INTERFEROMETRIC POWER OF GAUSSIAN SYSTEMS IN A SQUEEZED THERMAL BATH

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Abstract. We describe the Markovian dynamics of the Gaussian interferometric power of two-mode Gaussian states, for a system composed of two bosonic modes, each one interacting with its squeezed thermal bath, in the framework of the theory of open systems based on completely positive quantum dynamical semigroups. The time evolution of the interferometric power is described in terms of the covariance matrix for Gaussian initial states. The behaviour of the Gaussian interferometric power depends on the initial state of the subsystem (squeezing parameter and thermal photon numbers) and on the parameters characterizing the squeezed thermal reservoirs (temperatures, dissipation coefficient, squeezed parameters of the baths and squeezing angle). We show that, independently of the initial state, the Gaussian interferometric power is monotonically decreasing in time, and in the limit of large times it asymptotically decreases to a zero value.

Key words: Quantum interferometric power, Gaussian states, open systems, squeezed thermal baths.

1. INTRODUCTION

The research on the dynamics of open quantum systems is constantly receiving high attention, in the perspective of potential applications to quantum information theory and quantum technologies like cryptography, imaging, metrology, and optical interferometry. The dynamics of open systems explores the system-environment interaction, which can exist in a large variety of situations – it can be weak or strong, and one can have a long system relaxation, that involves a transfer of information from the system to its surrounding environment. In the weak coupling case the dynamics of open quantum systems is described using a master equation in the Lindblad form, applying the Born-Markov approximation, while in the strong coupling case the dynamics of open systems involves shorter relaxation times compared to the environment correlation time-scales. Recent studies, focused on the characterization of open non-Markovian quantum dynamics systems, highlighted the deviation from the dynamical semigroup properties of the completely positive trace preserving quantum

maps. The divisibility of completely positive trace preserving quantum maps is a defining feature of Markovian dynamics, and any deviation from this feature indicates a non-Markovian behaviour of quantum information properties, such as distinguishability, entanglement, quantum mutual information, interferometric power [1].

The interferometric power of a system consisting of two parties was analysed by Adesso in Ref. [2], using the quantum Fisher information, and it was obtained a closed formula for this operational quantifier for all two-mode Gaussian states, by restricting to Gaussian local dynamics.

The comprehension of different kinds of quantum correlations, like steering, entanglement, nonlocality is rapidly advancing. The genuine properties of quantum states, different form classical ones, are described by these quantum correlations, which became very useful in quantum information processing. The implementation of quantum information protocols and tasks into real systems is a challenging procedure, since they are always interacting with the surrounding environment. Therefore, it is compulsory to take into consideration decoherence and dissipation, in order to get a realistic description of quantum processes. In the last years decoherence and dynamics of quantum correlations in open systems of continuous variables have been intensively studied [3–11]. Recently we studied, in the framework of the theory of open systems based on completely positive quantum dynamical semigroups, the dynamics of quantum correlations of both uncoupled and, respectively, coupled two bosonic modes embedded in a common thermal reservoir, or in the two-reservoir model, for initial Gaussian states of the subsystem [12–17].

In the previous work [18] we described the dynamics of the interferometric power in a system composed of two bosonic modes immersed in a thermal reservoir, in the framework of the theory of open systems based on completely positive quantum dynamical semigroups. In the present work we study, in the same framework, the Gaussian interferometric power in a subsystem composed of two bosonic modes, each one interacting with its own squeezed thermal bath. The paper is organized as follows. In Sec. 2 we define the interferometric power and describe its properties. In Sec. 3 we write the Markovian master equation for the density operator of the considered open system interacting with a squeezed thermal environment and the evolution equation for the covariance matrix. From this equation we obtain the time evolution for the covariance matrix of the state of the bimodal bosonic system. Then we describe in Sec. 4 the time evolution of the Gaussian interferometric power and show that, independent of the initial state, the Gaussian interferometric power is monotonically decreasing in time, and in the limit of large times it asymptotically decreases to a zero value. A summary is given in Sec. 5.

