DYNAMICAL ASPECTS
OF A MASSLESS TENSOR GAUGE FIELD OF DEGREE $(k+1)$

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This paper aims to the determination of the number of degrees of freedom and also a generating set of gauge transformations for a theory whose dynamics is governed by a second-order Lagrangian that describes the evolution of an Abelian tensor gauge field of degree $(k+1)$

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1. MOTIVATION

The last decades have recorded a growing interest in the study of massless tensor gauge fields with the mixed symmetry [1]–[8] [here are only a few systematic contributions but the list is huge]. At present, the main reason for investigating such gauge fields is due to the fact that they relativistically describe particles with higher spins that naturally appear in string field theory [as vibrating modes]. Another reason for analyzing such entities comes from theirs implications in gravitational theories. Precisely, there are species of massless tensor gauge fields [among those that corespond to a Young diagram with two columns] that are duals of the Pauli-Fierz field [9], [10]. In this context it has been carefully analyzed the consistent interactions that involve the duals of the Pauli-Fierz field and various fields [11]–[13]. In all the above mentioned quests the massless tensor gauge fields that transforms under irreducible representations of the Lorentz group are involved alone, as given objects. Starting with this remark, the present attempt deals with a tensor gauge field that describes, as limit situations, a $(k+1)$-form or a massless tensor gauge field with the mixed symmetry $(k,1)$. The idea of such analysis finds its roots in a work of Einstein [14] [and exhausted by continuators [15], [16]] performed in order to unify gravity with electromagnetism.

The aim of this paper consists in the canonical analysis of a massless tensor gauge field of degree $(k+1)$, $A_{\mu_1\cdots\mu_k\|\alpha}$, that is antisymmetric in the first k Lorentz indices

$$A_{\mu_1\cdots\mu_k\|\alpha} = \frac{1}{k!}A_{[\mu_1\cdots\mu_k]\|\alpha}$$

and subject to the gauge transformations

$$\delta_\epsilon A_{\mu_1\cdots\mu_k\|\alpha} = \partial_{[\mu_1}\epsilon_{\mu_2\cdots\mu_k]\|\alpha}. \tag{1}$$

In the above, the bosonic gauge parameters $\epsilon_{\mu_1\cdots\mu_{k-1}\|\alpha}$ are completely antisymmetric in theirs first $(k-1)$ Lorentz indices

$$\epsilon_{\mu_1\cdots\mu_{k-1}\|\alpha} = \frac{1}{(k-1)!}\epsilon_{[\mu_1\cdots\mu_{k-1}]\|\alpha}.$$

The most general second-order Lagrangian density invariant under the gauge transformations (1) reads as

$$\begin{aligned}\mathcal{L}_0 = \tfrac{1}{2(k+1)(k+1)!}\Big(&-\tfrac{(-)^k}{k+1}F_{\mu_1\cdots\mu_{k+1}\|\alpha}F^{\mu_1\cdots\mu_{k+1}\|\alpha} + a_1 F_{\mu_1\cdots\mu_k\beta\|\alpha}F^{\mu_1\cdots\mu_k\alpha\|\beta}\\ &+a_2 F^{\mu_1\cdots\mu_k}F_{\mu_1\cdots\mu_k}\big)\end{aligned} \tag{2}$$

where a_1 and a_2 are arbitrary real constants and $F_{\mu_1\cdots\mu_{k+1}\|\alpha}$ is the field-strength of the tensor gauge field $A_{\mu_1\cdots\mu_k\|\alpha}$ defined in the usual manner

$$F_{\mu_1\cdots\mu_{k+1}\|\alpha} \equiv \partial_{[\mu_1}A_{\mu_2\cdots\mu_{k+1}]\|\alpha}. \tag{3}$$

By $F_{\mu_1\cdots\mu_k}$ we denoted the trace of the field-strength, $F_{\mu_1\cdots\mu_k} \equiv \sigma^{\alpha\beta}F_{\mu_1\cdots\mu_k\beta\|\alpha}$. Throughout the paper we work with the flat metric of 'mostly minus' signature $\sigma_{\mu\nu} = (+-\cdots-)$. The notation $[\mu\ldots\nu]$ $((\mu\ldots\nu))$ signifies full antisymmetry (symmetry) with respect to the indices between brackets without normalization factors [*i.e.* the independent terms appear only once and are not multiplied by overall numerical factors].

