THE APPEARANCE OF A SELF-FOCUSING NONLINEARITY IN A NEAR-SURFACE LAYER OF A CRYSTAL IN DEPENDENCE OF ELECTRIC FIELD

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Abstract. We proposed a model that describes the appearance of a self-focusing nonlinear response near the surface of the initially linear crystal, in dependence of electric field. We obtain exact analytical solution of the nonlinear wave equation describing the new type of nonlinear surface wave. The wave consists of sandwiched between two exponentially decaying fields, the nonlinear soliton-like part in the optical domain, when the electric field exceeds the critical value. The total power flow of the surface wave is calculated analytically. The redistribution of the wave power flows in the crystal regions is also analyzed.

Key words: nonlinear optics, nonlinear dielectric permittivity, nonlinear surface wave, stepwise nonlinearity.

1. INTRODUCTION

Nonlinear properties of crystals are often used in various optoelectronic technical applications [1–10]. Therefore the studies of various types of nonlinear surface waves and solitons have been actively continuing for decades [11–35].

The authors of [36, 37] reported that the laser beam radiation propagating along the boundaries of semiconductor crystals can significantly change the optical properties of the border regions and the condition of the wave propagation. The optical nonlinear properties of such semiconductor crystals are especially pronounced in the exciton region of the spectrum, when the characteristic relaxation times of excitons and biexcitons are very small and they are comparable in order with the picoseconds [38–40]. In order to theoretically describe such phenomena, the nonlinearity models with abruptly changing dielectric function in dependence of electric field were proposed [41–49].

In this paper, we describe the peculiarities of the formation of an optical domain with self-focusing optical property near the surface of the crystal, which initially has no nonlinear response. The nonlinear Kerr-type self-focusing response appears when the electric field amplitude exceeds the critical value (in what follows is called the switching field). We obtain a new type of nonlinear surface wave consisting of nonlinear soliton-like part sandwiched between two exponentially decaying fields.

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This paper is organized as follows. The main equations and the modified stepwise nonlinearity model are presented in Sec. 2. Here we derive the solutions of the formulated nonlinear equation describing the nonlinear surface. In addition, the power flow is calculated. In Sec. 3, we detaily compare the new type of surface wave and its properties with the previously obtained results. Finally, in Sec. 4 we present our conclusions.

2. SURFACE WAVE CHARACTERISTICS

Let us consider the planar contact (free surface) of linear optical medium characterized by the dielectric constant $\varepsilon_0$ with nonlinear crystal characterized by dielectric permittivity $\varepsilon_N$ depending on electric field amplitude $E$ as follows. Initially, the whole crystal is a linear one and is characterized by the dielectric constant $\varepsilon_1$. There is no nonlinear response in the crystal initially. The electric field strength increases after the surface wave is excited. After the electric field strength reaches the threshold value of the switching field $E_s$, a localized zone (optical domain) at a distance of $z_s$ from the surface is formed in the crystal. We denote by $z_s$ the point where the electric field is equal to the threshold value of the switching field: $|E(z_s)| = E_s$. Now the crystal inside the domain of $z_s$ width is characterized by a self-focusing nonlinearity with the nonlinear dielectric permittivity $\varepsilon_N(E) = \varepsilon_1 + \alpha_2 |E|^2$, where $\varepsilon_2$ is the unperturbed dielectric constant of domain, and $\alpha_2$ is the positive nonlinearity coefficient of domain. Hence, the linear crystal becomes a nonlinear one in a finite near-surface zone after the surface wave is excited.

