MINORITY CURRENT CARRIERS ARE RESPONSIBLE FOR THE SUPERCONDUCTING STATE

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Abstract. It is shown that minority current carriers in superconductors are those carriers that carry the superconducting current. This conclusion was made possible by the new method of minority charge carriers basic parameters determination in solid matter based on magnetoresistance curve analyses within the framework of the phenomenological semiclassical two-band model. These calculations also remove the problem of anomalous behaviour of the Hall Effect near the critical temperature of a superconductor.

Key words: superconductor, magnetoresistance, Hall effect.

1. INTRODUCTION

Galvanomagnetic phenomena from the beginning of their research in superconductors began to surprise researchers. First it was shown that the Hall Effect in superconductor Nb changes its sign near the critical temperature [1]. Further this phenomenon was observed in other superconductors [2–5]. This change of sign began to be considered an anomaly. With the discovery of high-temperature superconductivity [6] these studies have become more intensive. In high-temperature superconductors, a change in the sign of the Hall Effect was also observed [7–13], but not only from positive to negative, but also *vice versa* [8]. Both in conventional and high-temperature superconductors, the appearance of resistivity dependence on the magnetic field was observed [3, 5, 7–10, 14–16]. In some superconductors near the critical temperature, a maximum of Hall resistivity was observed, which the authors called a ghost critical field [17]. These features of galvanomagnetic phenomena in superconductors had different explanations, the most common of which is the motion of vortices. It was also suggested that these features are associated with changes in the basic parameters of current carriers – their concentration and mobility [9]. We will provide analytical and calculating support for this idea.

The purpose of this work is to apply the magnetoresistive method of determining basic parameters of minority current carriers in a solid matter [18] to a superconductor to calculate the mobility of minority current carriers at a gradual decrease in the temperature of the superconductor to a critical temperature.

2. THE MODEL

First, let us briefly outline what this method consists of.

Consider the phenomenological model of galvanomagnetic phenomena for a solid material with two types of current carriers (having opposite or the same sign).

It is known that the transverse resistivity ρ_{xx} for this case depends on the magnetic field inductance B in the following way:

$$\rho_{xx} = \frac{1}{e}\frac{n\mu_n + p\mu_p + \mu_n\mu_p(n\mu_p + p\mu_n)B^2}{(n\mu_n + p\mu_p)^2 + \mu_n{}^2\mu_p{}^2(n \pm p)^2 B^2}, \tag{1}$$

where e is charge of electron, n, p, μ_n, μ_p are concentrations and mobilities of two types current carriers and we use sign “+” if the carriers have the same sign and “–“ if the opposite sign.

Imposing the condition $\mathrm{d}^2\rho_{xx}/\mathrm{d}B^2 = 0$ on this equation gives us the opportunity to determine the position in the magnetic field of the flex point of ρ_{xx} curvature

$$B_f = \frac{1+ab}{\sqrt{3}(1 \pm a)\mu_n}, \tag{2}$$

where $a = n/p$, $b = \mu_n/\mu_p$.

Now the equation (1) can be introduced by the form (index xx in ρ_{xx} we omit for the sake of writing simplification):

$$\rho = \rho_0 \frac{1 + \dfrac{\mu_n(a+b)}{\sqrt{3}B_f(1 \pm a)b}B^2}{1 + \dfrac{1}{3B_f{}^2}B^2}, \tag{3}$$

where ρ_0 is resistivity at $B = 0$.

Suppose that one type of current carriers has a significant concentration (*e.g.* electrons) and the other has a much lower concentration (*e.g.* holes). Then $a << 1$. Since, the minority charge carriers mobility is, as the rule, higher or compared with majority ones, that is $b \geq 1$, then $a << b$ and the magnetic field dependence of ρ gets the form:

$$\rho = \rho_0 \frac{1 + \dfrac{\mu_n}{\sqrt{3}B_f}B^2}{1 + \dfrac{1}{3B_f{}^2}B^2}. \tag{4}$$

What to substitute into this equation $B = B_f$ we will get formula for calculating minority current carriers mobility

$$\mu_n = \frac{\sqrt{3}}{B_f}\left(\frac{4}{3}\frac{\rho_f}{\rho_0} - 1\right), \tag{5}$$

where we wrote for convenience $\rho(B_f) = \rho_f$.