The proposed tensor gauge field brings together some tensor gauge fields that transforms under irreducible representations of the Lorentz group as we shall see in the following. In view ot this we decompose the gauge field $A_{\mu_1\cdots\mu_k\|\alpha}$ into its "irreducible" components

$$\begin{aligned}A_{\mu_1\cdots\mu_k\|\alpha} &\equiv t_{\mu_1\cdots\mu_k|\alpha} + B_{\mu_1\cdots\mu_k\alpha}\\ &\equiv \left(A_{\mu_1\cdots\mu_k\|\alpha} - \tfrac{1}{k+1}A_{[\mu_1\cdots\mu_k\|\alpha]}\right) + \left(\tfrac{1}{k+1}A_{[\mu_1\cdots\mu_k\|\alpha]}\right),\end{aligned} \tag{4}$$

then the Lagrangian density (2) becomes

$$\begin{aligned}\mathcal{L}_0 = &-\tfrac{(-)^k+\left(k^2+k+1\right)a_1+a_2}{2(k+1)^2(k+2)!}\left(H_{\mu_1\cdots\mu_{k+2}}\right)^2 + \tfrac{a_1+a_2-(-)^k}{2(k+1)^2(k+1)!}\left(\partial_\alpha B_{\mu_1\cdots\mu_{k+1}}\right)^2\\ &+\tfrac{a_1-(-)^k}{2(k+1)^2(k+1)!}\left(\mathcal{F}_{\mu_1\cdots\mu_{k+1}|\alpha}\right)^2 + \tfrac{a_2}{2(k+1)(k+1)!}\left(\mathcal{F}_{\mu_1\cdots\mu_k}\right)^2\\ &+(-)^k\,\tfrac{a_1+a_2-(-)^k}{(k+1)^2(k+1)!}\mathcal{F}_{\mu_1\cdots\mu_{k+1}|\alpha}\partial^\alpha B^{\mu_1\cdots\mu_{k+1}} + \partial_\mu j^\mu.\end{aligned} \tag{5}$$

In the local function (5) $H_{\mu_1\cdots\mu_{k+2}}$ stands for the field-strength of the $(k+1)$-form denoted by $B_{\mu_1\cdots\mu_{k+1}}$

$$H_{\mu_1\cdots\mu_{k+2}} \equiv \partial_{[\mu_1}B_{\mu_2\cdots\mu_{k+2}]},$$

the tensor $\mathcal{F}_{\mu_1\cdots\mu_{k+1}|\alpha}$ is defined by

$$\mathcal{F}_{\mu_1\cdots\mu_{k+1}|\alpha} \equiv \partial_{[\mu_1} t_{\mu_2\cdots\mu_{k+1}]|\alpha}$$

and $\mathcal{F}_{\mu_1\cdots\mu_k}$ is nothing but the trace of $\mathcal{F}_{\mu_1\cdots\mu_{k+1}|\alpha}$

$$\mathcal{F}_{\mu_1\cdots\mu_k} \equiv \sigma^{\alpha\beta}\mathcal{F}_{\mu_1\cdots\mu_k\alpha|\beta}.$$