We consider only transverse-electric (TE) nonlinear waves propagating along the surface with the components of electromagnetic field: $E_x = E_z = 0$, $H_y = 0$ and $E_j(x, z) = E(z) \exp \left( i (kx - \omega t) \right)$, where $k$ is the wave number, $\omega$ is the wave frequency, $c$ is the speed of light. From Maxwell equations we derive the nonlinear differential equation describing the electric field distribution $E(z)$ in the transverse interface direction [47, 48]

$$E''(z) + \{\omega^2 \varepsilon(z, |E|)/c^2 - k^2\} E(z) = 0,$$

where the new form of the dielectric permittivity $\varepsilon(z, |E|)$ is defined by the function

$$\varepsilon(z, E) = \begin{cases} 
\varepsilon_1, & |E| < 0 \\
\varepsilon_N(|E|), & |E| > 0 
\end{cases},$$

where

$$\varepsilon_N(|E|) = \begin{cases} 
\varepsilon_1, & |E| < E_s \\
\varepsilon_2 + \alpha_2 |E|^2, & |E| > E_s 
\end{cases}.$$  

(1)
We use the boundary conditions derived in Refs [41, 45, 47, 48]. We consider the surface wave satisfying the condition of extinction at infinity: \( |E(z)| \to 0 \) at \( |z| \to \infty \).

Let the amplitude of electric field at the surface \( E_0 = E(z = 0) \) exceeds the value of the switching field \( E_s \), because it is necessary that \( E_0 > E_s \) for the optic domains formation [41].

Thus, in the range \( n^2 > \max\{\epsilon_{0,1,2}\} \) (\( n = ck/\omega \) is the effective refractive index) the nonlinear surface wave of TE polarization propagating along \( x \) direction consists of three parts corresponding to three different regions, and it can be exactly calculated as

\[
E(z) = \begin{cases} 
E_0 e^{\alpha z}, & z < 0, \\
\frac{\sqrt{2(n^2 - \epsilon_z)}}{\sqrt{\epsilon_z} \cosh(q_z(z - z_s))} |E| > E_s, & 0 < z < z_s, \\
E_s e^{-q_i(z-z_i)}, & |E| < E_s, z > z_s,
\end{cases}
\]  \hspace{1cm} (3)

where

\[
q_j = (n^2 - \epsilon_j)^{1/2} c / \omega, \quad j = 0, 1, 2,
\]  \hspace{1cm} (4)

\[
E_0 = \left( 2 \frac{\epsilon_0 - \epsilon_2}{\alpha_2} \right)^{1/2},
\]  \hspace{1cm} (5)

\[
z_s = \frac{c}{\omega \sqrt{(n^2 - \epsilon_2)}} \arctanh \left( \frac{n^2 - \epsilon_0}{n^2 - \epsilon_2} \right),
\]  \hspace{1cm} (6)

\[
z_s = \frac{c}{\omega \sqrt{(n^2 - \epsilon_2)}} \left\{ \arctanh \left( \frac{n^2 - \epsilon_0}{n^2 - \epsilon_2} \right) + \arctanh \left( \frac{n^2 - \epsilon_1}{n^2 - \epsilon_2} \right) \right\},
\]  \hspace{1cm} (7)

In addition, we obtain from the boundary conditions and the field (3) that the threshold value of the switching field is not an arbitrary parameter, but is completely determined by the crystal properties, as follows

\[
E_s = \left( 2 \frac{\epsilon_1 - \epsilon_2}{\alpha_2} \right)^{1/2}.
\]  \hspace{1cm} (8)

We derive from Eqs. (5) and (8) that the surface wave (3) exists only when \( \epsilon_2 < \epsilon_1 < \epsilon_0 < n^2 \).

The surface wave (3) field distributions are shown in Fig. 1 with different values of effective refractive index. The maximum of surface wave inside the domain increases with increasing the effective refractive index.
The distribution of the electric field $E$ along the $z$-axis of the surface wave (3) with $c = 1; \omega = 1; \varepsilon_0 = 2.5; \varepsilon_1 = 1.5; \varepsilon_2 = 1.8; \alpha_2 = 1$ in conventional units; line (1) – $n = 3$; line (2) – $n = 4$; line (3) – $n = 5$.

