TRAPPED ELECTRON MODE TURBULENCE: TEST MODES APPROACH

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Abstract. We perform a test mode analysis for the case of the dissipative TEM instability in slab geometry, by studying the influence of the statistical properties of a turbulent background on the frequencies and growth rates of the test modes. Our approach naturally incorporates the ion trajectory diffusion and ion stochastic trapping present already in the quasilinear and, respectively, weakly nonlinear stages of the turbulence evolution.

Key words: Trapped electron modes, turbulence evolution, test modes.

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1. INTRODUCTION

The understanding of turbulence evolution in tokamak plasmas is a very active topic in both theoretical and experimental fusion research. In magnetically confined plasmas, the particle and heat transport is strongly influenced by the low frequency drift type turbulence, driven by the gradients of the temperature, density, magnetic field, etc. The most relevant drift instabilities are the ion temperature gradient (ITG), the electron temperature gradient (ETG) and the dissipative and collisionless trapped electron modes (DTEM and CTEM) [1–6].

In this work, we will limit ourselves to the study of the trapped electron modes. DTEMs are caused by a strong temperature gradient and large collisionality, being associated with the edge of the tokamak, while CTEMs are generated by the electron curvature drift resonance, being relevant in the core [7]. In the context of tokamak plasma experiments, TEMs play an important role in advanced confinement regimes with electron transport barriers and in the hot electron regime, relevant for experiments with dominant central electron heating (see Ref. [8] and references therein).

We present a study of DTEMs in turbulent plasma that is focused on the effect of trajectory trapping in the structure of the background turbulence. The latter is due to the stochastic particle advection caused by the electric drift, which leads to trajectory eddying. The statistics of trajectories may strongly deviate from Gaussianity in the presence of trapping. We follow the philosophy of [12–14, 16]: starting from the drift kinetic equations, we obtain a dispersion relation for test modes which is dependent on the statistical properties of the background turbulence. In turn, the frequencies and growth rates of the test modes will provide the tendencies of the turbulence evolution.

In the first Section we discuss the electron and ion responses to a perturbation of the potential for the DTEM case, in a drift kinetic framework. In Section 2, we will analyze the resulting dispersion relation for the test modes, and in the final Section we will discuss the results.

2. TEM-SLAB

The basic physical mechanisms of the DTEM instability may be evidenced even in the absence of the complexities inherent to the toroidal geometry. Indeed, the simplest setup in which the instability manifests itself consists of an essentialy straight magnetic field, supplemented by two localised magnetic mirrors ensuring the population of trapped electrons [9, 10].

Our model consists of a low β plane plasma slab. The magnetic field is directed along the z axis, *i.e.* $\mathbf{B} = B\mathbf{e}_z$, and the plasma nonuniformity is considered along the x axis, *i.e.* $n_0 = n_0(x)$, $T_e = T_e(x)$. The characteristic density and temperature lengths, $L_n = n_0 \left|dn_0/dx\right|^{-1}$ and $L_{T_e} = T_e \left|dT_e/dx\right|^{-1}$ are much larger than the wavelengths of the drift modes. At $z = \pm L$, there are two well localized, perfectly reflecting magnetic mirrors which confine a fraction $\delta < 1$ of the total electron population. The untrapped electrons and ions are allowed to flow in the whole plasma volume, extending between $z = \pm L'$.

We start from the gyrokinetic equations for electrons and ions in the case of a Larmor radius smaller than the correlation length of the potential:

$$\partial_t f^\alpha - \frac{\nabla\phi \times \mathbf{b}}{B} \cdot \nabla f^\alpha + v_z \partial_z f^\alpha - \frac{q_\alpha}{m_\alpha} \partial_z \phi \frac{\partial}{\partial v_z} f^\alpha = \mathcal{C}^\alpha \,, \tag{1}$$

where $\alpha = e, i$, ϕ being the potential of the turbulence and $\mathcal{C}^\alpha$ the collision term. Here, we consider characteristic wavelengths larger than the ion Larmor radius, leading to negligible effects when averaging the potential over the gyromotion. In the following, we consider a background potential ϕ_0 and study test modes on the turbulent plasma.

