

NUMERICAL INVESTIGATION ON NEW MODELS OF THE INTERNAL ENERGY ΔU AND THE WORK W

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Abstract. In this paper, novel models of the internal energy variation ΔU and the work W were investigated numerically using FORTRAN software. These are currently developed by employing a real gas theory represented by Berthelot state equation, which allows appearing the thermal and caloric imperfections. As opposed to past methodologies and models, it was observed that the new form of internal energy variation ΔU and the work W are exceptionally simple to use at any condition, especially critical to figure the adjustment of the internal energy variation ΔU and the work W when they fluctuates with pressure, it was previously difficult to obtain. This work is based on high theoretical development and strong numerical methods, furthermore, usually solves more accurate results and a higher speed of convergence. Applications of thermodynamics and physics to a variety of worked examples, especially the computation remains difficult in most fields, but with this study we extend the calculation whatever the circumstances. The comparison of error between the current model, which is based on real gas theory, and the PG model presents that the pressure-temperature combinations and the thermal-caloric imperfections effects play a significant role, and the error can reach up to 20%.

Key words: internal energy, work, high temperatures, high pressures, computational thermodynamics, real gas model, thermal and caloric imperfections.

1. INTRODUCTION

Thermodynamics is a physics branch which construes the governing laws of work conversion process into heat and *vice versa*, and can explain the phenomena of temperature, heat etc. [1]. The terms internal energy and work are frequently and extensively utilized in scientific language and widely applied in physics, mechanics and heat transfer [2]. The work is common word and everyday hears about it; all are said to be working. However, in physics, the word 'Work' covers an unequivocal and exact significance. The energy is often defined as the capacity to produce work [3].

The study of internal energy is very carefully supported in Physics, for its effectiveness to describe the matter properties or describe processes in which the properties of matter undergo changes [4–5]. When the internal energy of a thermodynamic system is the energy it contains. It is an extensive state function, associated with this system [6]. It is equal to the sum of the kinetic energy and of all the potential energies of interaction of the elementary entities of this system [7].

In fact, it corresponds to the intrinsic energy of the system, defined at the microscopic scale, excluding the kinetic or potential energy of interaction of the system with its environment, at the macroscopic scale [8].

Returning to the eighteenth century, Carnot Sadi was the first to introduce the work, his book considered the founding the backbones of thermodynamics [9]. A few years later James Prescott Joule demonstrated that mechanical energy can be converted into internal energy and *vice versa* [10]. He explains that the gravitational potential energy of water is converted to internal energy. One year later, Rudolf Clausius Proved that the total energy of a system is always conserved, as known today by the first law of thermodynamics [11–12], he said ‘*Change in internal energy (ΔU) of the system is equal to heat supplied to the system (Q) minus the work done by the system (W) on the surroundings*’. It is mathematically into the equation (8).

The first law of thermodynamics is speaks about the relationship between his terms, and debated methods and ideas based on work and energy [13]. It is explained the meaning in terms of work in adia-thermal conditions. The unit of measure of enthalpy in the International System of Units (SI) is joule ($\text{kg}\cdot\text{m}^2\cdot\text{s}^{-2}$) [14], although other historical units are still sometimes in use. It is commonly used when studying state changes involving the energy of a system in many chemical, biological and physical processes [15].

The easiest relationship of the internal energy is based on the perfect gas theory with a constant specific heat C_V , represented by $\Delta U = C_V \Delta T$ [16], the real behavior was ignored, and this latter has been proven by many authors and researchers of its lack of seriousness and its unacceptable results, especially at high temperature [17].

The high temperature model *HT* which is based on the caloric imperfection *i.e.*, $C_p = C_p(T)$ and $\gamma = \gamma(T)$ [18], managed to obtain satisfactory results at high temperature. However, we fell into the question about the changes of the internal energy and the work at high pressure.

Authors in references [19–20] they have made an important leap in entering the field of real gases *via* quantum physics by using Gibbs’s works, their works focused on the zero of the internal energy especially for saturated Liquid and Vapor. As a result, there is no absolute zero point for internal energy (and thus enthalpy) and it can only be expressed relative to a human-chosen reference point. For this reason, their works remains theoretical because it is difficult to implement and contains a huge amount of assumptions.

