

ON CANONICAL TRANSFORMATION AND TACHYON-LIKE “PARTICLES” IN INFLATIONARY COSMOLOGY

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We examine a classical and quantum dynamics of systems described by the DBI-type Lagrangian with tachyon-like potentials and corresponding DBI Lagrangians on (non-)Archimedean spaces. The dynamic of tachyon fields in spatially homogeneous and in zero-dimensional limits is analysed. A formalism for connecting a wide class of potentials and DBI Lagrangians with the locally equivalent canonical Lagrangians is presented. The results for exponentially decaying and inverse cosine hyperbolic are reviewed and for the potentials of the form $V(x) = x^{-n}$ are discussed in more details. Classical actions and corresponding quantum propagators are calculated for these potentials in the Feynman path integral approach, on both Archimedean and non-Archimedean spaces.

Key words: DBI Lagrangians, tachyon cosmology.

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1. INTRODUCTION

The evolution of the early universe in its early stage should be best described by quantum cosmology [1]. Due to the size of the universe, which is related to the Plank scale, it is logical to consider various geometries (in particular nonarchimedean [2] and noncommutative [3] ones) and the parametrizations of the space-time coordinates: real, p-adic, or even adelic [4].

One of the most challenging periods of the evolution of the Universe, despite its shortness, is the inflation period, in particular its very beginning. Some of the best candidates to give some physical background and understanding of the quantum origin of inflation are string theory, M-theory and string field theory in general.

It can be considered that an inflationary phase is driven by potential energy of the scalar field (inflaton) whose dynamics is described by Klein-Gordon equation [5]. However, in recent years other non-standard scalar field actions are used in cosmology. One of those models, motivated by string theory, is k -inflation [6, 7].

K -inflationary model can be defined by local action for scalar field ϕ minimally coupled to Einstein gravity:

$$S = -\frac{1}{16\pi G} \int \sqrt{-g} R d^4x + \int \sqrt{-g} \mathcal{L}(\phi, X) d^4x, \quad (1)$$

where R is the Ricci scalar, g is the determinant of the metric tensor with the components $g_{\mu\nu}$ and signature $(-,+,+,+)$, \mathcal{L} is Lagrangian of the scalar field and X is “kinetic” term

$$X = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi. \quad (2)$$

One of the particularly attractive models of k -inflation is tachyonic inflation. A tachyonic Lagrangian

$$\mathcal{L}_{tach} = \mathcal{L}(T, \partial_\mu T) = -V(T)\sqrt{1 + g_{\mu\nu}\partial^\mu T\partial^\nu T}, \quad (3)$$

where T is tachyonic scalar field and $V(T)$ its potential, can be written as non-standard Lagrangian of DBI-type. The Lagrangian (3) contains potential as a multiplicative factor and a square root of derivatives (“kinetic” term).

This model of non-standard Lagrangian and tachyon-like “matter” was proposed by Sen [8]. In this paper we will discuss the simplified, zero-dimensional, model to understand better the evolution of the tachyon itself, and at least as a toy model, to explore classical and quantum dynamics of the corresponding “tachyon-like” particle [9]. At first glance, the “toy” model looks like the classical model of a particle moving in a constant external field with quadratic “damping”-like term [10]. However, Hamiltonian is conserved quantity for this system [11, 12]. The zero dimensional analogue of a tachyon field can be obtained by a standard correspondence: $x^i \rightarrow t, T \rightarrow x, V(T) \rightarrow V(x)$. The action and Lagrangians are:

$$L_{tach} = -V(x)\sqrt{1 - \dot{x}^2}, \quad (4)$$

$$S = \int L_{tach}dt = - \int dtV(x)\sqrt{1 - \dot{x}^2}. \quad (5)$$

Besides, the conjugate momenta and Hamiltonian are:

$$p = \frac{\partial L_{tach}}{\partial \dot{x}} = \frac{\dot{x}}{\sqrt{1 - \dot{x}^2}}V(x), \quad (6)$$

$$\mathcal{H}_{tach}(x, p) = \sqrt{p^2 + V^2(x)}. \quad (7)$$

The equation of motion for tachyonic scalar field in a flat space is

$$\ddot{x}(t) - \frac{1}{V(x)}\frac{dV}{dx}\dot{x}^2(t) = -\frac{1}{V(x)}\frac{dV}{dx}. \quad (8)$$

In this paper we will focus on a few very interesting tachyonic potentials, and also completely solvable ones. We will discuss classical and quantum dynamics of these tachyonic fields, on archimedean spaces described by real numbers. Also, we briefly discuss the same systems on nonarchimedean spaces (described by p-adic numbers). The classical canonical transformations are imposed in order to simplify the equation of motion and to find a locally equivalent “standard” Lagrangian with

canonical kinetic term which gives the same equation of motion. In a sense, it is further generalization of the procedure presented in [13].