Based on the local function (5) we get the Lagrangian action

$$\begin{aligned}
S_0^{\mathrm{L}}\left[t_{\mu_1\cdots\mu_k|\alpha}, B_{\mu_1\cdots\mu_{k+1}}\right] = \int \mathrm{d}^D x \Big[(-)^k \tfrac{a_1+a_2-(-)^k}{(k+1)^2(k+1)!}\mathcal{F}_{\mu_1\cdots\mu_{k+1}|\alpha}\partial^\alpha B^{\mu_1\cdots\mu_{k+1}} \\
-\tfrac{(-)^k k+\left(k^2+k+1\right)a_1+a_2}{2(k+1)^2(k+2)!}\left(H_{\mu_1\cdots\mu_{k+2}}\right)^2 + \tfrac{a_1+a_2-(-)^k}{2(k+1)^2(k+1)!}\left(\partial_\alpha B_{\mu_1\cdots\mu_{k+1}}\right)^2 \\
+\tfrac{a_1-(-)^k}{2(k+1)^2(k+1)!}\left(\mathcal{F}_{\mu_1\cdots\mu_{k+1}|\alpha}\right)^2 + \tfrac{a_2}{2(k+1)(k+1)!}\left(\mathcal{F}_{\mu_1\cdots\mu_k}\right)^2\Big]
\end{aligned} \tag{6}$$

that governs the dynamics of the 'irreducible' components $t_{\mu_1\cdots\mu_k|\alpha}$ and $B_{\mu_1\cdots\mu_k\alpha}$ that transform under irreducible representations of the Lorentz group. One can simply proof that the Lagrangian action (6) rightly describes the dynamics of the 'irreducible modes' of the starting gauge field. Indeed, if we set in (6)

$$a_1 = (-)^k, \quad a_2 = 0, \tag{7}$$

then the gauge field $t_{\mu_1\cdots\mu_k|\alpha}$ with the mixed symmetry $(k,1)$ becomes a pure one [it does not appear in the Lagrangian action]. Furthermore, by taking

$$a_1 = -\frac{(-)^k}{k}, \quad a_2 = (-)^k\,\frac{k+1}{k}, \tag{8}$$

in the local functional (6) then $(k+1)$-form $B_{\mu_1\cdots\mu_{k+1}}$ becomes a pure gauge field.

The above analysis suggests that two of the tensor gauge fields of degree equal to $(k+1)$ which transform under irreducible representations of the Lorentz group can be treated in unified manner through the gauge field $A_{\mu_1\cdots\mu_k\|\alpha}$ whose dynamics is governed by the Lagrangian density (2). This remark justifies the Dirac analysis of the starting gauge model (2).

2. DIRAC ANALYSIS

In this section we perform the canonical analysis [17]–[19] of the model with the Lagrangian density (2). In view of this, if we denote by $\pi_{\mu_1\cdots\mu_k\|\alpha}$ the canonical momenta associated with the fields $A^{\mu_1\cdots\mu_k\|\alpha}$, the definitions of the formers lead to

$$\begin{aligned}
\pi_{\mu_1\cdots\mu_k\|\alpha} &\equiv \frac{1}{k!}\frac{\partial\mathcal{L}_0}{\partial\dot{A}^{[\mu_1\cdots\mu_k]\|\alpha}} \\
&= \tfrac{(-)^{k+1}}{(k+1)(k+1)!}\left(F_{0\mu_1\cdots\mu_k\|\alpha} - a_1 F_{\alpha[0\mu_1\cdots\mu_{k-1}\|\mu_k]} - a_2\sigma_{\alpha[0}F_{\mu_1\cdots\mu_k]}\right).
\end{aligned} \tag{9}$$

In the above we denoted by dot the derivative in respect with the temporal coordinate x^0. From the definitions (9) we infer the primary constraints

$$G^{(1)}_{i_1\cdots i_{k-1}} \equiv \pi_{0i_1\cdots i_{k-1}\|0} \approx 0, \quad G^{(1)}_{i_1\cdots i_{k-1}\|j} \equiv \pi_{0i_1\cdots i_{k-1}\|j} \approx 0 \tag{10}$$

and also the relations

$$\pi_{i_1\cdots i_k\|0} = \tfrac{1}{(k+1)(k+1)!}\left[\left(a_1 + a_2 - (-)^k\right) F_{0i_1\cdots i_k\|0} + (-)^k a_2 F'_{i_1\cdots i_k}\right], \tag{11a}$$

$$\pi_{i_1\cdots i_k\|j} = \tfrac{(-)^{k+1}}{(k+1)(k+1)!}\left(F_{0i_1\cdots i_k\|j} - a_1 F_{j[0i_1\cdots i_{k-1}\|i_k]} - (-)^k a_2 \sigma_{j[i_1} F_{i_2\cdots i_k]0}\right). \tag{11b}$$