To solve the thermal-caloric imperfections, high pressure and temperature effects on all thermodynamics issues, many researchers have proposed various methods and theories about. The first author who tried to break through the thermal and the caloric imperfections is Janet Bainbridge [21]. He developed an easy method to evaluate the change in internal energy, except it is not general for all fluids, but is limited to simple-fluids, and it is noted that, the author used many approaches and approximations to compute the integrations; he also used a polynomial state equation known today as Viriel state equation [22]. It is also, his equations for C_p

and C_V are in fact, they come back to high temperature model, which is very far from real gas behavior. Therefore, its results are not up to the desired level.

Two years early, we have solved the problem of entropy under high conditions of pressure and temperature [23], the results are very satisfying and fitted for all conditions, and the solution was made in a short time. This is what encourages us to complete our research work by developing a new general formula of enthalpy valid at any condition.

Quite recently, considerable jump has been gaining in thermodynamics by the development of a general form of enthalpy variation, which is a thermodynamic potential which synthesizes in a single function the internal energy of the system (related to its temperature and to its quantity of matter) and the work (related to its pressure) required to occupy its volume. The results were very satisfactory and the error between the perfect gas model PG and the real gas model was great then 10% [24].

The thermal-caloric imperfection effect in thermodynamics has been scarcely investigated from the point of view numerically, due to the difficulty of solving nonlinear equations and complex integrals. In the last few years there has been a growing interest in computational thermodynamics with the rapid development of technology in the field of computers as well as software [25]. The current study focused on the investigation of new models of internal energy and the work numerically using FORTRAN power station software. With assumption of real gas model, the thermodynamics relationships are developed by using of Berthelot's equation of state [26–27]. This state equation is used to be into account the effects of intermolecular forces. It is expected that caloric imperfection might be represented by incorporating the expression of Planck in the C_p and C_V .

In this work we will investigate the thermal and caloric imperfection effects caused by the molecular interactions on new models of internal energy and work for air. A large number of empirical state equations have been proposed to describe the real gases, among which the most accurate is the Berthelot state equation [28]. The investigation covers thermodynamics, physics and gas dynamics fields and offers a solution to a problem in thermodynamics. A comparison was made for each case in order to notice the difference to take an idea about the limitation of previous studies and the effectiveness of the new models.

2. MATHEMATICAL FORMULATIONS

2.1. REAL GAS MODEL

For this work the real gas theory described by Berthelot state equation is presented [29]:

$$P(T, \rho) = \frac{\rho RT}{(1 - \rho b)} - \frac{a \rho^2}{T}. \quad (1)$$

Or, the equation of state relative to one mole of the gas is written:

$$\left(P + \frac{a}{TV^2}\right)(V - b) = RT. \quad (2)$$

Then

$$P = \frac{RT}{(V - b)} - \frac{a}{TV^2}, \quad (3)$$

where: $a = 3 P_c V_c^2 T_c$ and $b = \frac{V_c}{3}$ with P_c , V_c and T_c being the critical pressure, volume and temperature, respectively.

The specific heat at constant volume C_V and pressure C_P are given by [26, 29]:

$$C_v(T, \rho) = C_{v,PG} \left\{ 1 + (\gamma_{PG} - 1) \left[\left(\frac{\theta}{T} \right)^2 \frac{z}{(1-z)^2} + \frac{2a\rho}{RT^2} \right] \right\}, \quad (4)$$

$$C_p(T, \rho) = C_{p,PG} \left[1 + \left(\frac{\gamma_{PG} - 1}{\gamma_{PG}} \right) \left\{ \left(\frac{\theta}{T} \right)^2 \frac{Z}{(1-Z)^2} + \frac{2a\rho}{RT^2} \left[1 + \frac{\left(\frac{2-b\rho}{1-b\rho} + \frac{a\rho}{2RT^2} \right)}{\frac{1}{(1-b\rho)^2} - \frac{2a\rho}{RT^2}} \right] \right\} \right]. \quad (5)$$

We have $z = e\left(\frac{\theta}{T}\right)$, $z_0 = e\left(\frac{\theta}{T_0}\right)$.