The paper is organized as follows. Following the Introduction, in Section 2 we introduce the canonical transformation to simplify the equation of motion and enable us to find the standard form Lagrangian, locally equivalent to the initial DBI-tachyonic one. Section 3 represents a short review of dynamics of systems described by exponential-like potentials and original results for power function potentials. In Section 4 we show that all these systems can be considered on non-Archimedean spaces, *i.e.* on p -adic number fields, and the main results are presented. We complete this paper with concluding remarks and suggestions for further investigation.

2. CANONICAL TRANSFORMATIONS

We develop here a “mathematical” method of transforming a class of non-standard Lagrangians to a canonical form, proposed by Musielak [13] and considered in [14], where even quadratic Lagrangians were obtained. An extended method based on the classical canonical transformation (CCT) seems to be very useful, because the quantization of tachyonic systems, described by a highly nonlinear Lagrangian (3), is an old and obviously hard problem.

It is well known from classical Hamiltonian mechanics [15–17] that a canonical transformation is a change of canonical variables, for instance (x, p) to new variables (\tilde{x}, \tilde{p}) , that preserves Hamilton’s equations:

$$\dot{x} = \frac{\partial \mathcal{H}(x, p)}{\partial p} \quad \rightarrow \quad \dot{\tilde{x}} = \frac{\partial \tilde{\mathcal{H}}(\tilde{x}, \tilde{p})}{\partial \tilde{p}}, \quad (9)$$

$$\dot{p} = -\frac{\partial \mathcal{H}(x, p)}{\partial x} \quad \rightarrow \quad \dot{\tilde{p}} = -\frac{\partial \tilde{\mathcal{H}}(\tilde{x}, \tilde{p})}{\partial \tilde{x}}. \quad (10)$$

To preserve the Hamilton’s equation the transformation $(x, p) \rightarrow (\tilde{x}, \tilde{p})$ must satisfies two conditions: to preserve the Poisson brackets $(\{.\}_{PB})$:

$$\{x, p\}_{PB} = \{\tilde{x}, \tilde{p}\}_{PB} = 1, \quad (11)$$

and Jacobian of the transformation has to be equal to one

$$J = \frac{\partial(x, p)}{\partial(\tilde{x}, \tilde{p})} = 1. \quad (12)$$

In the theory of CCT there are particular types of functions that lead to canonical transformations. These types of functions are known as generating functions. In this paper we will use the generating function that depends on an old momenta p and a new coordinate \tilde{x} defined as

$$G(\tilde{x}, p) = -pF(\tilde{x}), \quad (13)$$

where $F(\tilde{x})$ is an arbitrary function in the new coordinate. For the particular generating function, defined in (13), relations between old coordinates x and new momenta \tilde{p} are given by

$$x = -\frac{\partial G}{\partial p} = F(\tilde{x}) \quad (14)$$

$$\tilde{p} = -\frac{\partial G}{\partial \tilde{x}} = pF'(\tilde{x}). \quad (15)$$

The relations that connect old and new variables are

$$\tilde{x} = F^{-1}(x), \quad (16)$$

$$\tilde{p} = pF'(\tilde{x}), \quad (17)$$

where $F^{-1}(x)$ is the inverse function of $F(\tilde{x})$. While

$$F'(\tilde{x}) = \frac{dF(\tilde{x})}{d\tilde{x}}. \quad (18)$$

The canonical transformation chosen in this form transforms the equation of motion (8) to

$$\ddot{\tilde{x}} + \left(\frac{F''(\tilde{x})}{F'(\tilde{x})} - F'(\tilde{x}) \frac{d \ln V(F(\tilde{x}))}{dF(\tilde{x})} \right) \dot{\tilde{x}}^2 + \frac{1}{F'(\tilde{x})} \frac{d \ln V(F(\tilde{x}))}{dF(\tilde{x})} = 0. \quad (19)$$

