

AN UNPAIRED STUDY ON THE ISOMERIC STATES 8^- AND 16^+ IN ^{178}Hf

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Abstract. The isomeric states 8^- and 16^+ in ^{178}Hf are analyzed within the framework of unpaired Cranked Nilsson-Strutinsky (CNS) model including Strutinsky renormalization and the Lublin-Strasbourg Drop (LSD) and Finite Range Liquid Drop (FRLDM) liquid drop methods. Special attention is given to the potential energy surfaces to determine the deformation parameters in the ground and isomeric states. Our calculations show that the hexadecapole deformations in the isomeric states are somewhat larger than those in previous study. Also, the pairing correlation has no effects on the isomeric states shape, while has a significant effects on the excitation energies.

Key words: Isomeric states, ^{178}Hf , Unpaired Cranked Nilsson-Strutinsky model, LSD and FRLDM models, deformation parameters.

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1. INTRODUCTION

An excited nucleus can decay to the ground state by electromagnetic radiation directly or *via* the decay to the other states. However, the structure and features of some of the nuclear excited states are in a way that forbid or decrease the possibility of the electromagnetic decay and result in a longer half-life up to 1 ns [1]. These semi-stable excited states with an up-to-1ns half-life are called isomers or isomeric states [2–5]. Isomers are categorized into three groups according to the formation or decay mechanism: shape, spin and K isomers that are formed respectively upon a significant change in nucleus shape, spin and spin orientation due to the decay [3, 6]. Shape isomer, which fission isomers are in this category, is an excited state with an unusual short half-life and a larger elongation relative to the ground state [7]. Creation of the spin isomers depends on the selection rules since the decay from an excited state to states with less energy needs an enormous change in angular momentum [4]. Such a change in the spin leads to an electromagnetic radiation with a high multi-polarity radiation.

The next type of isomers is the K isomers in which the quantum number K is the projection of total angular momentum onto the nucleus symmetry axis. If changes in the K is larger than the multipole order λ , the selection rule of K ($\lambda > \Delta K$) is violated

and decay is forbidden; as a result its half-life gets longer and thus a K isomer is created. A most interesting example for the K isomers are the states 6^+ , 8^- , 14^- and 16^+ in ^{178}Hf [4]. The state $K^\pi = 16^+$ with a half-life of 31 y, has the maximum excitation energy of 2.4 MeV among the nuclear isomers [6]. In the structure of Hf nucleus, orbitals close to the Fermi surface have a high Ω . These high Ω orbitals play an important role in creating isomeric states. If pairing between protons or neutrons is broken, these high Ω orbitals will be occupied and thus nuclear states with greater values of K and longer half-life will be created [8]. Isomeric states in ^{178}Hf have 2-quasi-particle and 4-quasi-particle configurations created by the combination of the single-particle orbitals [9, 10]. It is found that there are two 8^- bands which are of mixed neutron and proton characters $\nu[514]_{\frac{7}{2}}^- [624]_{\frac{9}{2}}^+$ and $\pi[404]_{\frac{7}{2}}^+ [514]_{\frac{9}{2}}^-$ and a 16^+ band which has the structure $\nu[514]_{\frac{7}{2}}^- [624]_{\frac{9}{2}}^+ \pi[404]_{\frac{7}{2}}^+ [514]_{\frac{9}{2}}^-$ [11, 12].

The motivation for the present work is to determine the aired Cranked shape of the isomeric states of 8^- and 16^+ in ^{178}Hf using the unpaired Cranked Nilsson-Strutinsky (CNS) model [13–15]. In this microscopic-macroscopic approach, the shell correction derived from the phenomenological Nilsson potential with the macroscopic Lublin-Strasbourg Drop (LSD) or Finite Range Liquid Drop (FRLDM) liquid drop model are combined.