The enthalpy variation is given by [24]:

$$\Delta H = \int_{T_1}^{T_2} C_p(T, \rho)|_{\rho=\rho_2} dT + \int_{\rho_1}^{\rho_2} C_T(T, \rho)|_{T=T_2} d\rho, \quad (6)$$

where

$$C_T(T, \rho) = \left(\frac{3ab^2\rho^3 - 6ab\rho - RT^3b + 3a}{2T\rho b - T b^2\rho^2 - T} \right). \quad (7)$$

2.2. THE INTERNAL ENERGY

During an elementary reversible process, the internal energy variation of the gas is [30]:

$$dU = \delta Q + \delta W, \quad (8)$$

where the heat is [15]:

$$\delta Q = C_V dT + l dV, \quad (9)$$

and the work is given by [3]:

$$\delta W = -P dV. \quad (10)$$

So,

$$dU = C_V dT + (l - P) dV. \quad (11)$$

In thermodynamics, we can characterize or present each state function by two state variables, then:

- The differential of U gives [16]:

$$dU = \left(\frac{\partial U}{\partial T} \right)_V dT + \left(\frac{\partial U}{\partial V} \right)_T dV. \quad (12)$$

- Clapeyron's first relationship is presented by [30]:

$$l = T \left(\frac{\partial P}{\partial T} \right)_V \quad (13)$$

- Using Clapeyron's relationship on Berthelot state equation we deduce

$$l = \frac{RT}{V-b} + \frac{a}{TV^2} \quad (14)$$

and

$$l - P = \frac{2a}{TV^2}. \quad (15)$$

So

$$dU = C_V dT + \left(\frac{2a}{TV^2} \right) dV. \quad (16)$$

On the other hand, for a unit mass of a gas, we have [23]:

$$V = \frac{1}{\rho}. \quad (17)$$

The derivative of the volume gives:

$$dV = -\frac{1}{\rho^2} d\rho. \quad (18)$$

We replace equation (18) in equation (16), we get:

$$dU = C_V dT - \frac{2a}{T} d\rho. \quad (19)$$

Substituting equation (19) between the state (1) given by (T_1, P_1, ρ_1, V_1) , and state (2) given by (T_2, P_2, ρ_2, V_2) , we find:

$$\Delta U = \int_{T_1}^{T_2} C_V(T, \rho) \Big|_{\rho=\rho_2} dT - \int_{\rho_1}^{\rho_2} \left(\frac{2a}{T} \right) \Big|_{T=T_2} d\rho. \quad (20)$$

Otherwise, the first law of thermodynamics explains in detail the relationship between the enthalpy variation and the internal energy variation, as shown in the equation (21).

$$\Delta H = \Delta U + P \Delta V. \quad (21)$$

2.3. THE WORK

A. Isothermal Process. The work received during an isothermal process is [17]:

$$W = \int_{V_1}^{V_2} -P dV, \quad (22)$$

$$W = - \int_{V_1}^{V_2} \left(\frac{RT}{V-b} + \frac{a}{TV^2} \right) dV. \quad (23)$$

After substituting equation (23) between the states (1) and (2) we find:

$$W_{1 \rightarrow 2} = RT_1 \ln \left(\frac{V_1 - b}{V_2 - b} \right) + \left[\frac{a(V_2 - V_1)}{V_1 V_2 T_1} \right]. \quad (24)$$

B. Isobaric Process. In this case, the perfect gas relationship for the work is valid, provided that the volumes used in the work relationship is extracted or computed from the real gas state equation.

The work given by the perfect gas model for isobaric process is [30]:

$$W = -P(V_2 - V_1). \quad (25)$$

C. Adiabatic Process. In this case, the perfect gas relationship for the work is valid, provided that the volumes and the specific heat ratio γ used in the work relationship are extracted or computed from the real gas state equation.