Therefore, to calculate the mobility of minority current carriers of a semiconductor or conductor, it is necessary to measure the magneto field dependence of its resistance and find three values on it: resistivity in zero field ρ_0, magnetoresistance flex field B_f as well as resistivity in this field ρ_f.

Based on equation (2) for $a << 1$, taking into account $ab = \sigma_n / \sigma_p$, we obtain the ratio of minority charge carriers conductivity to majority ones

$$\frac{\sigma_n}{\sigma_p} = 4\left(\frac{\rho_f}{\rho_0} - 1\right). \tag{6}$$

In practice in formula (5) and (6) we can substitute ρ_f / ρ_0 on corresponding ratio of the measured potential differences U_f / U_0 because $\rho_f / \rho_0 = U_f / U_0$.

Now let's derive the formula for calculating the concentration of minority current carriers. For this purpose, in equation (1) we put the strong field condition, *i.e.* $B \to \infty$.

Also taking into account that $a << 1$ and $a << b$ we get saturation resistivity

$$\rho_\infty = \frac{1}{ep\mu_p} = \frac{1}{\sigma_p}, \tag{7}$$

where we wrote for convenience $\rho(B_f) = \rho_f$.

Hence, as well as from (5) and (6), taking into account that $\sigma_n = en\mu_n$ we obtain formula for determining minority current carriers concentration

$$n = \frac{4B_f}{\sqrt{3}e\rho_\infty} \frac{\frac{\rho_f}{\rho_0} - 1}{\frac{4}{3}\frac{\rho_f}{\rho_0} - 1}. \tag{8}$$

The value of ρ_∞ can often be difficult to access because it must be measured in a strong magnetic field, but there is a possibility it should be eliminated.

Substituting (7) into (6) and then the result in (8) we obtain the formula without ρ_∞

$$n = \frac{4\sqrt{3}B_f}{e\rho_0} \frac{\frac{\rho_f}{\rho_0} - 1}{\left(4\frac{\rho_f}{\rho_0} - 3\right)^2} . \tag{9}$$

Let us ask the question: how can we know whether the real experimental curve obeys the classical model we used. To answer this question we consider equation (5) in a strong field limit neglecting 1 in comparison with the *B* containing summand in the numerator and denominator of (4) getting another formula for determining of μ_n

$$\mu_n = \frac{\rho_\infty}{\sqrt{3}\rho_0 B_f} . \tag{10}$$

Combining the latter formula with (5) we get the relation between the experimentally measured values ρ_0, ρ_f and ρ_∞ :

$$\rho_\infty = 4\rho_f - 3\rho_0 . \tag{11}$$

This equation can serve as a test equation to check whether the experimental curve obeys equation (1) and, accordingly, all the formulas derived from it. If it does, then the calculation of minority current carriers parameters by the formulas (5), (6), (8), and (9) will be correct.

The described method has been tested on all types of solids: metal, semiconductor and superconductor [18].

3. RESULTS AND DISCUSSION

The most impressive result of the application of this method was found for the superconductor, namely calculating the minority charge carriers mobility of its in the critical temperature range.

Figure 1 shows the experimental dependences of transverse resistivity on the magnetic field inductance for several temperatures for high temperature multilayer superconductor [$YBa_2Cu_3O_7/PrBa_2Cu_3O_7]_{25}$ near critical point T_c=89.9 K, namely for $T > T_c$. The figure shows that at a temperature that is only one degree higher than the critical, resistivity is almost independent of the magnetic field. This independence of resistivity on the field indicates that only one type of current carriers in this superconductor reveals itself. Indeed, from Eq. (1), provided that $p = 0$, we obtain $\rho = 1/en\mu_n$ which shows that the resistivity does not depend on the magnetic field. When the temperature drops by tenths of a degree starting from 91 K, magnetoresistance rapidly appears. Obviously, the reason is that the second type of current carriers having low concentration begins to manifest itself.

Let us calculate the mobility of these minority current carriers having determined for each curve the flex point B_f , the zero field resistivity ρ_0, flex point resistivity ρ_f and high field limit resistivity ρ_∞. We note, that applying of the above equations which origin from semiclassic analyses of galvanomagnetic phenomena is reasonable for this case, since experimental curves showed on the Fig. 1 are in good agreement with the test equation (11).