Unfortunately, this equation of motion still looks quite complicated. However, it was already said that the function $F(\tilde{x})$ was arbitrary. Now, we can choose the function $F(\tilde{x})$ in such a way that the second term in (19) vanishes. If the potential $V(x) \neq 0$, it can be shown that a good choice is

$$F^{-1}(x) = \int_{x_0}^x \frac{dx}{V(x)}. \quad (20)$$

where x_0 can be chosen arbitrary. When we define the function $F(\tilde{x})$, using its inverse function $F^{-1}(x)$, the equation of motion (19) simplifies to

$$\ddot{\tilde{x}} + \frac{1}{F'(\tilde{x})} \frac{d \ln V(F(\tilde{x}))}{dF(\tilde{x})} = 0. \quad (21)$$

Now, it can be easily seen [13, 18] that the canonical Lagrangian which corresponds to this equation of motion can be written as

$$L_{quad}(\tilde{x}, \dot{\tilde{x}}) = \frac{1}{2} \dot{\tilde{x}}^2 + \frac{1}{2V(F(\tilde{x}))^2}. \quad (22)$$

3. SOME CLASSES OF TACHYONIC POTENTIALS

In this paper we assume that a tachyonic potential $V(x)$ satisfies the following properties [19]

$$V(0) > 0, \quad V'(x > 0) < 0, \quad V(x \rightarrow \infty) \rightarrow 0. \quad (23)$$

There are lot of different potentials of this kind. Here, we apply the “new” CCT method on already considered tachyonic potentials [10, 14] and discuss one particular, the power potential case ($V(x) = \frac{1}{\kappa x}$), in more details.

3.1. EXPONENTIAL POTENTIAL

As exponentially increasing potential of the form $V(x) = e^{\alpha x}$ does not meet the conditions (23) we discuss only the case of the exponentially decreasing potential of the form

$$V(x) = e^{-\alpha x}, \quad \alpha - const > 0. \quad (24)$$

Though the equation (20), we introduce the function $\tilde{x} = F^{-1}(x)$ as

$$F^{-1}(x) = \frac{1}{\alpha} e^{\alpha x}, \quad (25)$$

which leads us to the function $x = F(\tilde{x})$ in the form of

$$F(\tilde{x}) = \frac{1}{\alpha} \ln(\alpha \tilde{x}). \quad (26)$$

Now, the full generating function (13) is

$$G(\tilde{x}, p) = -pF(\tilde{x}) = -\frac{p}{\alpha} \ln(\alpha \tilde{x}), \quad (27)$$

and it reduces the equation of motion to the well known form

$$\ddot{\tilde{x}} - \alpha^2 \tilde{x} = 0. \quad (28)$$

It can easily be shown that after applying the transformation (25) and returning to the original variable x , the equation (28) transforms to

$$\ddot{x} + \alpha \dot{x}^2 = \alpha, \quad (29)$$

which is equivalent to equation (8) that was obtained in [10].

The “standard” Lagrangian that corresponds to this equation of motion can be written [13, 14] as

$$L_{quad}(\tilde{x}, \dot{\tilde{x}}) = \frac{1}{2} \dot{\tilde{x}}^2 + \frac{1}{2} \alpha^2 \tilde{x}^2 \quad (30)$$

Because of the character of equation (20), which connects Lagrangians (4), including potential (24), and Lagrangian (30) we consider the last one as being “locally” equivalent to the one presented in (4).

We have already shown [10, 14, 20] that the corresponding classical action is

$$S_{cl}(\tilde{x}_2, T; \tilde{x}_1, 0) = \frac{\alpha}{2} (\operatorname{csch}(\alpha T)) ((\tilde{x}_1^2 + \tilde{x}_2^2) (\cosh(\alpha T)) - 2\tilde{x}_1\tilde{x}_2). \quad (31)$$

Besides, it is known that the corresponding transition amplitude (quantum propagator) [21] for the quadratic action can be directly written down

$$K(\tilde{x}_2, T; \tilde{x}_1, 0) = \sqrt{\frac{1}{2\pi i \hbar} \frac{\alpha}{\sinh(\alpha T)}} e^{i \frac{S_{cl}(\tilde{x}_2, T; \tilde{x}_1, 0)}{\hbar}} \quad (32)$$