The work given by the perfect gas model for isobaric process is [4]:

$$W = \frac{P_2 V_2 - P_1 V_1}{\gamma - 1}, \quad (26)$$

where

$$\gamma = \frac{C_p(T, \rho)}{C_v(T, \rho)}. \quad (27)$$

Furthermore, we can calculate the work from the first law of thermodynamics for an adiabatic transformation ($Q = 0$) only.

$$\Delta U = W + Q. \quad (28)$$

In this case, the work equals the internal energy variation, which is developed previously and presented by the relationship (20).

D. Isochoric Process. For isochoric process the work as in perfect gas model equals zero $W = 0$, in this case, the internal energy variation equal the heat $\Delta U = Q$, the heat is supplied by heat transfer modes or Joule heating and equal the enthalpy variation given by the relationship (6), so for this process the internal energy variation equals the enthalpy variation.

$$\Delta U = \Delta H. \quad (29)$$

3. CALCULATION PROCEDURE

It is often required to know the thermodynamic functions such as the internal energy and the work at temperatures and pressures other than those available from tabulated data. Fortunately, the new models of internal energy and the work employ both at the same time with a simple method. The static state (1) presented in relationship (20) is always given by $T_1 = 298.15$ K and $P_1 = 1$ atm and $\rho_1 = \rho_1(T_1, P_1)$.

A numerical integration is required for the enthalpy variation given by the relationship (20) of both functions $C_V(T, \rho)$ and $(2a/T)$ in intervals $[T_1, T_2]$ and $[\rho_1, \rho_2]$ respectively.

In order to reduce the time required for the calculations with high accuracy, we used the Gauss Legendre quadrature [31], this algorithm is better than Simpson's method [32], which is an exact quadrature method for a polynomial of degree $(2n-1)$ with n points taken on the domain of integration. If the function which allows to calculate over a finite interval $[a, b]$, the definite integral $\int_a^b f(x)dx$ of a given continuous function f [33].

The method is of the form:

$$\int_a^b f(x)dx = \sum_{i=1}^n w_i f(x_i), \quad (30)$$

where $f(x_i)$ is the ponderation function, which can ensure the integrability of f . The coefficients of w_i are called the quadrature coefficients (or weights quadrature). The points x_i or nodes, are real, distinct, unique and are the roots of polynomials for the scalar product [34].

The coefficients of the quadrature w_i as well as the abscissas x_i for our integration of order 64 are presented in Table 1 below.

Table 1

Abscissas and associated coefficients of the 64th Gauss Legendre integration quadrature

i	$\pm x_i$	w_i
1	.006368073	0.003509312
2	.0071942277	0.0091372115
3	0.0276188818	0.0156959771
4	0.0325469453	0.0171370005
5	0.0518394344	0.0234179378
6	0.0653161897	0.0254990642
7	0.1227581033	0.0283420289
8	0.1539089403	0.0329111118
9	0.1984778666	0.0371728923
10	0.2361421214	0.0390969435
11	0.2665500455	0.0416559573
12	0.2993243619	0.0438260415
13	0.3640656989	0.0455869341
14	0.3803563189	0.0469221942
15	0.4577640192	0.0478193546
16	0.4758461672	0.0482700387
17	0.5541538328	0.0482700387
18	0.5722359808	0.0478193546
19	0.6496436811	0.0469221942
20	0.6659343011	0.0455869341
21	0.7406756381	0.0438260415
22	0.7534499545	0.0426559573
23	0.8038578786	0.0390969435
24	0.8315221334	0.0361728923
25	0.8860910597	0.0349111118
26	0.8972418967	0.0303420289
27	0.9346838103	0.0274990642
28	0.9481605656	0.0234179378
29	0.9774530547	0.0191370005
30	0.9823811182	0.0156959771
31	0.9828057723	0.0091372115
32	0.9986319268	0.0015093120

In the aim to validate the proposed model RG, a comparison between our model and previous models, for this we present the high temperature model HT, which is developed on the basis to interpolate the specific heat C_p and γ , this interpolation is given by polynomial of the n^{th} degree of the temperature T [35].

