}_I = y\bar{\Psi}h\chi + y\bar{\chi}h\Psi = y\bar{\Psi}h'\chi + y\bar{\chi}h'\Psi + y\bar{\Psi}\frac{1}{\partial^2}J(x)\chi + y\bar{\chi}\frac{1}{\partial^2}J(x)\Psi. \tag{13}$$

Practically in this approach we obtained a source term for the scalar and a new interaction term for the the two species of fermions. First we notice that $J(x)$ is not a quantum field in the partition function. Then we consider the infinite series expansion

of $J(x)$ around $x = x_0$ (where x_0 is considered fixed) to get:

$$\frac{1}{\partial^2}J(x) = \frac{1}{\partial^2}J(x_0) + \frac{1}{\partial^2}\partial_\rho J(x)|_{x=x_0}(x^\rho - x_0) + \frac{1}{\partial^2}\partial_\rho\partial_\sigma J(x)|_{x=x_0}(x^\rho - x_0)(x^\sigma - x_0) + ... \quad (14)$$

The whole point of the expansion above is that there is a term which does not depend on x at all and can be assimilated to a constant s be that infinite. Then there is a new mass term in the action in the form:

$$y\bar{\Psi}s\chi + y\bar{\chi}s\Psi, \quad (15)$$

where s is a constant with mass dimension 1.

In the mass eigenstate basis which will be the correct basis in the path integral approach one has:

$$\Psi' = \cos\theta\Psi + \sin\theta\chi$$
$$\chi' = -\sin\theta\Psi + \cos\theta\chi, \quad (16)$$

with the mass eigenvalue $m + s$ for Ψ' and the mass eigenvalue $m - s$ for χ'. One may also write the initial basis in terms of the mass eigenstates:

$$\Psi = \cos\theta\Psi' - \sin\theta\chi'$$
$$\chi = \sin\theta\Psi' + \cos\theta\chi'. \quad (17)$$

The full spin operator then for the quantum field theory with two fermion discussed here will thus become in the mass eigenstate basis:

$$S_t = \int \frac{d^3p}{(2\pi)^3}\int\frac{dp_0}{2\pi}\int\frac{dq_0}{2\pi}\langle\Psi'^\dagger(p)\frac{1}{2}\Sigma_z\Psi'(q)\rangle + \int \frac{d^3p}{(2\pi)^3}\int\frac{dp_0}{2\pi}\int\frac{dq_0}{2\pi}\langle\chi'^\dagger(p)\frac{1}{2}\Sigma_z\chi'(q)\rangle. \quad (18)$$

We are interested in determining the spin separately for the fermions Ψ and χ as external states. By the LSZ theorem apart for some factors these states can be introduced inside in the path integral approach. This would lead for fermion Ψ to:

$$S_z(\Psi) \propto \left[\Psi'^\dagger(p)\frac{1}{2}\Sigma_z\Psi'(q)\bar{\Psi}(k) + \chi'^\dagger(p)\frac{1}{2}\Sigma_z\chi'(q)\bar{\Psi}(k)\right]|0\rangle, \quad (19)$$

for the external state $\bar{\Psi}(k)$ and to:

$$S_z(\Psi) \propto$$
$$\Big[\Psi'^{\dagger}(p)\frac{1}{2}\Sigma_z\Psi'(q)\bar{\chi}(k) +$$
$$\chi'^{\dagger}(p)\frac{1}{2}\Sigma_z\chi'(q)\bar{\chi}(k)\Big]|0\rangle, \tag{20}$$

for the external state $\bar{\chi}(k)$. Note again that $\vec{q}=\vec{p}$.

In order to get a meaningful result in both cases the external states must contract with the state q and must be expressed in the new mass eigenstate basis. Note also that by the LSZ theorem the external states satisfy $\gamma^\mu k_\mu - m = 0$. The contraction of the states k with the states q will lead to the propagator times $\delta(k-q)$. Then one remains with the state with four momentum $(p_0,\vec{k})$. Since this state will be put on shell it is required to satisfy $p_0^2 - m^2 - \vec{k}^2 = 0$, therefore will colapse into the full state $|k\rangle$. For simplicity we omit all these detailed calculations.

Finally one obtains for the projection of the spin for $\Psi(k)$:

$$S_z(\Psi) \propto$$
$$[(\cos\theta)^2\frac{1}{\gamma^\mu k_\mu - m - a}|_{\gamma^\mu k_\mu - m=0} +$$
$$(\sin\theta)^2\frac{1}{\gamma^\mu k_\mu - m + a}|_{\gamma^\mu k_\mu - m=0}]|\Psi(k)\rangle =$$
$$-[(\cos\theta)^2 - (\sin\theta)^2]\frac{1}{a}|\Psi(k)\rangle. \tag{21}$$

Similarly for $\chi(k)$ one gets:

$$S_z(\chi) \propto$$
$$[(\cos\theta)^2\frac{1}{\gamma^\mu k_\mu - m + a}|_{\gamma^\mu k_\mu - m=0} +$$
$$(\sin\theta)^2\frac{1}{\gamma^\mu k_\mu - m - a}|_{\gamma^\mu k_\mu - m=0}]|\chi(k)\rangle =$$
$$[(\cos\theta)^2 - (\sin\theta)^2]\frac{1}{a}|\chi(k)\rangle. \tag{22}$$

Equations (21) and (22) show without any doubt that the projections of the spins of the two separated fermions on the z axes have opposite signs no matter how far they are in time or space.

In this paper we first briefly reviewed the EPR paradox and the controversies associated to it. Next we considered as an illustration a system consisting of two fermions interacting with a scalar meson. We placed this system in a quantum field

theoretical approach to determine the eigenvalues of S_z spin of the fermions long time after the interaction between them ceased to exist. These fermions may be spatially separated or not. The novelty of the approach consists in the intermediate steps in which we assimilate the exponential $\exp[i\Delta p_0 x^0]$ characterizing the wave functions of the fermions with an extra term in the actual action for the fermions. We showed that this inclusion changed the mass eigenstates for the two fermions thus establishing a kinetic interference between them at all times now matter how largely separated they are. This entanglement manifested itself through a clear and definite correlation between the quantum spins of two on-shell separated fermions.

Our method can explain the EPR paradox and shows that in a quantum physics context there is no definite distinction between space time and fields and that there is always an alternative description in which their roles may be switched. Glimpses of this idea were known early on from the dawn of quantum field theories when physicists made a decision and considered more amenable the theories in which fields or particles were considered as main variables and space-time as a framework in which they existed.

In the end the conclusion of this work is that if the particles are in contact and interact at a moment in space time there is always a quantum mechanical description in which they are in contact at all times. The measure in which this statement violates causality is a complex topic which will not be discussed here.

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