3.2. INVERSE cosh POTENTIAL

As the second example lets us discuss tachyonic potential in the form

$$V(x) = \frac{1}{\cosh(\beta x)} = \frac{2}{e^{\beta x} + e^{-\beta x}}, \quad \beta - const, \quad (33)$$

and the function $\tilde{x} = F^{-1}(x)$ is

$$F^{-1}(x) = \frac{1}{\beta} \sinh(\beta x), \quad (34)$$

which means that the function $x = F(\tilde{x})$ is

$$F(\tilde{x}) = \frac{1}{\beta} \operatorname{arcsinh}(\beta \tilde{x}). \quad (35)$$

The full generating function is given by

$$G(\tilde{x}, p) = -pF(\tilde{x}) = -\frac{p}{\beta} \operatorname{arcsinh}(\beta \tilde{x}). \quad (36)$$

Again, the generating function reduces the equation of motion to the well known form

$$\ddot{\tilde{x}} - \beta^2 \tilde{x} = 0. \quad (37)$$

This equation of motion can be derived the from quadratic Lagrangian [13, 14]

$$L_{quad}(\tilde{x}, \dot{\tilde{x}}) = \frac{1}{2} \dot{\tilde{x}}^2 + \frac{1}{2} \beta^2 \tilde{x}^2. \quad (38)$$

As in the previous case the classical action can be presented as [14, 20]

$$S_{cl}(\tilde{x}_2, T; \tilde{x}_1, 0) = \frac{\beta}{2} \operatorname{csch}(\beta T) ((\tilde{x}_1^2 + \tilde{x}_2^2) (\cosh(\beta T)) - 2\tilde{x}_1\tilde{x}_2). \quad (39)$$

The corresponding quantum propagator (on Archemedean spaces - real numbers) is

$$K(\tilde{x}_2, T; \tilde{x}_1, 0) = \sqrt{\frac{1}{2\pi i \hbar} \frac{\beta}{\sinh(\beta T)}} e^{i \frac{S_{cl}(\tilde{x}_2, T; \tilde{x}_1, 0)}{\hbar}}. \quad (40)$$

The propagator can also be written in the form [20, 22, 23], more suitable for p -adic and adelic quantisation and applications.

$$K_{\infty}(\tilde{x}_2, T; \tilde{x}_1, 0) = \lambda_{\infty} \left(\frac{\beta}{2h \sinh(\beta T)} \right) \left| \frac{\beta}{h \sinh(\beta T)} \right|_{\infty}^{1/2} \times \chi_{\infty} \left(-\frac{\beta}{2h} \operatorname{csch}(\beta T) ((\tilde{x}_1^2 + \tilde{x}_2^2) (\cosh(\beta T)) - 2\tilde{x}_1 \tilde{x}_2) \right), \quad (41)$$

where an arithmetic λ -function and additive character χ_{∞} are defined as

$$\lambda_{\infty}(b) = e^{-\frac{i\pi}{4} \operatorname{sgn}(b)}, \quad \chi_{\infty}(a) = e^{-2\pi i a}. \quad (42)$$

It can easily be seen that in both examples the equations of motions (28) and (37) and Lagrangians (30) and (38) have the same form which corresponds to an inverted harmonic oscillator system [24].

3.3. TACHYON POTENTIAL AS A POWER FUNCTION CASE

Power potential is presented as $V(x) \sim x^n$ where n can be any real number. However, regarding (23), the potential is tachyonic when $n < 0$. We will discuss here the simplest case, when $n = -1$, *i.e.* the potential is of the form

$$V(x) = \frac{1}{\kappa x}, \quad \kappa - \text{const} > 0, \quad (43)$$

A more general case: $n \in \mathbb{Q}$ or $n \in \mathbb{Z}$ and $n < 0$ will be discussed elsewhere [25].

The equation of motion (8) for this potential becomes

$$\frac{x(t)x''(t) + x'(t)^2 - 1}{\kappa x(t)\sqrt{1 - x'(t)^2}} = 0, \quad (44)$$

and its solution is

$$x(t) = \pm \sqrt{(C_2 + t)^2 - e^{2C_1}}, \quad (45)$$

where constants C_1 and C_2 can be determined from the initial and the final conditions $x(0) = x_1$ and $x(T) = x_2$.

