GRAVITY AND MATTER REINTERPRETED

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Abstract. In this work we propose a universal action for gravity and matter which in first orders leads to standard gravity and QFT theories. In our theory matter acts as conformal factors of generic conformal transformations applied to the metric. In this framework we derived easily the Schwarzschild metric with cosmological constant and the Reissner-Nordstrom metric. Moreover we estimate the value of the cosmological constant in very good agreement with the natural value.

Key words: Gravity, Quantum Field Theory, cosmological constant, unified model.

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1. INTRODUCTION

The gravitational interaction has similarities and differences with the other three main interactions described by the gauge fields. The gauge field is the main mediator in theoretical particle physics and its properties are largely known and understood and encapsulated in the quantum theory of fields which is unitary and renormalizable. In contrast the gravity action is non renormalizable through the standard methods and is constructed on a different conceptual framework.

Incorporating all the interactions in an universal theory is a difficult task which over the years lead to many alternatives and extensions of Einstein theory of relativity [1–4]. Even more a quantum theory of gravitation has been tackled in many attempts, most notable being loop quantum gravity [5] and string theory [6].

There are various formulations and extensions of general relativity. In [7] and [8] we propose two distinct formulations, the latter containing the electromagnetic field and the possibility to add extra matter or gauge fields. These formulations were in a certain limit completely equivalent to the Einstein gravitational theory. In [9] we included the electromagnetic tensor in a non symmetric metric tensor in the same framework.

In this work based on the main ideas introduced in [7–9] we propose a universal action that considers all the interactions on the same footing. In section II, IV we derive in our set-up the Schwarzschild and Reissner-Nordstrom solutions whereas in section III we discus the cosmological constant solution and find a natural answer for the cosmological constant problem. In section V we justify our choice of coordinates which is related to a specific observer. Section VI is dedicated to the conclusions.

2. THE SET-UP

We start by considering the following action:

$$\sqrt{-g(x)}\sqrt{\eta(x+y)}, \tag{1}$$

where $g(x)$ is the determinant of the metric and $\eta(x+y)$ is the determinant of the metric in normal coordinates around point x.

All matter contributions and constants that appear in the theory are implemented as infinitesimal or finite special conformal transformations acting on $\eta(x+y)^{-\frac{1}{2}}$. An infinitesimal conformal transformation of parameter a_{μ} in x acts as:

$$\exp[-i[a^{\mu}(2x_{\mu}x^{\nu})\partial_{\nu}-a^{\mu}x^2\partial_{\mu}]], \tag{2}$$

whereas in y acts as:

$$\exp[-i[a^{\mu}(2y_{\mu}y^{\nu})\partial_{\nu}-a^{\mu}y^2\partial_{\mu}]], \tag{3}$$

One can include or drop the i factor.

In the following we will show that this set-up leads to the metric for a cosmological constant, the exterior Schwarzschild solution and in the presence of an electrostatic potential to the Reissner-Nordstrom metric.

We start with the Ansatz:

$$\sqrt{-g(x)}\sqrt{\eta(\frac{x^{\mu}-\Lambda^{\mu}x^2}{1-2(\Lambda x)+x^2\Lambda^2}+y^{\mu}+2(yA)y^{\mu}-y^2A^{\mu})}, \tag{4}$$

which correspond to a finite transformation of parameter Λ^{μ} with all components different than zero and an infinitesimal conformal transformation with parameter A^{μ} discussed before on y^{μ}.

If Λ is the Planck scale or close to it and x is large then:

$$\frac{x^{\mu}-\Lambda^{\mu}x^2}{1-2(\Lambda x)+x^2\Lambda^2}\approx-\frac{\Lambda^{\mu}}{\Lambda^2}. \tag{5}$$

One can switch at any point x and y in the description of the action we considered. In the case here however y^{μ} should be regarded as a field. This field can be quantized case which will correspond to quantum gravity. Note that in our picture the carrier of the gravitational interaction is a spin one field and not the graviton. In the classical case the field y^{μ} should satisfy the equation of motion. There are various solutions which lead to different equivalent results (all solutions amount to the statement that the argument of η depends only on x). Here we shall consider the solution for which:

$$y^{\mu}=-\frac{x^{\mu}-\Lambda^{\mu}x^2}{1-2(\Lambda x)+x^2\Lambda^2}\approx+\frac{\Lambda^{\mu}}{\Lambda^2}. \tag{6}$$

In the case of a static spherical symmetry we thus get $y_r^2=\frac{1}{\Lambda^2}$.