In formula (11a) were made use of the notations

$$F'_{i_1\cdots i_k} \equiv \sigma^{jl} F_{i_1\cdots i_k j\|l}, \quad \pi'_{i_1\cdots i_{k-1}} = \sigma^{jl}\pi_{i_1\cdots i_{k-1}j\|l} \tag{12}$$

Starting with the equations (11b), only by algebraic computations, we derive

$$\pi'_{i_1\cdots i_{k-1}} = \tfrac{a_1 + a_2(D-k) - (-)^k}{(k+1)(k+1)!} F_{0i_1\cdots i_{k-1}}, \tag{13a}$$

$$\pi_{[i_1\cdots i_k\|i_{k+1}]} = \tfrac{(-)^k}{(k+1)!}\left(a_1 F_{i_1\cdots i_{k+1}\|0} - (-)^k \tfrac{k a_1 + (-)^k}{(k+1)} F_{0[i_1\cdots i_k\|i_{k+1}]}\right). \tag{13b}$$

The first step in the canonical analysis is completed by solving the equations (11a)–(11b) in respect with some of the generalized velocities. In view of this, the results (13a)–(13b) lead to seven distinct situations [dictated by the factors that multiply the *temporal components* of the field-strength in (11a) and (13a)–(13b)], namely

$$a_1 = (-)^k, \quad a_2 = 0; \tag{14}$$

$$a_1 = -\frac{(-)^k}{k}, \quad a_2 = (-)^k \frac{k+1}{k}; \tag{15}$$

$$a_1 = -\frac{(-)^k}{k}, \quad a_2 = (-)^k \frac{k+1}{k(D-k)}; \tag{16}$$

$$a_1 = -\frac{(-)^k}{k}, \quad a_2 \equiv a \in \mathbb{R}\setminus\left\{(-)^k \frac{k+1}{k}, (-)^k \frac{k+1}{k(D-k)}\right\}; \tag{17}$$

$$a_1 \equiv \bar{a} \in \mathbb{R}\setminus\left\{-\frac{(-)^k}{k}, (-)^k\right\}, \quad a_2 = \frac{(-)^k - \bar{a}}{D-k}; \tag{18}$$

$$a_1 \equiv \tilde{a} \in \mathbb{R}\setminus\left\{-\frac{(-)^k}{k}, (-)^k\right\}, \quad a_2 = (-)^k - \tilde{a}; \tag{19}$$

$$a_1 + a_2 \neq (-)^k \neq a_1 + (D-k)\, a_2, \quad a_1 \in \mathbb{R}\setminus\left\{(-)^k\right\}. \tag{20}$$

In the remaining part of this section we will complete the canonical analysis of the model in each of the seven situations delimited in the above.

2.1. CASE I

In the situation (14) the Lagrangian density (2) becomes

$$\mathcal{L}_0^{(I)} = \frac{(-)^k}{2(k+1)(k+1)!}\left[-\frac{1}{k+1}(F_{\mu_1\cdots\mu_{k+1}\|\alpha})^2 + F_{\mu_1\cdots\mu_k\beta\|\alpha}F^{\mu_1\cdots\mu_k\alpha\|\beta}\right] \tag{21}$$

and the definitions of the canonical momenta (9) lead to the independent primary constraints (10) and

$$\gamma^{(1)}_{i_1\cdots i_k} \equiv \pi_{i_1\cdots i_k\|0} \approx 0, \tag{22a}$$

$$\gamma^{(1)}_{i_1\cdots i_k|j} \equiv \pi_{i_1\cdots i_k\|j} - \frac{1}{k+1}\pi_{[i_1\cdots i_k\|j]} \approx 0. \tag{22b}$$