2.1. SHORT TIME EQUILIBRIUM

In order to study the electron and ion responses to a test mode in a turbulent background potential, we need to evaluate the deviations from an equilibrium distribution function, even if the latter is valid only for a short time.

In the case of the electrons the parallel dynamics dominates, and the last two terms in (1) are much larger than the first two terms. The electron solution of (1) for short time (smaller than the characteristic times of the first two terms) may be obtained by neglecting them:

$$f_0^e = n_0(x) F_M^e \exp\left(\frac{e\phi_0}{T_e}\right), \qquad F_M^e = \left(\frac{m_e}{2\pi T_e}\right)^{3/2} \exp\left(-\frac{m_e \mathbf{v}^2}{2T_e}\right), \quad (2)$$

where $e = |e|$ and F_M^e is the Maxwell distribution of the electron velocities.

In the case of the ions, the perpendicular dynamics is dominant, and the last two terms in Eq. (1), corresponding to large characteristic times, can be neglected for the short time solution. Because the potential frequencies are low enough compared to the plasma frequency, the Laplace equation reduces to the quasineutrality condition, *i.e.* the equality of electron and ion density perturbations. The small time equilibrium distribution function of the ions is obtained from the electron distribution:

$$f_0^i = n_0(x)\ F_M^i\ \exp\left[e\phi_0(x, y - V_* t, z)/T_e\right], \quad (3)$$

where F_M^i is the Maxwell distribution of the parallel ion velocities. The integral over the perpendicular velocity was already performed since the dependence of the gyro-averaged potential on the Larmor radius is small at the wavelengths considered here. It can easily be checked that this function is a small time (smaller than the characteristic times of the last two terms in Eq. (1)) solution of the ion equation (1) if V_* is the diamagnetic velocity

$$V_* = -\frac{T_e}{eB}\frac{1}{n_0}\frac{dn_0}{dx} = \frac{T_e}{eBL_n} = \frac{c_s \rho_s}{L_n}\ , \quad (4)$$

where $c_s = \sqrt{T_e/m_i}$, $\rho_s = c_s/\Omega_i$, and $\Omega_i = eB/m_i$.

2.2. PERTURBATION OF THE BACKGROUND POTENTIAL

The change of the potential with $\delta\widetilde{\phi} = \varphi(z)\exp(ik_x x + ik_y y - i\omega t)$ leads to a variation of the distribution functions of both electrons and ions. Besides the adiabatic response obtained simply by replacing $\phi_0 \to \phi_0 + \delta\widetilde{\phi}$ in the equilibrium expressions, a new term h appears in the distribution functions:

$$f_t^\alpha = f_1^\alpha + h^\alpha\ , \qquad f_1^\alpha = n_0(x)\ F_M^\alpha \exp\left[e\left(\phi_0 + \delta\widetilde{\phi}\right)/T_e\right]\ , \quad (5)$$

where $\alpha = e, i$ and h is the non-adiabatic term. It is useful at this point to rewrite Eq. (1) as $\mathcal{O}^\alpha[\phi] f^\alpha = \mathcal{C}^\alpha$, where the operator $\mathcal{O}^\alpha[\phi]$ is the derivative along trajectories in the potential ϕ, defined by:

$$\mathcal{O}^\alpha[\phi] \equiv \partial_t - \frac{\nabla\phi \times \mathbf{b}}{B}\cdot\nabla + v_z\partial_z - \frac{q_\alpha}{m_\alpha}\partial_z\phi\frac{\partial}{\partial v_z}\ . \quad (6)$$

Expanding in the first order in the potential perturbation, we are left with:

$$\mathcal{O}^{\alpha}[\phi_0+\delta\widetilde{\phi}]\left(f_0^{\alpha}(1+e\delta\widetilde{\phi}/T_e)+h^{\alpha}\right)=\mathcal{C}^{\alpha}\ . \tag{7}$$

In the following, we shall evaluate this expression for electrons and ions separately.