2. GAUSSIAN INTERFEROMETRIC POWER

The interferometric power (IP) and its properties are defined here according to Ref. [2]. We denote by ρ_{AB} the input state of the interferometer, corresponding to a bosonic continuous variable system of modes A and B. Mode A, which enters a black box, undergoes the following unitary transformation:

$$U_A^{\phi} = \exp(i\phi H_A)\,. \tag{1}$$

We denote by ρ_{AB}^{ϕ} the transformed state of the two modes after the black box. In order to construct an estimator $\phi_{\rm est}$ for the parameter ϕ, one performs a measurement on the output state, that can be iterated κ times for the purpose of a higher statistical accuracy of the estimator. The Cramér-Rao bound [19] introduces the following constraint on the variance $\Delta\phi^2 \equiv \langle(\phi_{\rm est}-\phi)^2\rangle$ of any estimator for the parameter ϕ:

$$\kappa\Delta\phi^2 \geq \frac{1}{\mathcal{F}(\rho_{AB}^{\phi})}\,, \tag{2}$$

where the quantum Fisher information [19, 20] is defined by [21, 22]

$$\mathcal{F}(\rho_{AB}^{\phi}) = -2\lim_{\epsilon\to 0}\frac{\partial^2 F(\rho_{AB}^{\phi},\rho_{AB}^{\phi+\epsilon})}{\partial\epsilon^2}\,, \tag{3}$$

using Uhlmann fidelity [23]

$$F(\rho_1,\rho_2) = \left\{\mathrm{Tr}\left[(\sqrt{\rho_1}\rho_2\sqrt{\rho_1})^{\frac{1}{2}}\right]\right\}^2. \tag{4}$$

Quantum Fisher information quantifies the precision for the estimation of the parameter ϕ, using the input probe state ρ_{AB} and a specific optimized measurement on the output state ρ_{AB}^{ϕ}.

The expression of IP of the state ρ_{AB} with respect to the probing mode A is given by

$$P^A(\rho_{AB}) = \frac{1}{4}\inf\mathcal{F}(\rho_{AB}^{\phi})\,. \tag{5}$$

Here the minimization runs over all possible choices of local dynamics generated by a Hamiltonian H_A with harmonic spectrum, so that $P^A(\rho_{AB})$ measures the worst-case precision guaranteed by using ρ_{AB} as a probe. Probe states ρ_{AB} with higher IP provide more reliable resources to be used in metrology, since they ensure a smaller variance in the estimation of ϕ.

We assume that the probe states are two-mode Gaussian states and the local dynamics U_A^{ϕ} is Gaussian, which keeps the Gaussian form of the states. This restriction implies that the generator H_A has to be at most quadratic in the canonical operators. We introduce the vector of quadrature operators $\boldsymbol{O} = \{q_A, p_A, q_B, p_B\}$, which satisfy the commutation relations $[O_j, O_k] = i\Omega_{jk}$ ($\hbar = 1$), where $\Omega = \left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)^{\oplus 2}$ is the

two-mode symplectic form [24, 25]. The first and second moments of the quadrature operators completely specify a Gaussian state ρ_{AB} [24]. Without loss of generality, one can consider states with zero first moments, since the first moments can be adjusted by local displacements and the IP is invariant under local unitary operations. Then a Gaussian state can completely be determined by its covariance matrix

$$\sigma_{AB} = (\sigma_{jk}) \equiv \begin{pmatrix} \alpha & \gamma \\ \gamma^{\mathrm{T}} & \beta \end{pmatrix}, \tag{6}$$

where $\sigma_{jk} = \mathrm{Tr}[\rho_{AB}\{O_j, O_k\}_+]$ (with $j,k = 1,\ldots,4$), 2×2 Hermitian matrices α and β are the covariance matrices for the single modes, and γ contains the cross-correlations between the modes. The Robertson-Schrödinger uncertainty relation

$$\sigma_{AB} + i\Omega \geq 0 \tag{7}$$

ensures the physicality of the states.