Expressing from the equations (9) [corresponding to the choice (14) of the real parameters a_1 and a_2] some of the generalized velocities, we get the canonical Hamiltonian density

$$\begin{aligned}\mathcal{H}_0^{(I)} &= -kA^{0i_1\cdots i_{k-1}\|j}\partial^l\pi_{li_1\cdots i_{k-1}\|j} - \frac{(-)^k k!}{2}\pi_{i_1\cdots i_k\|j}\pi^{[i_1\cdots i_k\|j]} \\ &+ \frac{(-)^k}{(k+1)^2}\pi^{[i_1\cdots i_k\|j]}F_{i_1\cdots i_k j\|0} + \frac{(-)^k}{2(k+1)^2(k+2)!}\left(F_{[i_1\cdots i_{k+1}\|j]}\right)^2.\end{aligned} \tag{23}$$

Direct computations show the Abelian character of the constraints (10) and (22a)–(22b). Therefore, the consistency of the primary constraints reduces only to the computation of the Poisson brackets between the canonical Hamiltonian and the primary constraints. It can be checked the consistency of the primary constraints produces secondary constraints

$$\gamma^{(2)}_{i_1\cdots i_k} \equiv \frac{1}{k+1}\partial^j\pi_{[i_1\cdots i_k\|j]} \approx 0, \tag{24}$$

that are off-shell reducible of order $L = k$, with the reducibility functions

$$\left(Z_{j_1\cdots j_{k-p-1}}\right)^{i_1\cdots i_{k-p}} = \partial^{[i_1}\delta^{i_2}_{j_1}\cdots\delta^{i_{k-p}]}_{j_{k-p-1}}, \quad p = \overline{0,k-1}. \tag{25}$$

Asking for the secondary constraints (23) to be preserved in time we get no more tertiary constraint as the constraints (22a)–(22b) and (24) are Abelian and

$$\left[\gamma^{(2)}_{i_1\cdots i_k}, H_0^{(I)}\right] = 0. \tag{26}$$

The arguments in the above allow us to conclude that the canonical Hamiltonian is exactly the first-class Hamiltonian of the system. The irreducible character of the first-class constraints (10) and (22a)–(22b), together with the reducibilities of order $L = p$ displayed by the constraints (24) allow us to compute the number of degrees of freedom for the model under study

$$N^{(I)}_{DOF} = \begin{pmatrix} D-2 \\ k+1 \end{pmatrix}. \tag{27}$$

Finally, on behalf of the Dirac's conjecture [according to which any first-class constraint generates gauge transformations], if we pass again to the Lagrangian formulation [*via* extended action], we derive for the functional associated with (21) the generating set of gauge transformations

$$\delta_{\epsilon,\xi}^{(I)} A_{\mu_1\cdots\mu_k\|\alpha} = \partial_{[\mu_1}\epsilon_{\mu_2\cdots\mu_k\alpha]} + \xi_{\mu_1\cdots\mu_k|\alpha}, \tag{28}$$

where the gauge parameters $\epsilon_{\mu_1\cdots\mu_k}$ are completely antisymmetric

$$\epsilon_{\mu_1\cdots\mu_k} = \tfrac{1}{k!}\epsilon_{[\mu_1\cdots\mu_k]} \tag{29}$$

and $\xi_{\mu_1\cdots\mu_k|\alpha}$ have the mixed symmetry $(k,1)$

$$\xi_{\mu_1\cdots\mu_k|\alpha} = \tfrac{1}{k!}\xi_{[\mu_1\cdots\mu_k]|\alpha}, \quad \xi_{[\mu_1\cdots\mu_k|\alpha]} = 0. \tag{30}$$

2.2. CASE II

In this part, we complete the canonical analysis of the model (2) in which the real parameters a_1 and a_2 are taken as in (15). For this choice, the Lagrangian density (2) becomes

$$\begin{aligned}\mathcal{L}_0^{(II)} = \tfrac{(-)^{k+1}}{2(k+1)(k+1)!}\Big[&\tfrac{1}{k+1}\left(F_{\mu_1\cdots\mu_{k+1}\|\alpha}\right)^2 + \tfrac{1}{k}F_{\mu_1\cdots\mu_k\beta\|\alpha}F^{\mu_1\cdots\mu_k\alpha\|\beta} \\ &-\tfrac{k+1}{k}\left(F_{\mu_1\cdots\mu_k}\right)^2\Big].\end{aligned} \tag{31}$$