For air the 9th and 10th polynomials of C_p and γ respectively are given by Tables 2 and 3 sequentially [36]:

$$C_p(T) = \sum_{i=1}^{10} a_i T^{(i-1)}, \quad (31)$$

$$\gamma(T) = \sum_{i=1}^{11} b_i T^{(i-1)}. \quad (32)$$

Table 2

Polynomial Coefficients of $C_p(T)$

i	a_i
1	1001.1058
2	$4.0661289 \cdot 10^{-2}$
3	$-6.3376997 \cdot 10^{-4}$
4	$2.7474759 \cdot 10^{-6}$
5	$-4.0338459 \cdot 10^{-9}$
6	$3.0697736 \cdot 10^{-12}$
7	$-1.3509355 \cdot 10^{-15}$
8	$3.472262 \cdot 10^{-19}$
9	$-4.8467531 \cdot 10^{-23}$
10	$2.8411878 \cdot 10^{-27}$

Table 3

Polynomial Coefficients of $\gamma(T)$

i	b_i
1	1.4030049
2	$-4.8380251 \cdot 10^{-05}$
3	$5.7713164 \cdot 10^{-07}$
4	$-2.4333247 \cdot 10^{-09}$
5	$4.1173757 \cdot 10^{-12}$
6	$-3.8217601 \cdot 10^{-15}$
7	$2.1579012 \cdot 10^{-18}$
8	$-7.6237753 \cdot 10^{-22}$
9	$1.6491564 \cdot 10^{-25}$
10	$-1.999789 \cdot 10^{-29}$
11	$1.0420059 \cdot 10^{-33}$

The specific heat at volume constant C_V for HT model is:

$$C_V(T) = \frac{C_P(T)}{\gamma(T)}. \quad (33)$$

The Gibbs free energy evaluation

At this point, when we have reached this phase in the equations development, we can simply deduce the value of the **Gibbs free energy**, this latter given by [16]:

$$\Delta G_{\text{Sys}}(T, \rho) = \Delta H_{\text{Sys}}(T, \rho) - T \Delta S_{\text{Sys}}(T, \rho), \quad (34)$$

when the entropy variation is given by [23]:

$$\Delta S(T, \rho) = R \int_{T_1}^{T_2} \left(\frac{C_V(T, \rho)}{RT} \right) dT + R \int_{\rho_1}^{\rho_2} \left(\frac{1}{\rho(\rho b - 1)} - \frac{a}{RT^2} \right) d\rho. \quad (35)$$

4. RESULTS AND COMMENTS

Figure 1 represents the internal energy variation with temperature for models *HT*, *GP* and *RG* respecting the initial conditions of temperature and pressure of 298.15 K and 1 atm respectively. The high temperature curve is presented using C_p and γ interpolation polynomials. With low temperatures (bellow 1000 K), we notice that the three models have the same behaviour, however, with temperature increasing the difference become obvious especially for *GP* model who describe the lowest values. We can also notice that perfect and real gaz curves are represented with straight lines, on the other hands, and with higher temperatures, it takes a shape of a wavy curve adjacent to real gas.

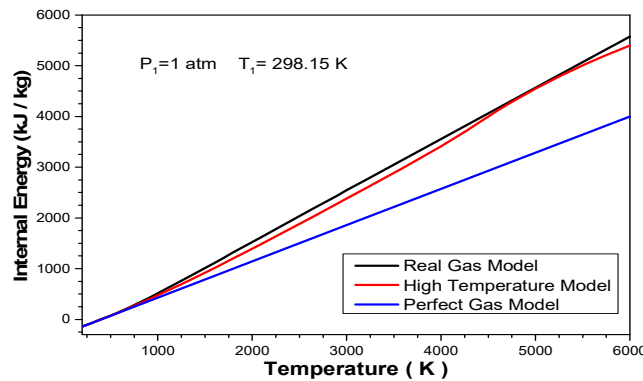


Fig. 1 – Internal energy variation vs temperature for HT, RG and GP models.