The following formula was obtained in Ref. [2] for the Gaussian IP of a two-mode Gaussian probe with covariance matrix σ_{AB}, consisting of four local symplectic invariants $A = \det\alpha$, $B = \det\beta$, $C = \det\gamma$ and $D = \det\sigma_{AB}$:

$$P_G^A(\sigma_{AB}) = \frac{X + \sqrt{X^2 + YZ}}{2Y}, \tag{8}$$

where

$$\begin{aligned} X &= (A+C)(1+B+C-D) - D^2, \\ Y &= (D-1)(1+A+B+2C+D), \\ Z &= (A+D)(AB-D) + C(2A+C)(1+B). \end{aligned}$$

IP has a series of interesting properties. Gaussian IP takes a zero value if and only if the state is a zero-discord state (classical-quantum state) [26]. It is known that in the Gaussian case, under the constraint of bounded mean energy per mode, the only classical-quantum states are product states [27–29]. In the case of two-mode Gaussian states, Gaussian IP vanishes only if they are product states, characterized by the invariants C = 0 and D = AB. Therefore, all correlated two-mode Gaussian states are useful for black-box optical interferometry, since quantum Fisher information has nonzero values for any possible local Gaussian dynamics. In addition, Gaussian IP is invariant under local unitary operations, and it is monotonically nonincreasing under arbitrary Gaussian quantum operations on subsystem B, so that the Gaussian IP is a faithful measure of bipartite discord-type correlations [26] for Gaussian states. The Gaussian IP is not symmetric under swapping of the two modes A and B for general two-mode Gaussian states, and it reduces to a Gaussian entanglement monotone [24] on pure states.

3. MASTER EQUATION FOR TWO BOSONIC MODES INTERACTING WITH SQUEEZED THERMAL BATHS

The time evolution of a system consisting of two bosonic modes, each one interacting weakly with its squeezed thermal bath, is given by the following Markovian master equation for the density operator $\rho(t)$ in the interaction representation [30–33]:

$$\dot{\rho(t)} = \sum_{k=1}^{2} \frac{\lambda}{2} \{(N_k+1)\mathcal{L}[a_k] + N_k \mathcal{L}[a_k^\dagger] - M_k^* \mathcal{D}[a_k] - M_k \mathcal{D}[a_k^\dagger]\}\rho, \tag{9}$$

where a_k and $a_k^\dagger$ are the annihilation and, respectively, creation operators of the two modes, and λ is the overall dissipation rate. In Eq. (9) $\mathcal{L}[O]\rho = 2O\rho O^\dagger - O^\dagger O\rho - \rho O^\dagger O$ and $\mathcal{D}[O]\rho = 2O\rho O - OO\rho - \rho OO$ are the Lindblad superoperators. $N_k \in \mathcal{R}$ and $M_k \in \mathcal{C}$ represent the effective photon numbers and the squeezing of the baths, respectively. The positivity of the density matrix imposes the constaint $|M_k|^2 \leq N_k(N_k+1)$. For $M_k = 0$ the bath is at thermal equilibrium, and N_k coincides with the average number of thermal photons in the bath. Otherwise, the bath is said to be squeezed, or phase sensitive.

The Hamiltonian of two bosonic modes A and B of identical mass $m_A = m_B = 1$ and frequencies $\omega_A = \omega_B = 1$ is given by

$$H = \frac{1}{2}(p_A^2 + p_B^2 + q_A^2 + q_B^2). \tag{10}$$

The equation of motion for the covariance matrix of the canonical observables q_A, q_B and p_A, p_B is the following (T denotes the transposed matrix) [33]:

$$\frac{d\sigma(t)}{dt} = Z\sigma(t) + \sigma(t)Z^{\mathrm{T}} + 2D, \tag{11}$$

where

$$Z = \begin{pmatrix} -\lambda & 1 & 0 & 0 \\ -1 & -\lambda & 0 & 0 \\ 0 & 0 & -\lambda & 1 \\ 0 & 0 & -1 & -\lambda \end{pmatrix} \tag{12}$$

and D is the matrix of diffusion coefficients.

The time-dependent solution of Eq. (11) is given by [33]

$$\sigma(t) = e^{-\lambda t}\sigma(0) + (1 - e^{-\lambda t})\sigma(\infty), \tag{13}$$

where the asymptotic covariance matrix

$$\sigma_\infty = \begin{pmatrix} 1+2(N_1+M_{1R}) & 2M_{1I} & 0 & 0 \\ 2M_{1I} & 1+2(N_1-M_{1R}) & 0 & 0 \\ 0 & 0 & 1+2(N_2+M_{2R}) & 2M_{2I} \\ 0 & 0 & 2M_{2I} & 1+2(N_2-M_{2R}) \end{pmatrix} \tag{14}$$