Setting (15) in the definitions of the canonical momenta (9) one obtains, besaides the independent primary constraints (10), also the constraints

$$\bar{\gamma}^{(1)}_{i_1\cdots i_k} \equiv \pi_{i_1\cdots i_k\|0} - \tfrac{1}{k(k+1)!}F'_{i_1\cdots i_k} \approx 0, \tag{32a}$$

$$\bar{\gamma}^{(1)}_{i_1\cdots i_{k+1}} \equiv \pi_{[i_1\cdots i_k\|i_{k+1}]} + \tfrac{1}{k(k+1)!}F_{i_1\cdots i_{k+1}\|0} \approx 0. \tag{32b}$$

Expressing from the equations (11b) some of the generalized velocities, we derive the canonical Hamiltonian density

$$\begin{aligned}\mathcal{H}_0^{(II)} = &-kA^{0i_1\cdots i_{k-1}\|\mu}\left(\partial^l \pi_{li_1\cdots i_{k-1}\|\mu}\right) + \tfrac{(-)^k}{2(k+1)^2(k+1)!}\left(F_{i_1\cdots i_{k+1}\|\mu}\right)^2 \\ &+ \tfrac{(-)^k}{2k(k+1)(k+1)!}F_{i_1\cdots i_k j\|l}F^{i_1\cdots i_k l\|j} - \tfrac{(-)^k}{2k(k+1)!}\left(F'_{i_1\cdots i_k}\right)^2 \\ &- \tfrac{(-)^k k(k+1)!}{2}\left(\pi_{i_1\cdots i_k\|j}\right)^2 - \tfrac{(-)^k}{2(k+1)}\pi_{i_1\cdots i_k\|j}F^{i_1\cdots i_k j\|0} \\ &+ \tfrac{(-)^k k^2(k+1)!}{2(D-k-1)}\left(\pi'_{i_1\cdots i_{k-1}}\right)^2.\end{aligned} \tag{33}$$

It can be checked that the primary constraints set [consisting in (10) and (32a)–(32b)] is an Abelian one such that the consistency of the primary constraints reduces only to the calculations of the Poisson brackets between them and canonical Hamiltonian (33)

$$\left[G^{(1)}_{i_1\cdots i_{k-1}}, H^{(II)}_0\right] = \partial^j \pi_{ji_1\cdots i_{k-1}\|0} \equiv G^{(2)}_{i_1\cdots i_{k-1}} \approx 0, \tag{34a}$$

$$\left[G^{(1)}_{i_1\cdots i_{k-1}\|j}, H^{(II)}_0\right] = \partial^l \pi_{li_1\cdots i_{k-1}\|j} \equiv G^{(2)}_{i_1\cdots i_{k-1}\|j} \approx 0, \tag{34b}$$

$$\left[\bar{\gamma}^{(1)}_{i_1\cdots i_k}, H^{(II)}_0\right] = \tfrac{2k+1}{2(k+1)} \partial^j \bar{\gamma}^{(1)}_{i_1\cdots i_k j} - G^{(2)}_{[i_1\cdots i_{k-1}\|i_k]} \approx 0. \tag{34c}$$

The results (34a)–(34c) display the secondary constraints

$$G^{(2)}_{i_1\cdots i_{k-1}} \approx 0, \quad G^{(2)}_{i_1\cdots i_{k-1}\|j} \approx 0, \tag{35}$$

that together with (10) and (32a)–(32b) constitute an Abelian set of constraints. The secondary constraints displayed in the above are off-shell reducible of order $L = k-1$ with the reducibility functions given by

$$\left(Z_{j_1\cdots j_{k-p-2}}\right)^{i_1\cdots i_{k-1-p}} = \partial^{[i_1} \delta^{i_2}_{j_1} \cdots \delta^{i_{k-p}]}_{j_{k-p-1}}, \quad p = \overline{0, k-2}, \tag{36a}$$

$$\left(Z_{j_1\cdots j_{k-p-2}\|j}\right)^{i_1\cdots i_{k-1-p}\|i} = \delta^i_j \left(Z_{j_1\cdots j_{k-p-2}}\right)^{i_1\cdots i_{k-1-p}}, \quad p = \overline{0, k-2}. \tag{36b}$$

The consistency of the secondary constraints (35) does not produce any new constraint so the Dirac algorithm stops at this level.