Consider the action of the special conformal generator K_μ of parameter a^μ. If we define:

$$\exp[ia^\mu K_\nu] = K, \tag{7}$$

then the action on the the metric is:

$$K\eta_{\mu\nu}(x+y)K^\dagger. \tag{8}$$

If we consider the operator real then the action will be a product of matrices:

$$K\eta_{\mu\nu}(x+y)K^t. \tag{9}$$

It is evident that the determinant of the quantity $K\eta_{\mu\nu}(x+y)K^t$ is just $\eta(x^\mu + 2y^\mu(ya) - y^2a^\mu)$. Here we considered the conformal operator acting on y. Noting that the operators K and K^t are actually exponentials and that in the end their expansion should lead to the QFT Lagrangian we infer that for the fields with positive kinetic terms the description in Eq. (9) is adequate whereas for the gauge fields which have negative kinetic terms the correct description would be:

$$K\eta_{\mu\nu}(x+y)K^{'t}. \tag{10}$$

where in $K^{'t}$ the generator K_μ is replaced by $-K_\mu$.

First we notice:

$$K^\mu_\mu X = \exp[-(2(ay)y^\mu - y^2a^\mu)\partial_\mu]X \approx$$
$$\exp[8a^\mu y_\mu]X + ..., \tag{11}$$

where we differentiated by parts. Then one infers:

$$K^\mu_\nu = (\exp[k(a^\rho y_\sigma])^\mu_\nu, \tag{12}$$

so we get just the exponent of a matrix (The constant factor k will be determined by considering the adequate symmetry of the exponent). Then,

$$K\eta_{\mu\nu}(x+y)K^t \approx$$
$$(\exp[k(a^\mu y_\nu)]\eta_{\alpha\beta}(\exp[k(a^\mu y_\nu)])^t)_{\rho\sigma} = \eta'_{\rho\sigma}. \tag{13}$$

The form of K will depend on the symmetry chosen.

Then one may write the action corresponding to the conformal transformation of parameter a^μ in y as:

$$\sqrt{-g(x)}[\det[K\eta(x)K^t]]^{\frac{1}{2}}. \tag{14}$$

Then the equation of motion for the metric leads quite naturally to:

$$g_{\mu\nu}(x) = [K\eta(x)K^t]_{\mu\nu}. \tag{15}$$

Equation (15) should be regarded as a matrix equation.

Next we shall apply our findings to the case when $a^i = \frac{A^i}{2} = 0$ and $a^0 = \frac{A^0}{2} = \frac{q}{2r}$ where $A^0(x)$ is the electrostatic potential. According to equation of motion for y_r Eq. (6) and the conditions of spherical symmetry and time independence (static metric) we get:

$$y_r = \frac{1}{\Lambda_r}. \tag{16}$$

From Eq. (15) one gets the following matrix equation:

$$K\eta(x)K'^t = \begin{bmatrix} (1+y_r^2(A^0)^2) & -y_r A^0 & 0 & 0 \\ +y_r A^0 & -1 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2(\theta) \end{bmatrix}, \tag{17}$$

where we used:

$$K = \exp\left[\begin{bmatrix} 0 & y_r A^0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right], \tag{18}$$

$$K'^t = \exp\left[\begin{bmatrix} 0 & -y_r A^0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right], \tag{19}$$

and,

$$\eta(x) = \left[\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2(\theta) \end{bmatrix}\right]. \tag{20}$$

Finally the interval corresponding to the metric $g_{\mu\nu}(x)$ would be:

$$ds^2 = (1+y_r^2(A^0)^2)dT_+dT_- - y_r A^0 dr dT_+ + y_r A^0 dr dT_- - dr^2 - d\Omega^2 =$$
$$(1+y_r^2(A^0)^2)dt^2 - \frac{1}{(1+y_r^2(A^0)^2)}dr^2 - d\Omega^2. \tag{21}$$

Here one takes $y_r = \frac{1}{\Lambda_r}$ and after a slight tuning of the constant in front notices that the metric obtained is identical to the Reissner-Nordstrom metric provided that:

$$dT_- = dt - \frac{y_r A^0}{1+y_r^2(A^0)^2}dr$$
$$dT_+ = dt + \frac{y_r A^0}{1+y_r^2(A^0)^2}dr, \tag{22}$$

which leads to a shift of the time variable. Note that in variable x we get the Reissner-Nordstrom solution without the source term in the Gullstrand-Painleve coordinates.