2.3. ELECTRON RESPONSE

We need to consider two electron subsystems: trapped (due to the confining effect of the magnetic mirrors) and untrapped populations. In between the magnetic mirrors, *i.e.* for $0<|z|<L$, where both species exist, the equilibrium densities are denoted by n_{0te} and, respectively, n_{0ue}, with $n_{0te}+n_{0ue}=n_0$. Here, we define the trapped electron fraction $\delta=n_{0te}/n_0<1$. Outside the magnetic mirrors, *i.e.* for $L<|z|<L'$, there are only untrapped electrons having density n_0.

In the case of trapped electrons, we employ the Krook model for the collision term [9, 10]: it describes the relaxation of the total distribution function to the adiabatic part with a velocity-dependent characteristic time $\nu(\mathbf{v})^{-1}$, thus $\mathcal{C}^{te}=-\nu_e(\mathbf{v})\ (f_t^{te}-f_1^{te})=-\nu_e(\mathbf{v})\ h^{te}$, where the superscript "te" stands for the trapped electron population. Eq. (7) becomes, to first order in the perturbations:

$$\mathcal{O}^{te}[\phi_0]\ h^{te}+\nu_e(\mathbf{v})\ h^{te}=\frac{ef_0}{T_e}\left[T^e_{\delta\phi}+T^e_{\phi_0}\right]\delta\widetilde{\phi}\ , \tag{8}$$

where;

$$T^e_{\delta\phi}=i\omega+i\frac{T_e}{eB}\frac{\partial_x\left[n_0F^e_M\right]}{n_0F^e_M}k_y\equiv i(\omega-\omega_e^*), \tag{9a}$$

$$T^e_{\phi_0}=\frac{\partial_y\phi_0}{B}\frac{\partial_x n_0}{n_0}\frac{\eta_e}{2}\left(-3+\frac{m_e\mathbf{v}^2}{T_e}\right)\ , \tag{9b}$$

with $\eta_e=L_n/L_{T_e}$. The solution obtained by integrating Eq. (8) along the straight zero order electron trajectories between the two mirrors, $z(\tau)=z+v_z(\tau-t)$:

$$h^{te}=f_0^{te}\ e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp(t)-i\omega t}\int_{-\infty}^{t}d\tau\ \left[T^e_{\delta\phi}+T^e_{\phi_0}\right]\ e^{-i(\omega+i\nu_e)(\tau-t)}\ \frac{e\varphi(z(\tau))}{T_e}\ . \tag{10}$$

The $T^e_{\phi_0}$ contribution vanishes upon averaging over the statistical realizations of the background turbulence, as $\langle\phi_0\rangle=0$. The trapped electrons undergo a periodic motion between the two magnetic mirrors at $z=\pm L$, with a very fast bounce frequency $\omega_{be}=v_z/2L$. This allows us to average Eq. (10) over the bounce time scale:

$$h^{te}=f_0^{te}\frac{e}{T_e}e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp(t)-i\omega t}\int_{-\infty}^{t}d\tau\ e^{-i(\omega+i\nu_e)(\tau-t)}\ T_{\delta\phi}\ \overline{\varphi}\ , \tag{11}$$

where $\overline{\varphi}=\frac{1}{2L}\int_{-L}^{L}dz'\varphi(z')$. Performing first the time integral, then the velocity integral and finally assembling together the adiabatic and non-adiabatic parts of the trapped electron density perturbations, we arrive at:

$$\delta n_{te} = n_{0te}\, e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp(t)-i\omega t}\ \frac{e}{T_e}\ \left(\varphi - \overline{\varphi}\ \left\langle \frac{\omega-\omega_e^*}{\omega+i\nu_e}\right\rangle_v\right)\ , \tag{12}$$

where the notation $\langle\cdots\rangle_v$ indicates the integration over the velocity space with the Maxwellian function. The imaginary part of the last term, which involves the electron collisionality, is the origin of the DTEM instability. Using the approximation $\nu_e(\mathbf{v}) = \nu v_{th,e}^3/(\epsilon|\mathbf{v}|^3)$ [11], together with the expressions of ω_e^* in Eq. (9a) and that of the Maxwellian function in Eq.(2) we evaluate the integral numerically, using the Simpson rule. A study of the integral's behaviour as a function of collisionality can be found in [17].