The tachyonic Lagrangian (3) for the potential (43) takes the form

$$L = \frac{1}{\kappa} \sqrt{1 - \frac{(T^2 - 2tT + x_1^2 - x_2^2)^2}{4T^2 \left(t^2 - \frac{t(T^2 + x_1^2 - x_2^2)}{T} + x_1^2 \right)}} \times \left(t^2 - \frac{t(T^2 + x_1^2 - x_2^2)}{T} + x_1^2 \right)^{-\frac{1}{2}}, \quad (46)$$

and it is obvious this Lagrangian is very complicated to be quantized, in particular, by path integral method. Because of that we use the CCT we introduced in Section

2, to find a locally equivalent canonical Lagrangian. We choose

$$F^{-1}(x) = \int^x \frac{dx}{V(x)} = \frac{1}{2}\kappa x^2, \quad (47)$$

which leads to the full generating function of the form

$$G(\tilde{x}, p) = -pF(\tilde{x}) = -p\sqrt{\frac{2}{\kappa}}\tilde{x}. \quad (48)$$

Applying this transformation, the equation of motion (44) takes a very simple form

$$\ddot{\tilde{x}}(t) - \kappa = 0, \quad (49)$$

and the corresponding quadratic Lagrangian [13, 14] is

$$L_{quad}(\tilde{x}, \dot{\tilde{x}}) = \frac{1}{2}\dot{\tilde{x}}^2 + \kappa\tilde{x} \quad (50)$$

The equation of motion (49) has the same form as the particle-system with constant (repulsive) force. The solution is well known and for the initial and final conditions $\tilde{x}(0) = \tilde{x}_1$ and $\tilde{x}(T) = \tilde{x}_2$ the classical trajectory can be written as

$$\tilde{x}(t) = \frac{\kappa}{2}(t^2 - tT) + \frac{t}{T}(\tilde{x}_2 - \tilde{x}_1) + \tilde{x}_1. \quad (51)$$

The classical action is

$$S_{cl} = \int_0^T L_{quad} dt = \frac{1}{2T}(\tilde{x}_1 - \tilde{x}_2)^2 + \frac{\kappa T}{2}(\tilde{x}_1 + \tilde{x}_2) - \frac{\kappa^2 T^3}{24}. \quad (52)$$

The action is quadratic and it can be quantized directly using the path integral method. The transition amplitude for the quadratic action is calculated form [21]:

$$K(\tilde{x}_2, T; \tilde{x}_1, 0) = \sqrt{-\frac{1}{2\pi\hbar i} \frac{\partial^2 S_{cl}}{\partial \tilde{x}_1 \partial \tilde{x}_2}} e^{i\frac{S_{cl}}{\hbar}}. \quad (53)$$

In our case, the propagator for the action (52) becomes

$$\begin{aligned} & K(\tilde{x}_2, T; \tilde{x}_1, 0) = \\ & = \sqrt{-\frac{i}{2\pi\hbar T}} \exp\left(-\frac{i\left(\kappa T^4 - 12\kappa T^2(\tilde{x}_1 + \tilde{x}_2) - 12(\tilde{x}_1 - \tilde{x}_2)^2\right)}{24\hbar T}\right), \end{aligned} \quad (54)$$

or using the equivalent form [22, 23]:

$$\begin{aligned}
K_\infty(\tilde{x}_2, T; \tilde{x}_1, 0) &= \lambda_\infty \left(-\frac{1}{2h} \frac{\partial^2 S_{cl}}{\partial \tilde{x}_1 \partial \tilde{x}_2} \right) \left| \frac{1}{h} \frac{\partial^2 S_{cl}}{\partial \tilde{x}_1 \partial \tilde{x}_2} \right|_\infty^{1/2} \times \\
&\quad \chi_\infty \left(-\frac{1}{h} S_{cl}(\tilde{x}_2, T; \tilde{x}_1, 0) \right) = \\
&\quad \lambda_\infty \left(\frac{1}{2hT} \right) \left| -\frac{1}{hT} \right|_\infty^{1/2} \chi_\infty \left(-S_{cl}(\tilde{x}_2, T; \tilde{x}_1, 0) \right). \quad (55)
\end{aligned}$$

It allows us, at least in principle, to describe quantum dynamics of a tachyonic system with potential (43), respectively (24) and (33), in non-relativistic quantum limit.