Next we shall consider the situation when the parameter factor of the special conformal transformation is b^μ. First we will assume that this b^μ is a mass vector associated to a fermion. We did not introduce in our set up the gamma matrices. We shall consider instead the fermion components as anticommuting scalars scaled by a factor $\frac{1}{(\Lambda_r)^{\frac{1}{2}}}$. We denote the scalars by $\Phi(x)_i$. To be consistent these scalars must be the parameters of extra conformal transformation associated to the theory. The exact implementation is irrelevant. Then the action should be of the form:

$$\sqrt{g(x)}[\eta(\frac{x^\mu-\Lambda^\mu x^2}{1-2(\Lambda x)+x^2\Lambda^2}+y^\mu+2(y(b+\Phi))y^\mu-y^2(b^\mu+\Phi^\mu))]^{\frac{1}{2}}. \tag{23}$$

Before determining the metric associated to this we need to consider that picture in which the solution for the equation of motion for y^μ is:

$$\begin{aligned}
&y^\mu=-\frac{x^\mu-\Lambda^\mu x^2}{1-2(\Lambda x)+x^2\Lambda^2}-2(y(b))y^\mu+y^2(b^\mu)\approx\\
&+\frac{\Lambda^\mu}{\Lambda^2}-2(y(b))y^\mu+y^2(b^\mu)=\\
&\delta(x^\mu)+\delta(y^\mu),\\
&y^\mu=-\delta x^\mu-2(\delta xb)(\delta x^\mu)+(\delta x)^2b^\mu.
\end{aligned} \tag{24}$$

The above result is obtained by applying the inverse conformal transformation with small parameter b^μ.

By simple inspection from Eq. (23) one notices that the propagator for the fermion fields is y^2 with adequate space time dependence which we shall ignore here. This is generally valid for any fields which are introduced in this framework. From Eq. (24) one deduces:

$$\begin{aligned}
&y^2\delta x=(\delta x)^2(1+(b^0)^2(\delta x)^2)\delta x=\\
&r^2\delta r\delta t(\frac{d}{dr}+(b^0)^2r).
\end{aligned} \tag{25}$$

Here we considered the approach adjusted for the static case and spherical symmetry. Then in order to get an adequate propagator one must have:

$$(b^0)^2=\frac{m}{r}, \tag{26}$$

where the result is written for the static case with spherical symmetry.

Next we shall consider the metric using the method described for an electrostatic field. We apply the procedure from the previous case with:

$$y_r^2 = \frac{1}{\Lambda^2}$$
$$(b_0)^2 = \frac{m}{r} \tag{27}$$

to determine:

$$ds^2 = (1 - \frac{m}{r\Lambda^2})dT^2 - \frac{1}{1 - \frac{m}{r\Lambda^2}}dr^2 - d\omega^2. \tag{28}$$

Here again:

$$dT = dt - \frac{\sqrt{\frac{m}{r\Lambda^2}}}{(1 - \frac{m}{r\Lambda^2})}. \tag{29}$$

By analogy with the previous case one notices that the metric in variable x is the Schwarzschild metric in Gullstrand-Painleve coordinates.

3. COSMOLOGICAL CONSTANT

Here we shall determine the metric in our approach corresponding to a cosmological constant term. In this case the conformal parameter is $a^0 = \Lambda_0$, $a^i = 0$ and the conformal operator is applied to x. The action of the conformal operator will lead to (by analogy with Eq. (17)):

$$K\eta(x)K^t = \begin{bmatrix} (1 - 4r^2(\Lambda^0)^2) & -2r\Lambda^0 & 0 & 0 \\ -2r\Lambda^0 & -1 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2(\theta) \end{bmatrix}, \tag{30}$$

where we used:

$$K = \exp\left[\begin{bmatrix} 0 & 2r\Lambda^0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right], \tag{31}$$

and $\eta(x)$ is defined in Eq. (20). Again one obtains the metric in x in the Gullstrand-Painleve coordinates from which the correspondence to the standard metric for a cosmological constant is obtained by making the substitution:

$$4(\Lambda^0)^2 = \frac{\Lambda^2}{3}$$
$$\Lambda^2 = 12(\Lambda^0)^2, \tag{32}$$

where Λ^2 is the standard cosmological constant $\Lambda^2 = 2.84 \times 10^{-122}\, l_P^{-2}$.

Note that our estimate for the metric is calculated considering the conformal parameter Λ^0 very small. In case in which it is large the contribution to the metric is just a constant translation. One may determine the value for Λ^0 at which this switch occurs from the conformal factor. If $\Lambda^0 r \gg 1$ the the metric is constant. If $\Lambda^0 r \ll 1$ then one obtains the usual metric with a cosmological constant. This explains naturally the unusual smallness of the cosmological constant. One can estimate $\Lambda^0 \approx \frac{1}{R}$ where R is the radius of the Universe $R = 2.773 \times 10^{61}\, l_P$. Then the standard cosmological constant will be given by:

$$\Lambda^2 = 12(\Lambda^0)^2 \approx 1.619 \times 10^{-122}\, l_P^{-2}. \tag{33}$$

which is very close to the value calculated from the experiment mentioned above.