The dependence of domain thickness (7) on the effective refractive index is shown in Fig. 2. The domain thickness monotonically decreases with increasing the effective refractive index.

The total power flow of the surface wave (3) can be written as $P = P_0 + P_1 + P_2$, where

$$P_0 = \frac{E_0^2}{2q_0} = \frac{c}{\alpha_2 \omega} \frac{\varepsilon_0 - \varepsilon_2}{\sqrt{n^2 - \varepsilon_0}}$$  (9)

is the power component in the linear dielectric;

$$P_1 = \frac{E_1^2}{2q_1} = \frac{c}{\alpha_2 \omega} \frac{\varepsilon_1 - \varepsilon_2}{\sqrt{n^2 - \varepsilon_1}}$$  (10)
is the power component outside the domain;

\[ P_0 = \frac{q_0 + q_1}{2} = \frac{2c}{\alpha_2} \left( \sqrt{n^2 - \varepsilon_0} + \sqrt{n^2 - \varepsilon_1} \right) \tag{11} \]

is the power component inside the domain.

The power components \( P_0 \) (9) and \( P_1 \) (10) monotonically decrease to zero value with increasing the effective refractive index. The power component \( P_2 \) inside the domain (11) monotonically increases with increasing \( n \). The total power \( P \) monotonically increases from its minimum value with increasing effective refractive index.

The dependences of relative power flows \( \delta P_j = P_j / P \) \( (j = 0, 1, 2) \), which allow us to analyze the redistribution of energy in the crystal regions, are shown in Fig. 3.

![Fig. 3 – The dependencies of relative power flows (9)–(11) on effective refractive index with values of the model parameters as in Fig. 1.](image)

We obtain that the largest percentage of radiation is concentrated within the domain, because \( \delta P_2 > \delta P_{0,1} \) (see Fig. 3). In addition, we note that \( \delta P_2 \gg \delta P_{0,1} \).

The fraction of radiation in the linear dielectric exceeds the fraction of radiation in the crystal outside the domain, because \( \delta P_0 > \delta P_1 \).

3. DISCUSSION

Now we want to compare the results obtained in this paper with the results reported in [47], where the Eq. (1) was solved in the case of a different form of nonlinearity. In [47] instead (2), the dielectric permittivity was written as:
Despite the fact that (12) transforms into (2) with \( \alpha_1 = 0 \), the solution of Eq. (1) with nonlinearity (12) with \( \alpha_1 > 0 \), has a different form outside the domain and it is expressed by hyperbolic cosine. The surface wave in this case was exactly calculated as \[47\]

\[
E(z) = \begin{cases} 
\sqrt{2} \frac{\varepsilon_0 - \varepsilon_2}{\alpha_2} e^{\varepsilon_2 z}, & z < 0, \\
\frac{\sqrt{2} (n^2 - \varepsilon_2)}{\sqrt{\alpha_2 \cosh(q_2 (z - z_s))}}, & \varepsilon_2 < E_s, \ 0 < z < z_s, \\
\frac{\sqrt{2} (n^2 - \varepsilon_2)}{\sqrt{\alpha_1 \cosh(q_1 (z - z_s))}}, & E_s < \varepsilon_2, \ z > z_s,
\end{cases}
\]

where

\[
z_s = \frac{c}{\omega \sqrt{n^2 - \varepsilon_2}} \left\{ \arccosh\left( \frac{\alpha_2 - \alpha_1}{\alpha_1} \frac{n^2 - \varepsilon_1}{\varepsilon_1 - \varepsilon_2} \right) + \arccosh\left( \frac{n^2 - \varepsilon_2}{\varepsilon_0 - \varepsilon_2} \right) \right\}.
\]

First, we note that the surface waves (3) and (13) exist under different relationship between unperturbed dielectric constants. We remind that the surface wave (3) exists only when \( \varepsilon_2 < \varepsilon_1 < \varepsilon_0 \) unlike the surface wave (13) exists under one of the following set of conditions: \( \varepsilon_2 > \varepsilon_1 \) and \( \alpha_1 > \alpha_2 \) or vice versa. Hence, the obtained in this paper surface wave can be excited only if the unperturbed dielectric constant of the initially linear crystal is greater than the one of the appearing domain with self-focusing nonlinear response. If the crystal initially had a nonlinear self-focusing response, then the opposite situation is also possible.