2. SUMMARY OF THE THEORY

In the present work, we investigate the isomeric states in ^{178}Hf using the potential-energy surfaces (PES) calculations with the CNS approach [13–15]. This model is presented in more details in Ref. [16]. In the CNS model, the single-particle levels are obtained from the deformed Nilsson potential with the standard single-particle parameters [13]. The diagonalization of the Cranked Nilsson Hamiltonian gives the eigenvalues as function of the deformation parameters $e_i^\omega = e_i^\omega(\varepsilon_2, \gamma, \varepsilon_4)$ which are referred to as the single-particle energies in the rotating system or Routhians [17]. The total energy of ^{178}Hf is calculated at the following grid points (numbers in brackets indicate the step length which the calculation is performed for a given variable):

$$\begin{aligned} x &= 0.0[0.04]0.36, \\ y &= -0.36[0.04]0.32, \\ \varepsilon_4 &= -0.08[0.02]0.08, \end{aligned} \quad (1)$$

where (x, y) are Cartesian coordinates that are connected with (ε_2, γ) by the expressions [17]

$$x = \varepsilon_2 \cos(\gamma + 30^\circ), \quad y = \varepsilon_2 \sin(\gamma + 30^\circ). \quad (2)$$

The total energy of a configuration consists of a macroscopic part, which is obtained from the LSD [18] or FRLDM [19] models, and a microscopic part resulting from

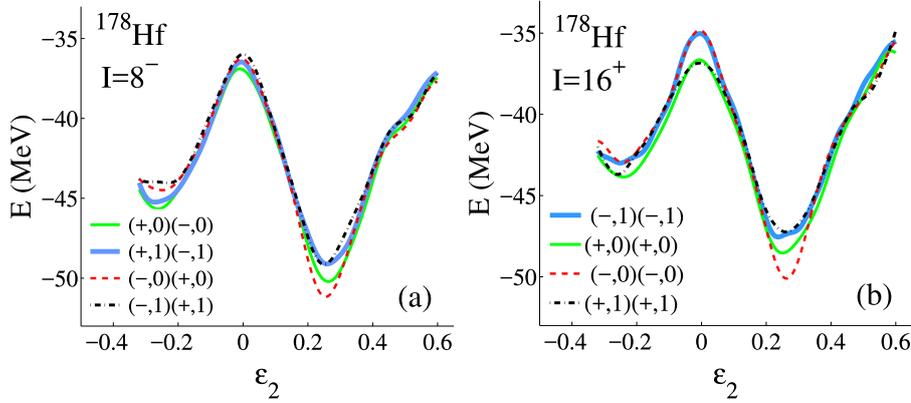


Fig. 1 – Total energy with respect to the axial quadrupole deformation for the configurations (a) at spin $I = 8^-$ for the configurations $(+,0)(-,0)$, $(+,1)(-,1)$, $(-,0)(+,0)$, $(-,1)(+,1)$ and (b) spin 16^+ for the configurations $(-,1)(-,1)$, $(+,0)(+,0)$, $(-,0)(-,0)$, $(+,1)(+,1)$ in ^{178}Hf . Note that the energies have been minimized with respect to the triaxial parameter γ as well as the hexadecapole deformation parameter ε_4 .

the Strutinsky shell correction model [20]. Minimizing the total energy with respect to the deformation parameter ε_4 for a given angular momentum gives the PES's. The minimum point in the PES's shows the energy and deformation (ε_2, γ) for the equilibrium state at each spin.

In the CNS model, good quantum numbers are parity, π and signature, α [17]. The quantum number α for nuclei with an even mass number is 0 or 1 and for a nuclei with an odd mass number is $+1/2$ and $-1/2$; additionally, the parity quantum number includes two values, positive and negative. Consequently, if the parity and α are kept constant for the neutrons and protons, there will be four yrast configurations (π, α) while if they are kept constant in proton and neutron states, separately, there will be 16 configurations as $(\pi_p, \alpha_p)(\pi_n, \alpha_n)$.