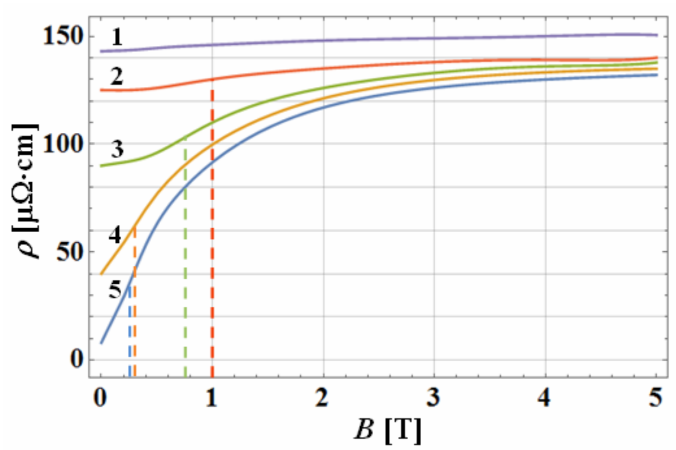

Fig. 1 – Magnetic field dependences of transverse resistivities for multilayer superconductor [$YBa_2Cu_3O_7$(72 Å)/$PrBa_2Cu_3O_7$(12 Å)]$_{25}$ at different temperatures near critical one: 1 – 92 K; 2 – 91 K; 3 – 90.5 K; 4 – 90.25 K; 5 – 90 K. Magnetic field is perpendicular to the layers. Dashed lines indicate flex points B_f. Experimental data are reconstructed from temperature dependences of transverse resistivity at different magnetic fields from [7] (Color online).

The results of calculations are shown in Figs. 2 and 3, which demonstrate a sharp increase in the mobility of minority current carriers when the temperature decreases towards the critical, and the same rapid increase in their contribution to the conductivity. At the same time, as can be seen from the Table 1 majority charge carriers conductivity remains constant what certainly shows their basic parameters to be constant.

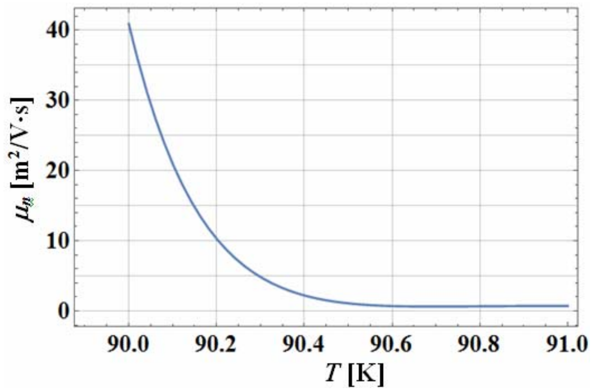

Fig. 2 – Temperature dependence of minority current carriers (electrons) mobility in layered superconductor $[YBa_2Cu_3O_7/PrBa_2Cu_3O_7]_{25}$ in the temperature range of superconductivity onset. In a narrow temperature range of 1K, the mobility of minority current carriers increases rapidly by almost two orders of magnitude (Color online).

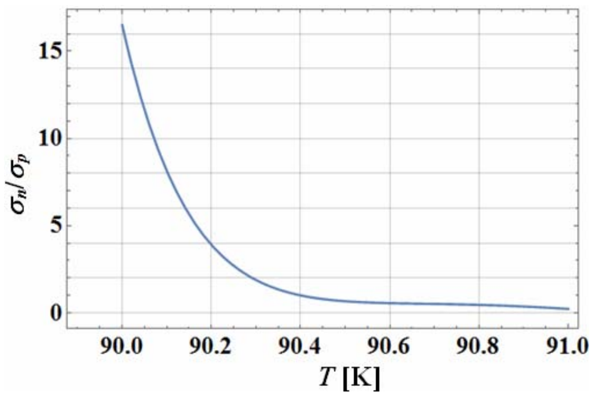

Fig. 3 – Temperature dependence of electron to hole conductivity ratio in layered superconductor $[YBa_2Cu_3O_7/PrBa_2Cu_3O_7]_{25}$ in the temperature range of superconductivity onset. In a narrow temperature range of 1K this ratio increases rapidly by almost two orders of magnitude changing the type of conductivity from hole-like to electron-like (Color online).