3.4. p -ADIC SHORT CONSIDERATION

Our consideration and calculation until now was done on the real spaces, *i.e.* when all quantities are parametrized by real numbers. However, as it was already stated it would be very important to develop such formalism on nonarchimedean spaces, *i.e.* on p -adic number fields (p is prime number). That can be achieved mathematically, *i.e.* formally, by changing the number field from $R = Q_\infty$ to Q_p and using p -adic numbers and complex (wave) functions with p -adic arguments [2, 4].

p -Adic string theory is a theory of a scalar field with infinitely many space-time derivatives [26]. Regarding cosmological inflation, this p -adic tachyon field model succeeds with inflation where inflatary (tachyonic) models based on the real string theory fail [26, 27]. In fact, tachyon-inflaton field, or equivalently a tachyon-like particle in these model rolls slowly in the conventional sense and drives a sufficiently long period of inflation. In short, for small p , the p -adic field potential is flat enough and slow roll inflation proceeds in the usual manner. In addition, for very big p the potential is very steep, but the p -adic scalar field rolls slowly, as a consequence of the nonlocal nature of the theory [27]. Despite some progress at a classical level, the quantum aspect of p -adic tachyon field (tachyon-like particle) is still unknown, *i.e.* the origin of tachyon driven inflation, in the concrete case.

Moving back to the theory we discussed here, it was shown [22, 23] that the propagator K_p for a quadratic action, and prime number p , can be written as

$$\begin{aligned}
K_p(\tilde{x}_2, T; \tilde{x}_1, 0) &= \\
&= \lambda_p \left(-\frac{1}{2} \frac{\partial^2 S_{cl}}{\partial \tilde{x}_2 \partial \tilde{x}_1} \right) \left| \frac{\partial^2 S_{cl}}{\partial \tilde{x}_2 \partial \tilde{x}_1} \right|_p^{1/2} \chi_p \left(-S_{cl}(\tilde{x}_2, T; \tilde{x}_1, 0) \right), \quad (56)
\end{aligned}$$

where we set $h = 1$ for simplicity. p -Adic additive character [4] is defined as

$$\chi_p(a) = e^{2\pi i \{a\}_p}, \quad (57)$$

where $\{a\}_p$ is the fractional part of the p -adic number a . The function λ_p is an arithmetic complex-valued function of a p -adic variable. The basic properties of the λ_p are

$$\lambda_p(0) = 1, \quad \lambda_p(a^2b) = \lambda_p(b), \quad |\lambda_p(b)|_\infty = 1, \quad (58)$$

$$\lambda_p(a) = 1, \quad |a|_p = p^{2\gamma}, \quad \gamma \in Z. \quad (59)$$

Finally, we can write the corresponding transition amplitude (56) for our model (43) in the p -adic case, more precisely for any prime p except $p = 2$, in the form

$$K_p(\tilde{x}_2, T; \tilde{x}_1, 0) = \lambda_p\left(\frac{1}{2T}\right) \left| -\frac{1}{T} \right|_p^{1/2} \chi_p\left(-S_{cl}(\tilde{x}_2, T; \tilde{x}_1, 0)\right), \quad (60)$$

where $S_{cl}(\tilde{x}_2, T; \tilde{x}_1, 0)$ is given in (52). It allows us to “determine”, or to describe quantum dynamics of a tachyon model, and to fix the vacuum state and consider its (in)stability and transformation. For some mathematical details, adelic generalization and constrains see for example [2, 4].

4. CONCLUSION

In this paper we discussed the dynamics of the tachyonic field possibly motivated by its important role in inflation. We started with the tachyonic (DBI type) Lagrangian, which is highly nonlinear and not suitable for quantization and showed that it is possible to find a locally equivalent canonical Lagrangian applying (local) classical canonical transformations.

We made a review of two well-known tachyonic models, discussed the power function case in more details and calculated the corresponding action. The classical action in all cases was transformed to a quadratic form, which allowed us to quantize these systems via the standard Feynman-path integral approach and to calculate the corresponding propagator in the real and p -adic case. The next natural step is adelic generalization of the model and consideration of vacuum state (in)stability.

Cosmological application of this investigation is not a straightforward task. For future work we propose the application of these results towards cosmology in the Friedmann-Robertson-Walker limits (3+1 dimensions instead of 1-dimensional case we considered) and “baby” universe approach [28] when instability of a physical vacuum can be driven by the p -adic sector of tachyon background field at the Planck scale.

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