3. RESULTS AND DISCUSSION

In the present research, parity and α are kept constant for proton and neutron states, separately. Therefore, among 16 configurations, four configurations: $(+,0)(-,0)$, $(-,0)(+,0)$, $(+,1)(-,1)$ and $(-,1)(+,1)$ can create the isomeric state 8^- . In Fig. 1(a), the total energy of these four configurations are drawn with respect to the quadrupole deformation ε_2 . As one can see, the lowest configuration is $(-,0)(+,0)$ as is about 500 keV lower than $(+,0)(-,0)$ in energy. Two isomeric states 8^- were observed in ^{178}Hf with close excitation energies, 1.147 and 1.479 MeV [11, 21]. Therefore, it is concluded that the two isomeric states 8^- are built on the configurations $(-,0)(+,0)$ and $(+,0)(-,0)$ corresponding to the bands with the

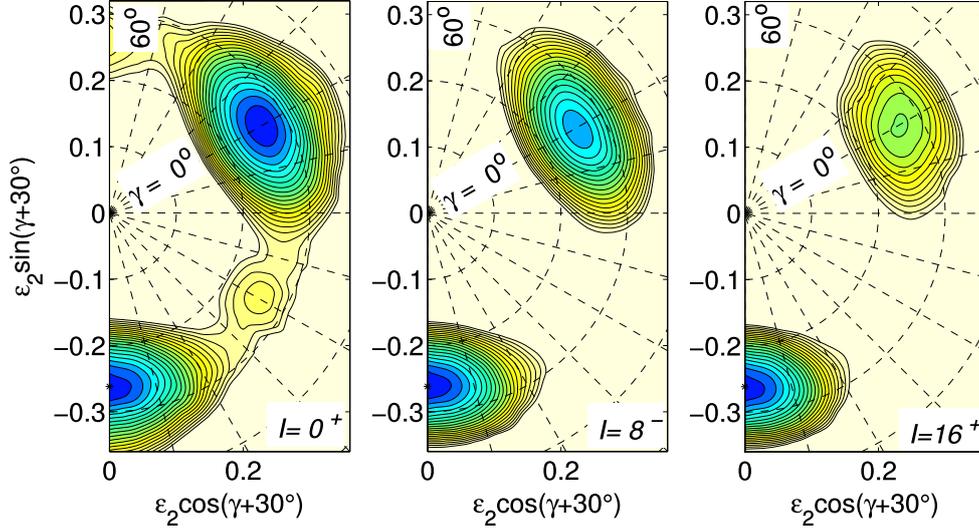


Fig. 2 – Calculated potential-energy surfaces versus quadrupole deformation ε_2 and the triaxiality parameter γ of ^{178}Hf for the ground state ($I = 0$), configuration $(-,0)(+,0)$ at spin $I = 8$ and the configuration $(-,0)(-,0)$ at spin $I = 16$. Contour lines are separated by 0.25 MeV and the γ plane is marked at 15° intervals. Dark regions represent low energy with absolute minima labeled with dots.

excitation energies 1.147 and 1.479 MeV, respectively. It is consistent to the experimental results including the configurations $\pi[404]_{\frac{7}{2}}^{+}[514]_{\frac{9}{2}}^{-}$ and $\nu[514]_{\frac{7}{2}}^{-}[624]_{\frac{9}{2}}^{+}$ as the $(-,0)(+,0)$ and $(+,0)(-,0)$, respectively, for the bands 8^- [11].

Also, four configurations $(+,1)(+,1)$, $(-,1)(-,1)$, $(+,0)(+,0)$ and $(-,0)(-,0)$ can create the isomeric state 16^+ . Fig. 1 (b) shows that the configuration $(-,0)(-,0)$ is the lowest in energy as is the best configuration for the band 16^+ . It is consistent to the experimental results that determine the structure $\pi[402]_{\frac{5}{2}}^{+}[514]_{\frac{9}{2}}^{-}$ $\nu[624]_{\frac{9}{2}}^{+}[514]_{\frac{7}{2}}^{-}$ for the band 16^+ with a negative parity for protons and neutrons [12].