Collisions are not important for the untrapped electrons and their density perturbation is considered adiabatic, as in [9, 10]:

$$\delta n_{ue} = n_{0ue}\ \ e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp(t)-i\omega t}\ \frac{e\varphi}{T_e}\ . \tag{13}$$

Trajectory eddying is negligible for both electron populations due to their fast parallel decorrelation.

2.4. ION RESPONSE

For the collisionless ions a similar treatment to the one presented above leads to the following expression for the linearized Eq. (7):

$$\mathcal{O}^i[\phi_0]\ h^i = \frac{ef_0^i}{T_e} e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp(t)-i\omega t}\left[T^i_{\delta\phi}+T^i_{\phi_0}\right]\varphi(z)\ , \tag{14}$$

where:

$$T^i_{\delta\phi} = i\omega - iV_* k_y - \left(1+\frac{T_e}{T_i}\right)v_z\partial_z + i\rho_s^2 k_\perp^2\omega\ , \tag{15a}$$

$$T^i_{\phi_0} = -\frac{e}{T_e}v_z\left(1+\frac{T_e}{T_i}\right)\partial_z\phi_0\ . \tag{15b}$$

In order to find the solution for the distribution function $h^i(\mathbf{x}_\perp, z; v_z; t)$, we take into account that the operator $\mathcal{O}^i[\phi_0]$ is the derivative along the ion trajectories in the background field ϕ_0, which are given by $d\tilde{\mathbf{x}}_\perp/d\tau = -\nabla\phi_0(\tilde{\mathbf{x}}_\perp(\tau))\times\mathbf{e}_z/B$, $z(\tau) = z + v_z(\tau-t)$. They are calculated backwards in time with the the boundary condition $\tilde{\mathbf{x}}_\perp(\tau = t) = \mathbf{x}_\perp$. We obtain the expression:

$$h^i = \frac{ef_0^i}{T_e} e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp-i\omega t}\int_{-\infty}^{t} d\tau\ \left[T^i_{\delta\phi}+T^i_{\phi_0}\right]\ \varphi(z(\tau))\ e^{i\mathbf{k}_\perp\cdot(\tilde{\mathbf{x}}_\perp(\tau)-\mathbf{x}_\perp)-i\omega(\tau-t)}\ . \tag{16}$$

Due to the fact that $v_{Ti}/(\omega L') \ll 1$ we may keep only the first term in the Taylor expansion of $\varphi(z(\tau))$, which allows for an easy evaluation of the parallel velocity integrals. The $T^i_{\phi_0}$ term contribution vanishes as the z-derivatives of ϕ_0 do not correlate with the perpendicular displacements, and we are left with:

$$\delta n_i = n_0 \frac{e}{T_e} e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp(t)-i\omega t}\left[\omega - V_* k_y - \left(1+\frac{T_e}{T_i}\right) v_{Ti}^2 \partial_z^2 \partial_\omega + \rho_s^2 k_\perp^2 \omega\right] \varphi(z)\, \Pi^i(\omega)\ , \tag{17}$$

where the ion propagator involves the average over the stochastic trajectories in the background field:

$$\Pi^i(\omega) = i\int_{-\infty}^{t} d\tau\ e^{-i\omega(\tau-t)} \left\langle e^{i\mathbf{k}_\perp\cdot(\tilde{\mathbf{x}}_\perp(\tau)-\mathbf{x}_\perp)}\right\rangle \tag{18}$$

2.5. DISPERSION RELATION

Taking into account Eqs. (12, 13, 17), the quasineutrality condition $\delta n_{te} + \delta n_{ue} = \delta n_i$ becomes:

$$-\delta\ \overline{\varphi} \left\langle \frac{\omega-\omega_e^*}{\omega+i\nu_e}\right\rangle_v = \left[(1+\rho_s^2 k_\perp^2)\omega - V_* k_y - \left(1+\frac{T_e}{T_i}\right) v_{Ti}^2\ \partial_z^2\ \partial_\omega\right] \varphi(z)\ \Pi^i(\omega)\ . \tag{19}$$