This such abrupt conductivity change from hole to electron-like induces us to conclude that electrons being minority current carriers in the normal state become superconducting ones in the superconductive state.

In other cuprate high-temperature superconductors (cuprate doped with Nd, Tm, Bi, Ca, Sr), as well as ordinary superconductors (including those that become superconductors under pressure), a similar behavior of magnetoresistance was found [7–10, 14, 15–17]. Therefore, it can be argued that this rapid increase in the mobility of minority current carriers when the temperature approaches critical one, is common, including the case when the minority current carriers are positive (as is the case for Nd-Ce-Cu-O [8]).

Table 1

Temperature dependence of current carriers basic parameters in layered superconductor $[YBa_2Cu_3O_7/PrBa_2Cu_3O_7]_{25}$. μ_n – electron mobility, μ_p – hole mobility, p – hole concentration, n – electron concentration, σ_n – electron conductivity, σ_p – hole conductivity, a – electron to hole concentration ratio, b – electron to hole mobility ratio

T [K]	μ_n [m^2/V·s]	μ_p [m^2/V·s]	$p\cdot10^{-27}$ [m^{-3}]	σ_n/σ_p	$\sigma_p\cdot10^{-5}$ [Ω·m]	$\sigma_n\cdot10^{-5}$ [Ω·m]	$n\cdot10^{-24}$ [m^{-3}]	a	b
91	0.71	0.0018	2.2	0.23	6.5	1.5	1.3	0.0059	390
90.5	1.3	0.0018	2.2	0.67	6.5	4.4	2.1	0.0095	720
90.25	7.1	0.0018	2.2	2.7	6.5	17.6	1.5	0.0068	3900
90	40.9	0.0018	2.2	16.5	6.5	107	1.7	0.0077	22000

We will show that the above analysis of magnetoresistance explains the phenomenon of the Hall Effect sign reversal near the critical temperature in both conventional [1–4] and high-temperature superconductors [5, 7–14].

One of such experiments is shown in Fig. 4a. The corresponding experiment on magnetoresistance is shown in Fig. 4b. These two figures show that the change of the sign of the Hall Effect appears (The Hall coefficient becomes field dependent) at the same temperature as the magnetoresistance *i.e.* at the temperature of 90.75 K. This change in the sign of the Hall Effect is easily explained by the obtained here great electron mobility. Indeed, from the expression for the Hall coefficient in the weak magnetic field for the case of two types of the opposite sign carriers

$$R_H = \frac{1}{e}\frac{p\mu_p^2 - n\mu_n^2}{(p\mu_p + n\mu_n)^2}. \quad (12)$$

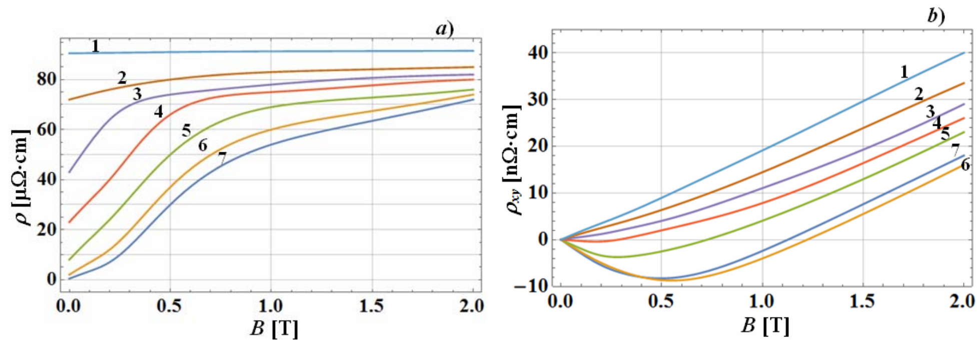

Fig. 4 – Magnetic field dependences of resistivity (a) and Hall resistivity (b) for epitaxial film superconductor $YBa_2Cu_3O_7$ at different temperatures if superconducting onset: 1 – 94 K; 2 – 93 K; 3 – 92.75 K; 4 – 92.5 K; 5 – 92.25 K; 6 – 92 K; 7 – 91.9 K. Experimental data are reconstructed from temperature dependences of resistivity and Hall resistivity at different magnetic fields from [8]. The magnetoresistance appears at the same temperature as the Hall Effect sign reversal (≈ 90.75 K) (Color online).