is determined only by the bath parameters (M_{kR} and M_{kI}, $(k = 1, 2)$ represent the real and the imaginary part of M_k). Here

$$N_k = n_{th,k}(\cosh^2 R + \sinh^2 R) + \sinh^2 R, \tag{15}$$

$$M_k = -(2n_{th,k} + 1)\cosh R \sinh R \exp i\varphi, \tag{16}$$

and (we set Boltzmann constant $k_B = 1$)

$$n_{th,k} = \frac{1}{2}(\coth \frac{1}{2T_k} - 1), k = 1, 2, \tag{17}$$

where R is the squeezing parameter of the baths, $n_{th,k}$ are the photon numbers of the baths, T_k their temperatures and φ is the squeezing phase.

4. DYNAMICS OF GAUSSIAN INTERFEROMETRIC POWER

We study the dynamics of the Gaussian IP for the considered system, by taking an initial squeezed thermal state, with the covariance matrix of the form:

$$\sigma(0) = \begin{pmatrix} a & 0 & c & 0 \\ 0 & a & 0 & -c \\ c & 0 & b & 0 \\ 0 & c & 0 & b \end{pmatrix}, \tag{18}$$

where

$$a = 2n_1 \cosh^2 r + 2n_2 \sinh^2 r + \cosh 2r, \tag{19}$$

$$b = 2n_1 \sinh^2 r + 2n_2 \cosh^2 r + \cosh 2r, \tag{20}$$

$$c = 2(n_1 + n_2 + 1)\sinh 2r \tag{21}$$

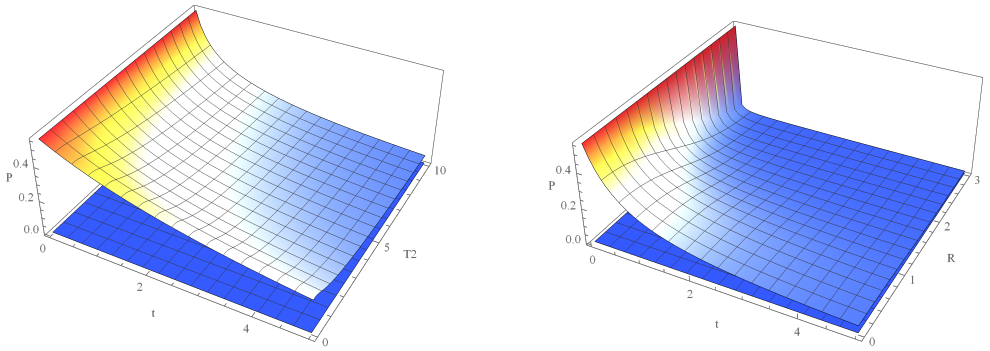
and r is the squeezing parameter of the initial state.

In Fig. 1 it is illustrated the dependence of the Gaussian IP P (8) as a function of time t and (a) temperature of the second squeezed thermal bath T_2 and (b) the squeezing parameter of the baths R. One can notice that the Gaussian IP is decreasing in time and is also decreasing by increasing the temperature of the second bath T_2 (by keeping fixed the temperature of the first bath) and the squeezing parameter R.

In Fig. 2 we represent the dependence of the Gaussian IP P as a function of squeezing angle φ and (a) time t and (b) thermal photon number n_1. One can notice that the Gaussian IP has an oscillating behaviour with squeezing angle φ and is increasing with thermal photon number of the first bosonic mode n_1 (by keeping fixed the thermal photon number of the second mode).

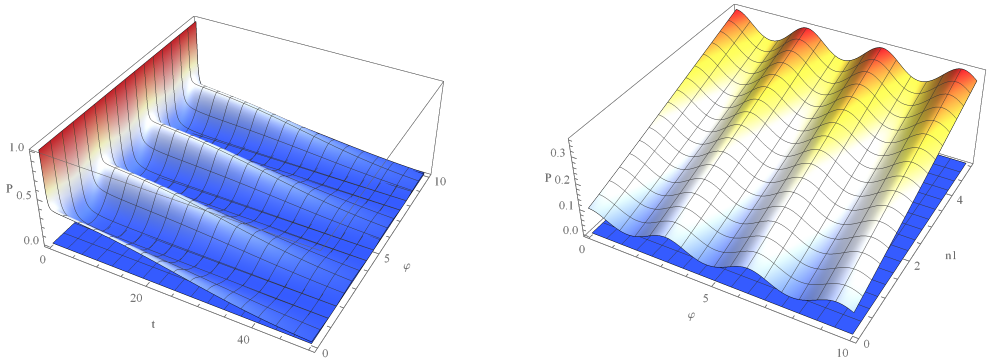
In Fig. 3 it is shown the dependence of the Gaussian IP P as a function of time t and (a) thermal photon number of the second bosonic mode n_2 (by keeping