Following [9, 10], we rearrange the previous equation as:

$$A(\omega)\varphi(z) - B(\omega)\varphi''(z) = C(\omega)\int_{-L}^{L} dz'\ \varphi(z')\ , \tag{20}$$

where:

$$A(\omega) = \left[\frac{V_* k_y}{\omega} - (1+\rho_s^2 k_\perp^2)\right]\cdot(-\omega)\,\Pi^i(\omega)\ ,$$
$$B(\omega) = \left(1+\frac{T_i}{T_e}\right) c_s^2\ \frac{\partial}{\partial\omega}\Pi^i(\omega)\ , \qquad C(\omega) = -\frac{\delta}{2L}\ \left\langle \frac{\omega-\omega_e^*}{\omega+i\nu_e}\right\rangle_v\ . \tag{21}$$

Eqs. (21) are in agreement with the corresponding expressions found in Refs. [9, 10], in the case of quiescent plasmas where $\Pi^i = -1/\omega$. Our treatment additionaly generates a finite Larmor radius correction $(1+\rho_s^2 k_\perp^2)$ and a temperature ratio correction $(1+T_i/T_e)$ to the ion terms.

Outside the magnetic mirrors there are no trapped electrons and thus their contribution to the right hand side of Eq. (20) vanishes. We may write the equation valid on the entire interval $-L' < z < L'$ as:

$$A(\omega)\varphi(z) - B(\omega)\varphi''(z) = C(\omega)\int_{-L}^{L} dz'\ \varphi(z')\ \mathrm{rect}(z;L) \tag{22}$$

where the rectangular function is defined as $\mathrm{rect}(z;L) = \theta(z-L) - \theta(z+L)$, with $\theta(z)$ the Heaviside step function.

2.6. EXACT SOLUTION

Usual treatments of the DTEM instability involve a harmonic expansion of the test mode [9, 10, 17, 18]. In the following we present the exact solution of Eq. (22), which allows the derivation of an explicit dispersion relation. By using the notations $\lambda^2 = A/B$, $\chi = -C/B$, we may rewrite the quasineutrality condition, Eq. (22), in a simplified form:

$$\varphi''(z) - \lambda^2\varphi(z) = \chi \int_{-L}^{L} dz'\ \varphi(z')\ \mathrm{rect}(z;L) \tag{23}$$

In order for the bounce averaged potential to be nonvanishing, we expect even solutions $\varphi(z) = \varphi(-z)$ and thus we restrict our attention to the interval $z \in [0, L']$. At the origin $z = 0$ we choose for simplicity the normalization $\varphi_1(0) = 1$, as the equation is homogeneous. By imposing $\varphi_1'(0) = 0$ we ensure the evenness of the solution. At $z = L$ the rectangular function on right hand side has a finite discontinuity, which preserves the continuity of the function itself and that of its first derivative at this point, thus $\varphi(L_-) = \varphi(L_+)$ and $\varphi'(L_-) = \varphi'(L_+)$. We obtain the following expression for the function $\varphi(z)$:

$$\varphi(z) = \begin{cases} \dfrac{\lambda\ (\lambda^2 + 2L\chi)\ \cosh(\lambda z) - 2\chi\ \sinh(\lambda L)}{\lambda\ (\lambda^2 + 2L\chi) - 2\chi\ \sinh(\lambda L)} & 0 \le z \le L \\ \dfrac{\lambda\ (\lambda^2 + 2L\chi)\ \cosh(\lambda z) - \chi\ [\sinh(2\lambda L - \lambda z) + \sinh(\lambda z)]}{\lambda\ (\lambda^2 + 2L\chi) - 2\chi\ \sinh(\lambda L)} & L < z \le L' \end{cases} \tag{24}$$

The condition $\varphi'(L') = 0$, consistent with the above mentioned harmonic expansion, leads to the dispersion relation:

$$\lambda\ (\lambda^2 + 2L\chi)\ \sinh(\lambda L') + \chi\ \left[\cosh(2\lambda L - \lambda L') - \cosh(\lambda L')\right] = 0\ . \tag{25}$$