Secondly, we note that the difference \( \Delta E \) between the field distributions of surface waves (3) and (13) outside the domain where \( E \approx E_s \) is very small compared to the average field amplitude in this region near the domain boundary (see Fig. 4). The ratio \( \Delta E / E \) is about 0.05–0.1%. However, the difference \( \Delta z_s \) between domain
thicknesses (7) and (16) is not a small quantity (see Fig. 5), and the ratio $\Delta z_s/z_s$ is about 10–12%. The domain thicknesses (7) formed at the surface of initially linear crystal is greater than the domain thicknesses (16) formed at the surface of initially nonlinear self-focusing crystal.

Finally, we compare our solution with the results reported in [41], where the linear medium with a jump change only in the unperturbed dielectric constant was considered. The authors of [41] used the simplest nonlinearity model with dielectric permittivity

$$
\varepsilon_{\kappa}(|E|) = \begin{cases} 
\varepsilon_1, & |E| < E_s, \\
\varepsilon_2, & |E| > E_s,
\end{cases}
$$

(18)
which can be obtained from (2) with $\alpha_2 = 0$. The equation (1) with (18) splits into two linear wave equations connected at the domain boundary in the crystal. Therefore, the surface wave inside the domain is described by the solution of linear wave equation, which can be written as [41]

$$E(z) = E_s \left( \frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_2 - n^2} \right)^{1/2} \cos(p_2 z - \varphi), |E| > E_s,$$

where $p_2 = (\varepsilon_2 - n^2)^{1/2} \omega/c$, $\varphi = \arccos((\varepsilon_2 - n^2)/(\varepsilon_2 - \varepsilon_0))^{1/2}$, and the switching field $E_s$ is an arbitrary parameter unlike that obtained in this paper. The obtained in [41] components of the surface wave in the linear medium and outside the domain are described by the same expression (3).

The main difference between the nonlinearity models (2) and (18) lies in the different ranges of existence of the surface waves. The surface wave (19) has the effective refractive index in the range $\varepsilon_1 < \varepsilon_0 < n^2 < \varepsilon_2$, unlike we have obtained for the surface wave (3) the following range of existence: $\varepsilon_2 < \varepsilon_1 < \varepsilon_0 < n^2$. Therefore, such nonlinearity models are applicable to the crystals with essentially different optical characteristics.

4. CONCLUSIONS

In this paper, we have put forward a model describing the appearance of a self-focusing nonlinear response near the surface of the crystal, which initially was a linear one. This phenomenon is caused by an increase in the strength of the electric field near the crystal surface. We find the analytical solution of the nonlinear wave equation describing this new type of nonlinear surface wave. Such surface wave consists of sandwiched between two exponentially decaying fields, the nonlinear soliton-like part in the optical domain, when the electric field exceeds the critical value. The domain thickness monotonically decreases with increasing the effective refractive index.

The power flow of the surface wave was calculated analytically. The redistribution of energy in the crystal regions was analyzed. The largest percentage of radiation is concentrated within the domain. The fraction of radiation in the linear dielectric exceeds the fraction of radiation in the crystal outside the domain.

A detailed comparison with previously obtained results has been carried out. We have compared the solution obtained in this paper with the solution derived for the case of a different nonlinearity model in which the Kerr nonlinearity coefficient is jump changed with the increasing electric field. In addition, we have compared our solution with the solution of a model of a linear medium with a jump change only in the unperturbed dielectric constant.
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