The sign of R_H will reverse negative when $p\mu_p^2 < n\mu_n^2$ what in its turn is provided by the calculated above great value of electron mobility μ_n. Thus, the behavior of the Hall resistivity near the critical temperature fits well into the framework of the usual semiclassical model of the Hall Effect, which means that there is no anomaly in its behavior.

However, one more thing should be explained. In some superconductors appears so-called ghost critical field what is Hall Effect maximum field near critical temperature [17] (Fig. 5a). We affirm that this curve corresponds to the classical behaviour of the Hall Effect with two types of holes

$$\rho_{xy} == \frac{1}{e}\frac{n\mu_n^2 + p\mu_p^2 + \mu_n^2\mu_p^2(n+p)B^2}{(n\mu_n + p\mu_p)^2 + \mu_n^2\mu_p^2(n+p)^2B^2}B, \tag{13}$$

where we have designated n, p, μ_n, μ_p concentrations and mobilities of two types of holes.

Acting similarly as in the case of magnetoresistance we obtain (for $n << p$) the formula that expresses the Hall resistivity through the experimental values

$$\rho_{xy} = \frac{R_0 + R_\infty/(3B_f^2)B^2}{1 + 1/(3B_f^2)B^2}B, \tag{14}$$

where R_0, R_∞ are Hall coefficients in weak and strong field limit correspondingly, B_f is the field of ρ_{xy} curvature change. Having found these quantities from Fig. 5b, we obtain the expression for the Hall resistivity curve shown in Fig. 5a

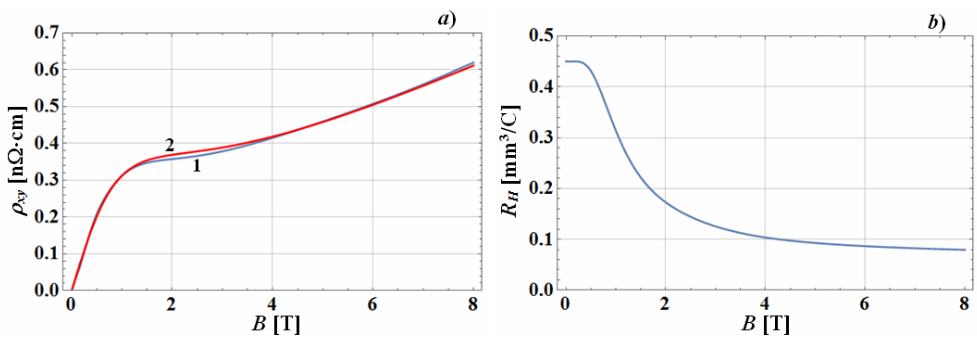

Fig. 5 – Magnetic field dependences of Hall resistivity (a) and Hall coefficient (b) *versus* magnetic field for disordered TaN thin film near critical temperature ($T_c \approx 2.75$ K): 1 – experiment, 2 – theory (eq. (15), the same as (14) or (13)). The experimental curve is in good agreement with the theoretical one. Experimental data are taken from the article [17] (Color online).

$$\rho_{xy} == \frac{0.45+0.037B^2}{1+0.56B^2}B \quad [\text{n}\Omega\cdot\text{cm}], \tag{15}$$

We see that this curve which corresponds to the theoretical formula (15) (the same as to (14) or (13)) coincides well with the experimental one. So, there is no need to talk about any ghost magnetic field since the mentioned experimental data fit into the framework of the usual model of the Hall Effect for a material with two types of current carriers of the same sign.

4. CONCLUSIONS

1. When the temperature of a superconductor, decreasing, becomes close to critical, the mobility of minority current carriers increases sharply, which leads us to conclude that exactly they carry superconducting current in superconductive state. This follows from the numerical analysis of the temperature stroke of the resistivity and the Hall Effect: the resistivity and the Hall coefficient rapidly become dependent on the magnetic field.

2. There is no anomaly or ghost in the behaviour of the Hall Effect in superconductors: the change of the sign of the Hall Effect fits well into the framework of the semiclassical phenomenological model with two types of current carriers of opposite signs, and the appearance of so-called ghost magnetic field fits into the framework of this model with two types of current carriers of the same sign.

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