At this stage, we can conclude that the canonical Hamiltonian (33) is exactly the first-class Hamiltonian of the system. The irreducible character of the first-class constraints (10) and (32a)–(32b) together with the $L = k-1$ reducibilities of the secondary constraints (35) allow us to count the number degrees of freedom for the model under study

$$N^{(II)}_{DOF} = D \begin{pmatrix} D-2 \\ k \end{pmatrix} - \begin{pmatrix} D \\ k+1 \end{pmatrix}. \tag{37}$$

Finally, if we pass again to the Lagrangian formulation [*via* extended action], we derive for the local function (31) the generating set of gauge transformations

$$\delta^{(II)}_{\epsilon,\xi} A_{\mu_1\cdots\mu_k\|\alpha} = \epsilon_{\mu_1\cdots\mu_k\alpha} + \partial_{[\mu_1}\epsilon_{\mu_2\cdots\mu_k]\alpha} - (-)^k k \partial_\alpha \epsilon_{\mu_1\cdots\mu_k} + \partial_{[\mu_1}\xi_{\mu_2\cdots\mu_k]|\alpha}. \tag{38}$$

In formula (38) the gauge parameters of ϵ-type are completely antisymmetric and $\xi_{\mu_1\cdots\mu_{k-1}|\alpha}$ display the mixed symmetry $(k-1,1)$

$$\xi_{\mu_1\cdots\mu_{k-1}|\alpha} = \tfrac{1}{(k-1)!}\xi_{[\mu_1\cdots\mu_{k-1}]|\alpha}, \quad \xi_{[\mu_1\cdots\mu_{k-1}|\alpha]} = 0. \tag{39}$$

2.3. CASE III

For the choice (16) of the parameters a_1 and a_2 the Lagrangian density (2) takes the form

$$\mathcal{L}_0^{(III)} = \frac{(-)^{k+1}}{2(k+1)(k+1)!}\left[\frac{1}{k+1}\left(F_{\mu_1\cdots\mu_{k+1}\|\alpha}\right)^2 + \frac{1}{k}F_{\mu_1\cdots\mu_k\beta\|\alpha}F^{\mu_1\cdots\mu_k\alpha\|\beta} - \frac{k+1}{k(D-k)}\left(F_{\mu_1\cdots\mu_k}\right)^2\right].$$

Invoking again the definitions (9) of the canonical momenta we derive the primary constraints set consisting in (10) and

$$\gamma^{(1)}_{i_1\cdots i_{k-1}} \equiv \pi'_{i_1\cdots i_{k-1}} \approx 0, \tag{40a}$$

$$\bar{\gamma}^{(1)}_{i_1\cdots i_{k+1}} \equiv \pi_{[i_1\cdots i_k\|i_{k+1}]} + \frac{1}{k(k+1)!}F_{i_1\cdots i_{k+1}\|0} \approx 0. \tag{40b}$$

Moreover, according with the general scheme [17]–[19], by solving the corresponding equations (9) in respect with some of the generalized velocities, we derive the canonical Hamiltonian

$$\begin{aligned}\mathcal{H}_0^{(III)} = & -kA^{0i_1\cdots i_{k-1}\|\mu}\left(\partial^l\pi_{li_1\cdots i_{k-1}\|\mu}\right) + \frac{(-)^k}{2(k+1)^2(k+1)!}\left(F_{i_1\cdots i_{k+1}\|\mu}\right)^2 \\ & - (-)^k\frac{k(k+1)!}{2}\left(\frac{D-k}{D-k-1}\left(\pi_{i_1\cdots i_k\|0}\right)^2 + \left(\pi_{i_1\cdots i_k\|j}\right)^2\right) \\ & - \frac{(-)^k}{2k(D-k-1)(k+1)!}\left(F'_{i_1\cdots i_k}\right)^2 + \frac{(-)^k}{2k(k+1)(k+1)!}F^{i_1\cdots i_kj\|l}F_{i_1\cdots i_kl\|j} \\ & + \frac{(-)^k}{D-k-1}\pi_{i_1\cdots i_k\|0}F'^{i_1\cdots i_k} - \frac{(-)^k}{2(k+1)}\pi_{i_1\cdots i_k\|j}F^{i_1\cdots i_kj\|0}.\end{aligned} \tag{41}$$