For simplicity we will restrict ourselves to the case $L' = 2L$, where the previous equation reduces to:

$$\frac{1}{\lambda L'}\ \tanh(\frac{\lambda L'}{2}) = 1 + \frac{\lambda^2}{\chi L'}\ . \tag{26}$$

2.7. RESULTS AND DISCUSSIONS

We have analyzed Eq. (26) in the case of quiescent plasmas, quasilinear and nonlinear regimes. In the following we use dimensionless quantities normalized using the units of length ρ_s and time L_n/c_s, *i.e.* $k_i \to k_i\rho_s$, $\omega \to \omega L_n/c_s$, $\Pi^i \to \Pi^i c_s/L_n$, $\nu \to \nu L_n/c_s$, $D_i \to D_i L_n/(\rho_s^2 c_s)$. The numerical values of the relevant quantities are: $L' = 2L = 2m$, $T_i/T_e = 1/3$, $L_n = L_{T_e} = 10cm$, $\epsilon = 0.63$.

Typical results in the case of quiescent plasmas with $\phi_0 = 0$ are presented in Fig. (1). Unstable modes are present in the low k_y and high k_y regions.

The frequencies decrease with the increase of trapped electron fraction δ and are weakly influenced by the electron collision frequency ν. The growth rates are increasing functions of both δ and ν, which shows that DTEMs are unstable due to the combined action of electron trapping by the magnetic mirrors and electron collisions.

In the weak turbulence regime, the Gaussian distribution of ion displacements leads to ion trajectory diffusion. The propagator of Eq. (18) becomes in this case $\Pi^i = -1/\left(\omega + ik_i^2 D_i\right)$, where the diffusion coefficient is defined as usual by $D_i = \left\langle [x_i(\tau) - x_i(t)]^2 \right\rangle / (t-\tau)$. Figure 2 shows that ion diffusion determines the damping of high k modes, in accordance with the well known results of Dupree. As a result, the increase of the turbulence amplitude is accompanied by the increase of its correlation length.

An increase in the amplitude of the stochastic potential causes deviations from the Gaussian distribution of the ion trajectories, due to the appearance of trapped trajectories leading to quasicoherent structures [12, 15]. Ion trajectories are of two types with different motions. The trapped trajectories have frozen mean square displacements S_i, while the free trajectories have a Gaussian distribution with diffusive evolution. The propagator is modified as [12] $\Pi^i = -e^{-k_i^2 S_i/2}/(\omega + ik_i^2 D_i)$.

The exponential factor may be absorbed into an effective, k-dependent, trapped electron fraction $\delta_{eff}(\mathbf{k}) = \delta\ e^{k_i^2 S_i/2}$. Being an increasing function of k, it has a destabilizing influence on high k modes, which is opposed by the damping due to ion diffusion. The competition of the two effects leads to the appearance of the maximum in the growth rates.

The increase of δ_{eff} due to ion trajectory structures also determines the decrease of the frequencies. As seen in Fig. (3), DTEMs that are damped by diffusion grow back due to trajectory structures. Unstable modes appear in turbulent plasmas in the range of wave numbers $k_x, k_y \sim 0.5$.

3. CONCLUSIONS

We have analyzed in a plasma slab model the influence of the background turbulence on the characteristics of the dissipative trapped electron instability. We have deduced the dispersion relation for test modes, dependent on the statistical properties of the background stochastic potential. We have shown that the turbulence does not change the structure of the dispersion relation of Eq. (23), but it influences the functions A, B and C. We have found an exact solution of the dispersion relation.

The background turbulence influences the DTEMS through the ion response. The stochastic potential ϕ_0 determines the diffusion of ion trajectories and, at larger amplitudes, the formation of ion trajectory structures due to ion eddying.