The potential energy surfaces are illustrated in Fig. 2 for the appropriate configurations, $(+,0)(+,0)$ for the ground state, the configurations $(-,0)(+,0)$ and $(-,0)(-,0)$ at spins $I = 8$ and $I = 16$, respectively, for the isomeric states. The same potential energy surfaces are obtained also for the configuration $(+,0)(-,0)$ at the spin $I = 8$. The nuclear shape in the ground and isomeric states are determined by the minimum points in the PES's. Fig. 2 shows that PES of ^{178}Hf in ground state has two minima at $(\varepsilon_2, \gamma) \sim (0.26, 0^\circ)$ and $(\varepsilon_2, \gamma) \sim (0.26, -120^\circ)$ with a little bit of difference in energy. In both cases, the nuclear shape in ground state is prolate. For the isomeric state 8^- , the energy difference between two minima increases to about 750 keV, thus the possibility of nucleus existence in the $\gamma \sim 0^\circ$ minimum decreases. Also in state 16^+ , the PES has a main minimum at $(\varepsilon_2, \gamma) \sim (0.26, -120^\circ)$ and a

Table 1

Calculated values of deformation parameters ε_2 , γ and ε_4 using the CNS model for the ground state $(+,0)(+,0)$, two configurations $(-,0)(+,0)$ and $(+,0)(-,0)$ at spin $I = 8$ and the configuration $(-,0)(-,0)$ at spin $I = 16$.

I^π	ε_2	γ	ε_4	$\varepsilon_2[11]$	$\gamma[11]$	$\varepsilon_4[11]$
0^+	0.263	0.30	0.0670	0.249	0°	0.044
8^-	0.260	-120°	0.058	0.247	0°	0.044
8^-	0.270	-120°	0.071	–	–	–
16^+	0.264	-120°	0.061	0.247	0°	0.044

local minimum at $(\varepsilon_2, \gamma) \sim (0.26, 0^\circ)$. The nucleus at the first minimum has an axial symmetric shape with a symmetry axis along the rotational axis. In quantum mechanics, rotation around the symmetry axis is forbidden, it leads to the nucleus treatment change from a collective motion to a single-particle one. However, in the second minimum, the nuclear shape is elongated as the nucleus rotates around the axis perpendicular to the symmetry axis, collectively. In Table 1, it is given the calculated deformation parameters for the equilibrium shapes in the ground states and the isomeric states 8^- and 16^+ and compared to the obtained values in the configuration-constrained calculations including the microscopic Woods-Saxon and macroscopic liquid drop models [11]. As one can see, the axial quadrupole deformation parameters ε_2 in our calculations are close to those in Ref. [11] while the ε_4 and γ values are different. Note, the nuclear shape is the same in $\gamma \sim 0^\circ$ and -120° while the nuclear motion is different (collective and single-particle motions, respectively). We have also performed the calculations using the macroscopic FRLDM model to search if different results are obtained in comparison to the LSD results. Our calculations show that the nucleus will have the same deformation also in the FRLDM calculations. The important point is a large hexadecapole deformation which is seen also in the macroscopic FRLDM calculations. However, we have minimized the energy with respect to the γ (ε_2) parameter as well as the hexadecapole deformation parameter ε_4 and drawn it with respect to the ε_2 (γ) in Fig. 3 (a) ((b)) at spins $I = 0, 8, 16$ for the configurations $(+,0)(+,0)$, $(-,0)(+,0)$ and $(-,0)(-,0)$, respectively.

In Fig. 3 (a) it is seen two prolate ($\gamma = 0^\circ$ and -120°) and oblate ($\gamma = -60^\circ$) minima at $\varepsilon_2 \sim 0.26$ for the spins $I = 0, 8, 16$ well. In Fig. 2, the oblate minimum is not drawn at spins $I = 8$ and 16 because is high in energy. As seen in Fig. 3 (b), in the ground state 0^+ , energy of the state $\gamma = 60^\circ$ is about 3 MeV higher than that of the $\gamma = 0^\circ$. It is the same also for the states at spins 8^- and 16^+ . It also is illustrated that at spin 0^+ , the two minima of $\gamma = 0^\circ$ and $\gamma = -120^\circ$ have approximately the same energy but as the spin increases, the minimum $\gamma = 0^\circ$ goes upper and thus