Also, by analysing the comparative curves presented in Figs. 2 and 3, the difference between the results is clearly noticed. Taking the example of the mean relative error between the real and perfect gas to be about 30% (Fig. 2) with 10% relative error between the two models used, namely RG and HT (Fig. 3). These errors were considered without taking into account low temperature variation because of the important relative errors in these conditions. These differences show the effects of the thermal and calorific imperfections presented by the molecular size a , the intermolecular force b , and the characteristic vibration temperature θ .

It is concluded from the figures discussed the interesting use of the HT model for high temperatures (above 3500 K) with a maximum error of 5%,

however, for low temperatures ranging from 500 to 700 K, the GP model is recommended.

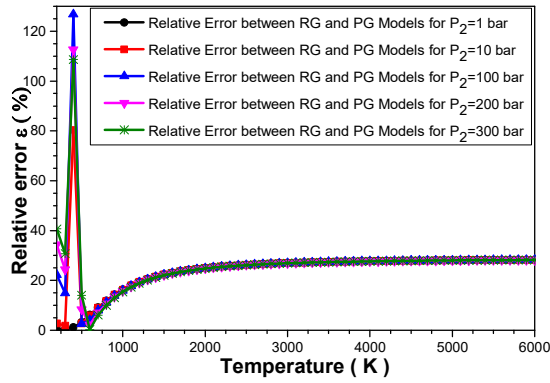


Fig. 2 – Internal energy variation vs temperature for RG model.

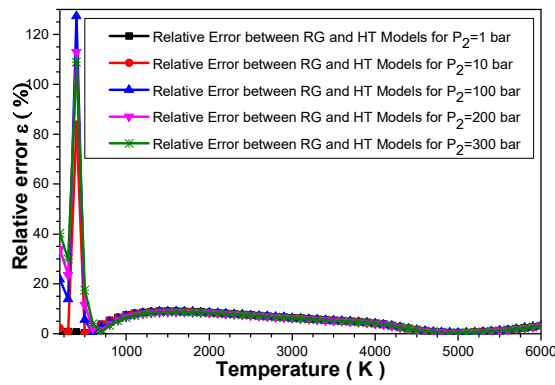


Fig. 3 – Internal energy's relative error vs temperature for RG-HT models.

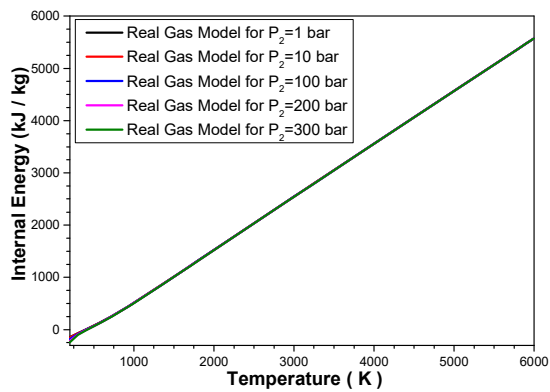


Fig. 4 – Internal energy's relative error vs temperature for RG-PG models.

The internal energy variation with temperature for the RG model is represented in Fig. 4 for pressure values of $P = 1$ atm, 10 atm, 200 atm and 300 atm. The internal energy variation is completely independent and doesn't depend on pressure, this is confirmed by thermodynamic references and gives more credibility to our calculation.

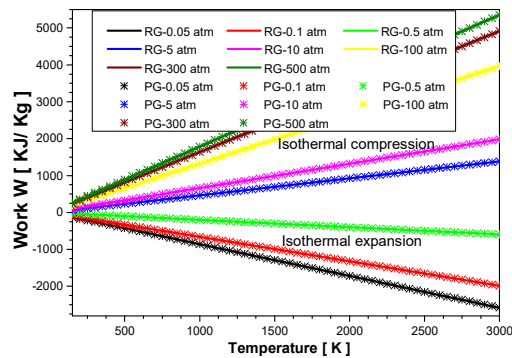


Fig. 5 – Delivered work vs temperature for an isothermal transformation.