4. THE FULL REISSNER-NORDSTROM SOLUTION

In the method introduced in section II and III there is one single gravitational source or field and no interaction. Consequently we had only one parameter y. Even in this case the set-up works with a particular adjustment of the conformal operator K. However in the case when there are multiple fields that interact the set-up must become more complicated and one should introduce a complete set of y_i adequate for each case.

Consider an electrostatic field produced by a source placed at the origin with a mass m. The metric produced in the Einstein theory in this case would be the Reissner-Nordstrom metric. With the set-ups in the section II and III we will get the Reissner-Nordstrom solution in the Gullstrand-Painleve coordinates of the form:

$$\begin{aligned} ds^2 = (1 - y_r^2(-(A^0)^2 + (b^0)^2))dT_+dT_- - \\ y_r(A^0 - b^0)drdT_+ + y_r(A^0 + b^0)drdT_- - dr^2 - d\Omega^2. \end{aligned} \tag{34}$$

Of course only experiment may decide the exact form of the metric.

Again acting to the left and approximated correctly these will lead to the full operator K in the the spherical static theory:

$$K = \exp\left[\begin{bmatrix} 0 & y_r A^0 + y_r b^0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right] \tag{35}$$

and,

$$K'^t = \exp\left[\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -y_r A^0 + y_r b^0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right] \tag{36}$$

The final form of the metric upon diagonalization will be exactly the Reissner-Nordstrom solution:

$$ds^2 = (1 - y_r^2((b^0)^2 - (A^0)^2))dt^2 - \frac{1}{(1 - y_r^2((b^0)^2 - (A^0)^2))}dr^2 - d\Omega^2, \tag{37}$$

provided that:

$$\begin{aligned} dT_+ &= dt - (b^0 - A^0)y_r dr \frac{1}{(1 - (y_r^2(b^0)^2 - y_r^2(A^0)^2))} \\ dT_- &= dt - (b^0 + A^0)y_r dr \frac{1}{(1 - (y_r^2(b^0)^2 - y_r^2(A^0)^2))}. \end{aligned} \tag{38}$$

Again the old coordinates are the correct Gullstrand-Painleve coordinates for the full Reissner-Nordstrom solution.

5. SYSTEM OF REFERENCE, OBSERVERS AND COORDINATES

In sections II, III and IV we discussed the spherical symmetric and static solutions of the metric tensor in our action. Our results are in perfect agreement with the Schwarzschild and Reissner-Nordstrom solutions but in Gullstrand-Paineleve coordinates for the former or the generalized Gullstrand-Painleve coordinates, that we introduced, for the latter. Further on we made the change of coordinates to the standard ones. In this section we shall justify our change of coordinates.

As it can be seen form Eq. (4) the matter is introduced in the action as conformal factors of the fields y_i. Then upon a series expansion it is evident that the square of the field (consider for simplicity a scalar) will be proportional to $y_s^{t\mu} g_{\mu\nu} y_s^{\nu}$ where the field is considered in the background gravity created by the rest of the fields. Therefore the y_s^{μ} for the scalar will depend on all the other y_i in the theory again introduced in the argument with the matter as conformal factors. In the end one solves the equation of motion for y_s as $y_s^{t\mu} = \frac{1}{\frac{1}{\partial^{\nu}}} K_{\nu}^{t\mu} \approx dx^{\nu} K_{\nu}^{t\mu}$ where K^t was introduced before. The transformation for y_s^{μ} will be obviously the transposed of it. It is easy to check then that if one introduces the finite conformal transformation in first orders then one gets exactly the change of coordinates from Gullstrand-Painleve coordinates to the standard coordinates for all the cases discussed in the paper.

6. CONCLUSIONS

In this work we discussed a unified model of matter and gravity which in its first stages was implemented in [7–9]. Here we applied directly the model to the exact solutions of the metric tensor. We obtained directly the Schwarzschild metric with a cosmological constant and the Reissner-Nordstrom solutions in generalized Gullstrand-Painleve coordinates. The change to the standard coordinates are natural in our framework.

The model embedded a natural solution to the cosmological constant problem. Using straightforward arguments we were able to estimate the cosmological constant in good agreement with the actual known value.

There are important consequences of our model both from the point of view of gravity and of quantum particles. Whereas, as shown, one expects that in first orders the model leads to established facts there are nevertheless differences that can determine the adequacy of our description with respect to the real world. These topics will be discussed in further work.

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