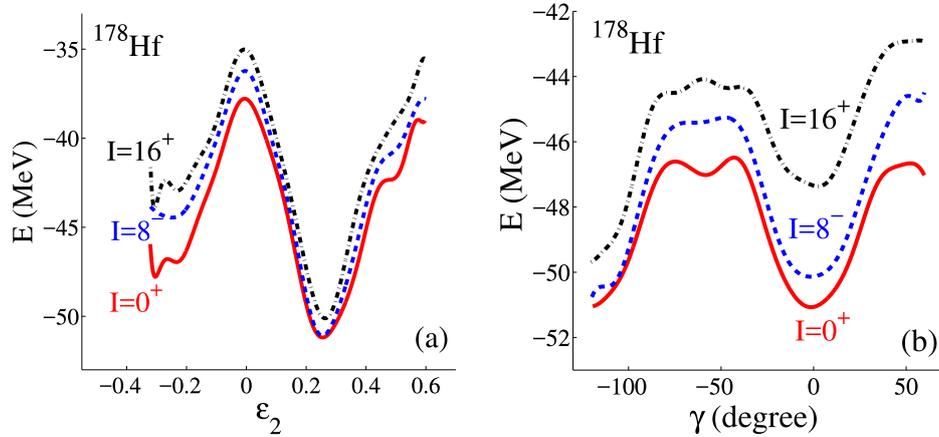


Fig. 3 – Total energy with respect to the (a) axial quadrupole parameter ε_2 and minimized with respect to the triaxial parameter γ (b) triaxiality parameter γ and minimized with respect to the ε_2 parameter for the ground state and isomeric states 8^- and 16^+ in ^{178}Hf . Note that the energies have been also minimized with respect to the hexadecapole deformation parameter ε_4 .

the minimum $\gamma = -120^\circ$ will be the lower state in energy. Recently, the energy of ^{178}Hf was calculated using the projected shell model and drawn with respect to the γ and a constant quadrupole deformation parameter $\beta_2=0.25$ ($\varepsilon_2 = 0.24$), and also with respect to the quadrupole parameters β_2 and a constant γ ($\gamma = 0^\circ$), see Fig. 4 in Ref. [12], also Fig. 3 in Ref. [11]. We have calculated the energy in a wider region of ε_2 and γ in comparison with previous studies [11, 12]. In the same region, our results are consistent with previous calculations.

One can also calculate the excitation energy of an isomeric state as the difference between the energies of the ground and the isomeric configurations. In our calculations, the excitation energies for two isomeric states 8^- , will be 0.669 MeV (0.481 MeV) for two configurations energies $(+,0)(-,0)$ and $(-,0)(+,0)$ if their percentage contribution are accounted %36 and %64 (%64 and %36), respectively [23]. In comparison with the experimental energy values for the 8^- bands, 1.479 and 1.147 MeV [9], difference is 0.81 and 0.666 MeV. Similarly for the isomeric state 16^+ , the excitation energy is calculated as 1.365 MeV for the configuration $(-,0)(-,0)$ that in comparison with experimental values 2.446 MeV the difference is 1.081 MeV. This discrepancy is because of neglecting the pairing correlations in the present calculations. Therefore, our calculations show that the pairing effect is important in the excitation energy calculations of the isomeric states.

In summary, the isomeric states 8^- and 16^+ have been studied using the unpaired Cranked Nilsson-Strutinsky approach including the macroscopic LSD and FRLDM models and the Strutinsky shell correction method. Our results on the quadrupole deformation parameters of equilibrium shapes have a good compati-

lity with obtained results using the paired Woods-Saxon [11] and also projected shell model [12]. While the hexadecapole deformation of isomeric states are larger in the LSD and FRLDM calculations than those in previous studies. It is also concluded that the pairing correlation has no an important role in the nuclear deformation calculations for the isomeric states. This result was also found from comparison between the paired and unpaired CNS calculations on the collective states in ^{161}Lu [22]. While, despite the existence of quasi-particles in the structure of isomeric states, our results are very different from the experiment in the excitation energy values which show the importance of the pairing correlations in the excitation energies.

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