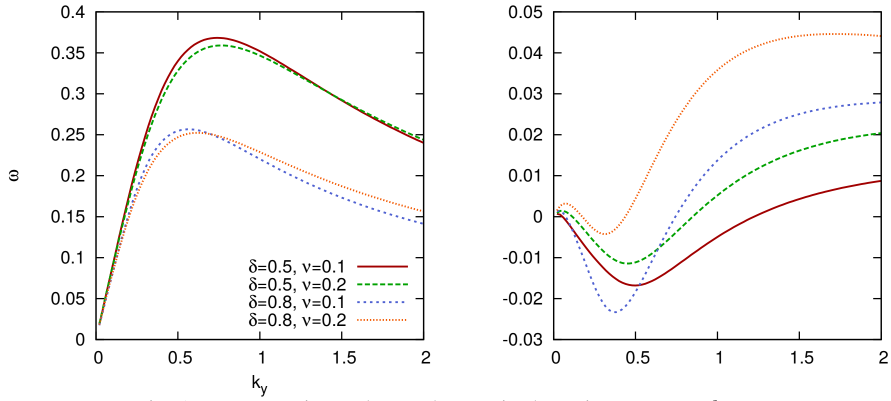

Fig. 1 – Frequencies and growth rates in the quiescent case, $k_x = 0$.

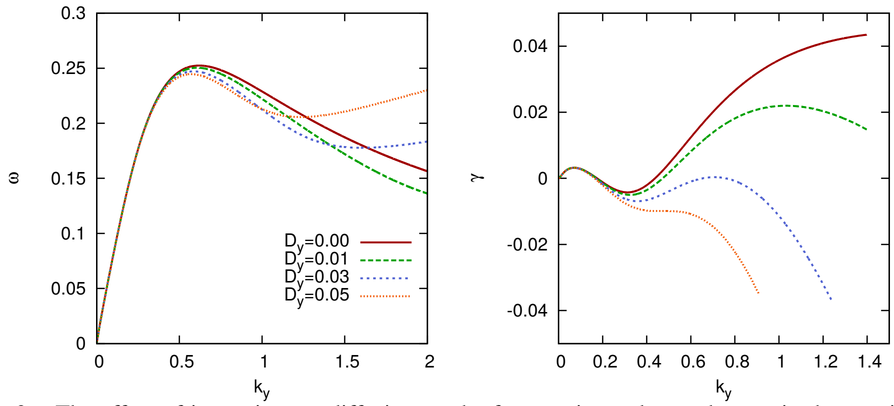

Fig. 2 – The effect of ion trajectory diffusion on the frequencies and growth rates in the quasilinear case, $\delta = 0.8$, $\nu = 0.2$, $k_x = 0$.

The diffusion has a damping effect on the large k_y modes, which is accompanied by the increase of the frequencies. At values of the order $D_i \sim 0.2\rho_s V_*$, the DTEMs are damped except for $k_y\rho_s < 0.1$.

Ion trajectory trapping determines the increase of the effective ratio of the electrons trapped by the magnetic mirrors, which leads to the increase of the growth rates. In this nonlinear regime, at structure sizes $S_i \sim 2 \div 3$, the ion eddying destabilizing effect overcomes the diffusive damping and leads to large growth rates, of the order of γ in the absence of turbulence.

A self consistent coupled test particle and test mode study similar to [19] will be developed in a future work. The effective interaction of the physical processes found here and the evolution of DTEM turbulence will be analyzed.

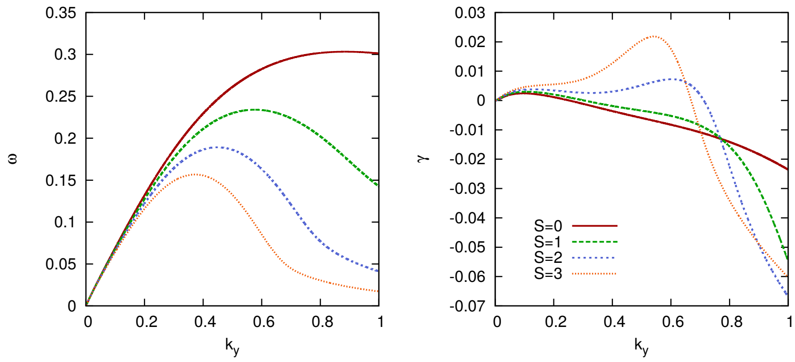

Fig. 3 – Frequencies and growth rates in the weakly nonlinear case, $\delta = 0.5$, $\nu = 0.2$, $k_x = 0.5$, $D_y = 0.05$, $S_x = S_y = S$.

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