For an isothermal transformation and initial pressure of 1 atm, the work variation with temperature for the GP and RG models is presented in Fig. 5 for several pressure values ranging from 0.05 to 500 atm. It is worthy to say that the temperature has a considerable impact on the work developed by both models, the latter is increasing when the pressure is higher than initial (isothermal compression), and decreasing when the pressure is lower than initial (isothermal expansion). Also, increasing the pressure permit to increase the work provided. In addition, both models describe the same behaviour with a slight difference in favour of the RG model, taking the example of state 2 with a pressure $P_2 = 300$ atm and temperature $T_2 = 350$ where the work developed is about 0.540 MJ/kg and 0.581 MJ/kg for the GP and RG models respectively, which also shows that the work provided by the RG model is always lower than that obtained by the PG model.

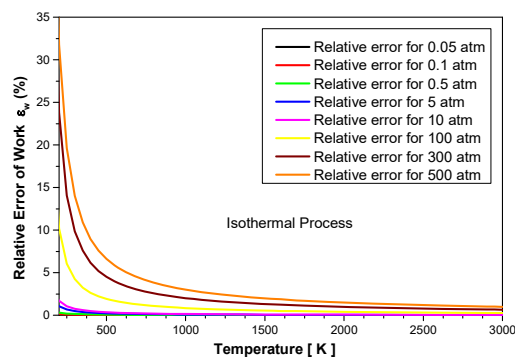


Fig. 6 – Work's relative error vs temperature for an isothermal transformation.

The two models were compared and their relative error is presented in Fig. 6. From this figure, increasing pressure leads to an important increase in relative error and could reach 35% for a pressure value of $P_2 = 500$ atm, the same remark for low temperatures and it's not the case for perfect gases. This difference clearly shows the thermal and calorific imperfections of the real gases.

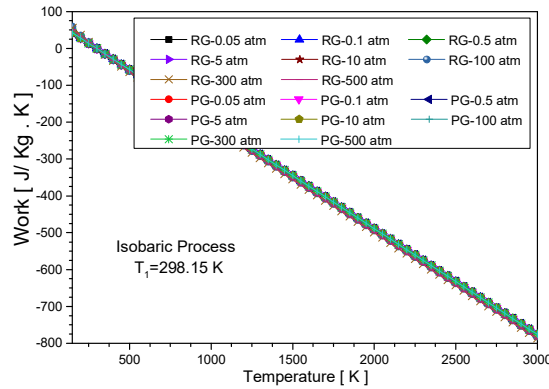


Fig. 7 – Delivered work vs temperature for an isobaric transformation.

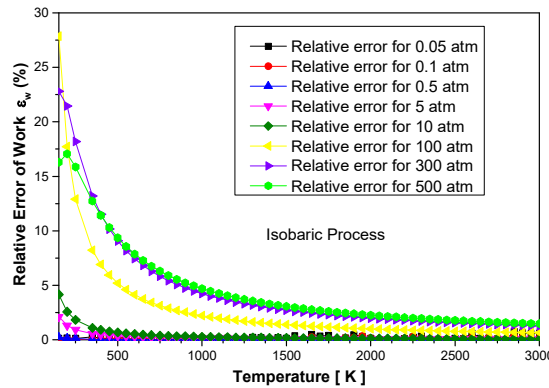


Fig. 8 – Work's relative error vs temperature for an isobaric transformation.

For the case of an isobaric transformation presented by Figs. 7 and 8, both models describe the same decreasing evolution with temperature increasing, noticing the minor impact of the pressure on the work developed. Considering these values, the work is positive when below temperature of 298.15 K and negative when increasing this latter.

Also, from Fig. 7, the provided work by RG model is constantly higher than that provided by PG model with a significant pressure effect showed in Fig. 8. The numerical data show in detail these differences, taking for example the temperature and pressure of state 2 of 250 K and 300 atm respectively, the calculated work values

are about 16.9 kJ/kg and 13.82 kJ/kg for RG and GP models respectively with an average error of 18.2%.