fixed the thermal photon number of the first mode) and (b) squeezing parameter r of initial state of the two bosonic modes. One can notice that the Gaussian IP is increasing with increasing both the photon number of the second bosonic mode n_2 and the squeezing parameter r.



(a) $P(t, T_2)$ (b) $P(t, R)$

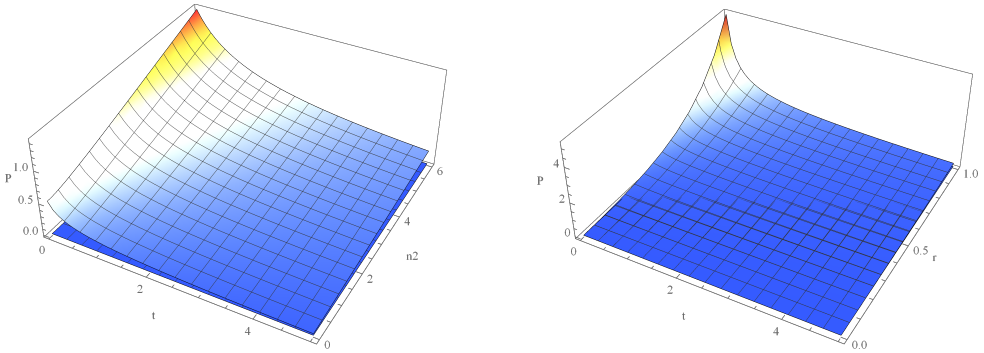
Fig. 1 – Gaussian IP P *versus* time t and (a) temperature T_2 of the second bath (for $r = 0.5, R = 0.5, T_1 = 0.5, \varphi = 5, n_1 = n_2 = 0.5, \lambda = 0.1$) and (b) squeezing parameter of the baths R (for $r = 0.5$, $T_1 = 1$, $T_2 = 10$, $\varphi = 5$, $n_1 = n_2 = 0.5$, $\lambda = 0.1$).



(a) $P(t, \varphi)$ (b) $P(\varphi, n_1)$

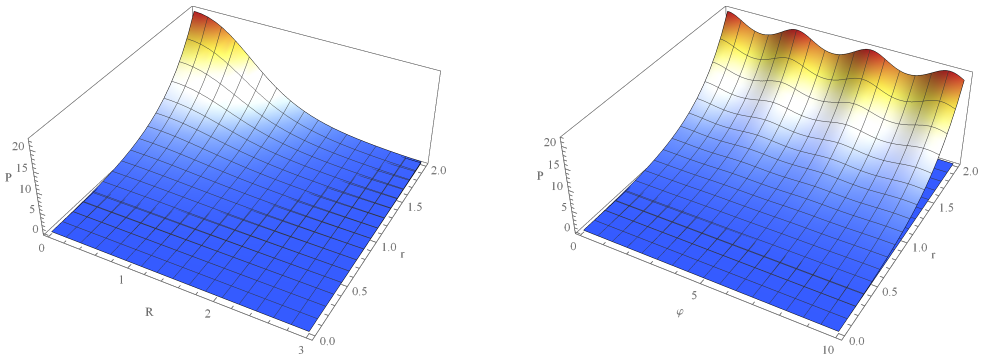
Fig. 2 – Gaussian IP P *versus* time t and (a) squeezing angle φ (for $r = 0.5, R = 2, T_1 = 1, T_2 = 10, n_1 = 1, n_2 = 5, \lambda = 0.1$) and (b) thermal photon number n_1 (for $t = 5, r = 0.5, R = 0.5, T_1 = T_2 = 2, n_2 = 0.5, \lambda = 0.1$).