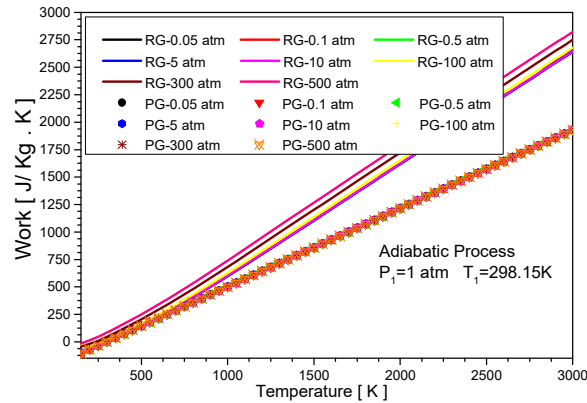


Fig. 9 – Delivered work vs temperature for an adiabatic transformation.

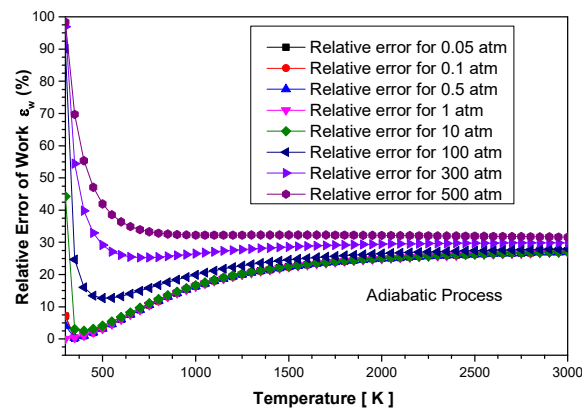


Fig. 10 – Work's relative error vs temperature for an adiabatic transformation.

For initial conditions of pressure and temperature of $P_1 = 1$ atm and $T_1 = 298.15$ K respectively, Figs. 9 and 10 present the work delivered variations and their relative errors for both models GP and RG and with pressure variation from 0.05 to 500 atm.

We notice that the work delivered in both models is increasing when increasing the temperature. In low temperatures, both models describe the same behaviour until reaching temperature of 775 K, after this value, the RG results considerably diverge from those of PG model. Another observation in the same figure is about the minor influence of the pressure on the delivered work using the PG model contrary to RG model where the delivered work increases when increasing pressure.

The relative error analysis showed in Fig. 10 describes clearly the important value, reaching 30% at high temperatures, which requires the use of real gas theory in thermodynamic problems to correct and improve equations and results.

5. CONCLUSIONS

From the outcome of our investigation it is possible to conclude that the new form of the internal energy and the work are very encouraging and uplifting, we have presented many examples of investigation, isothermal, isobaric and adiabatic process. The evidence from this study suggests intimates that the possibility of used the new models for any area of research or under any condition of pressure-temperature.

This paper has highlighted on the importance of thermal-caloric imperfections and pressure-temperature effects. We have found a cutting-edge solution for the enthalpy under various effects, as results we have obtained an accurate, satisfactory and comprehensive results proving the effectiveness of the new forms.

Air is used as fluids in this study; the results of the present work suggest that an extension to other fluids is necessary in the future to further investigate the robustness of the method. However, it would also be necessary to determine the intermolecular force a , molecular size b and molecular vibrational energy constant Θ .

The equations of the HT model can be obtained from the REAL GAS model, by cancelling the intermolecular force a , molecular size b . At this moment if the molecular vibrational energy constant Θ is also cancelled, the model is becoming the perfect gas model GP .

The possibility of use the HT model in any study provided that the temperature is between 500 K and 3000 K, outside this interval of temperatures the error of HT model becomes greater than 5%, which is the value of the acceptable error in gas dynamics and computational thermodynamics.

The strength of this study lies in the correction and the improvement of the internal energy and the work especially in special circumstances. For that this paper provides the framework and the backbone to encourage researchers to dive on this field or working on them without limitations. At this moment, we can say this work has solved a strong point on thermodynamics and gas dynamics, and gave us a high contribution for many fields or subjects still ongoing.

The current findings indicate that an extension to the use of this form fits the conditions of supersonic flight, especially at high altitude would be of interest. We can highly recommend using the new models because we find that a better understanding of the behavior of thermal machines and physical phenomena like cold and high pressure separators, for wind tunnel research works, for refrigeration and oil refinery processes and for research studies on pumps and rotating machinery.

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