In Fig. 4 it is shown the dependence of the Gaussian IP P as a function of squeezing parameter r of the initial state of the two bosonic modes and (a) squeezing parameter of the baths R and (b) squeezing angle φ. One can notice again that the Gaussian IP is increasing with the squeezing parameter r, is decreasing with increasing the squeezing parameter R and is oscillating with squeezing angle φ.



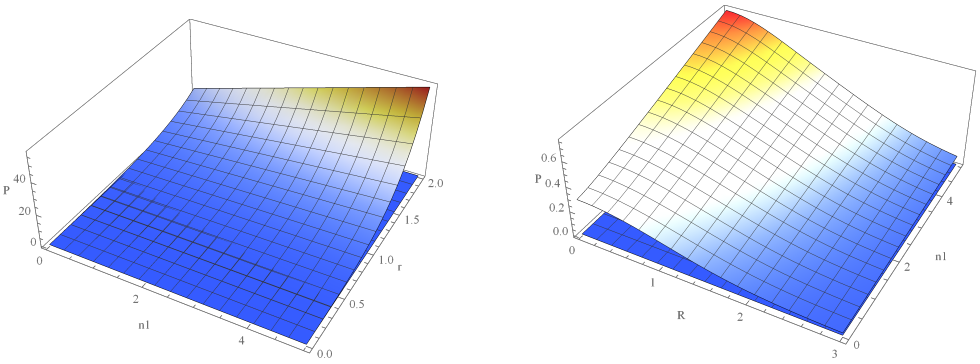
(a) $P(t, n_2)$ (b) $P(t, r)$

Fig. 3 – Gaussian IP P *versus* time t and (a) thermal photon number of the second bosonic mode n_2 (for $r = 0.5, R = 0.5, T_1 = 1, T_2 = 10, \varphi = 5, n_1 = 0.5, \lambda = 0.1$) and (b) squeezing parameter of the initial state r (for $R = 0.5, T_1 = 1, T_2 = 10, \varphi = 5, n_1 = n_2 = 0.5, \lambda = 0.1$).



(a) $P(r, R)$ (b) $P(r, \varphi)$

Fig. 4 – Gaussian IP P *versus* squeezing parameter of initial state of two bosonic modes r and (a) squeezing parameter of the baths R (for $t = 0.5, T_1 = 1, T_2 = 10, \varphi = 5, n_1 = n_2 = 0.5, \lambda = 0.1$) and (b) squeezing angle of the baths φ (for $t = 0.5, R = 0.5, T_1 = 1, T_2 = 10, n_1 = n_2 = 0.5, \lambda = 0.1$).



(a) $P(n1, r)$ (b) $P(n1, R)$

Fig. 5 – Gaussian IP P *versus* thermal photon number n_1 and (a) squeezing parameter r of the initial state of the system (for $t = 0.5, R = 0.5, T_1 = 1, T_2 = 10, \varphi = 5, n_2 = 0.5, \lambda = 0.1$) and (b) squeezing parameter of the baths R (for $t = 0.5, r = 0.5, T_1 = 1, T_2 = 10, \varphi = 5, n_2 = 0.5, \lambda = 0.1$.)

In Fig. 5 it is shown the dependence of the Gaussian IP P as a function of thermal photon number of the first bosonic mode n_1 and (a) squeezing parameter r of initial state of the two bosonic modes and (b) the squeezing parameter of the baths R. One can notice once more that the Gaussian IP is increasing with the thermal photon number of the first bosonic mode n_1 (by keeping fixed the thermal photon number of the second mode n_2), with the squeezing parameter r and is decreasing with increasing the squeezing parameter of the baths R.

5. SUMMARY

We investigated the Markovian dynamics of the Gaussian interferometric power of two-mode Gaussian states, for a system composed of two bosonic modes, each one interacting with its squeezed thermal bath, in the framework of the theory of open systems based on completely positive quantum dynamical semigroups. This dynamics was described in terms of the covariance matrix for Gaussian input states. The behaviour of the Gaussian interferometric power depends on the initial state of the subsystem (squeezing parameter and thermal photon numbers) and on the parameters characterizing the squeezed thermal reservoirs (temperatures, dissipation coefficient, squeezed parameter of the baths and squeezing angle). Independently of the initial state, in the limit of large times the Gaussian interferometric power decays asymptotically to a zero value.

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