As in the other two cases, the primary constraints set (10) and (40a)–(40b) is Abelian. Due to this, the compelling for the primary constraints to be preserved in time reduces only to the computation of the Poisson brackets between them and canonical Hamiltonian (41)

$$\left[G^{(1)}_{i_1\cdots i_{k-1}}, H_0^{(III)}\right] = \partial^j\pi_{ji_1\cdots i_{k-1}\|0} \equiv G^{(2)}_{i_1\cdots i_{k-1}} \approx 0, \tag{42a}$$

$$\left[G^{(1)}_{i_1\cdots i_{k-1}\|j}, H_0^{(III)}\right] = \partial^l\pi_{li_1\cdots i_{k-1}\|j} \equiv G^{(2)}_{i_1\cdots i_{k-1}\|j} \approx 0, \tag{42b}$$

$$\left[\gamma^{(1)}_{i_1\cdots i_{k-1}}, H_0^{(III)}\right] = (-)^k G^{(2)}_{i_1\cdots i_{k-1}} \approx 0, \tag{42c}$$

$$\begin{aligned}\left[\bar{\gamma}^{(1)}_{i_1\cdots i_{k+1}}, H_0^{(III)}\right] & = -(-)^k\left(\partial_{[i_1}\pi_{i_2\cdots i_{k+1}]\|0} - \frac{(-)^k}{k(k+1)!}\partial^jF_{i_1\cdots i_{k+1}\|j}\right) \\ & \equiv -(-)^k\bar{\gamma}^{(2)}_{i_1\cdots i_{k+1}} \approx 0.\end{aligned} \tag{43}$$

The results (42a)–(43) put into evidence the secondary constraints

$$G^{(2)}_{i_1\cdots i_{k-1}} \approx 0, \quad G^{(2)}_{i_1\cdots i_{k-1}\|j} \approx 0, \quad \bar{\gamma}^{(2)}_{i_1\cdots i_{k+1}} \approx 0. \tag{44}$$

that together with the primary constraints (10) and (40a)–(40b) constitute an Abelian set of constraints.

Combining now the Abelian character of the constraints set (10), (40a)–(40b) and (44) with the results

$$\left[G^{(2)}_{i_1\cdots i_{k-1}}, H^{(III)}_0\right] \approx 0,\ \left[G^{(2)}_{i_1\cdots i_{k-1}\|j}, H^{(III)}_0\right] \approx 0,\ \left[\bar{\gamma}^{(2)}_{i_1\cdots i_{k+1}}, H^{(III)}_0\right] \approx 0$$

we conclude that the Dirac algorithm stops at this stage.

The concrete expressions of the first-class constraints (10), (40a)–(40b) and (44) evidence that firstly, the constraints $\bar{\gamma}^{(2)}_{i_1\cdots i_{k+1}} \approx 0$ are off-shell reducible of order $(D-k-2)$ with the reducibility functions

$$\left(Z_{j_1\cdots j_{k+p+1}}\right)^{i_1\cdots i_{k+p}} = \partial_{[j_1}\delta^{i_1}_{j_2}\cdots\delta^{i_{k+p}}_{j_{k+p+1}]},\quad k=\overline{1,D-k-2} \tag{45}$$

secondly, the constraints (35) are $L=k-1$ off-shell reducible with the reducibility functions given in (36a)–(36b), thirdly the constraints $\gamma^{(1)}_{i_1\cdots i_{k-1}} \approx 0$ and $G^{(2)}_{i_1\cdots i_{k-1}\|j} \approx 0$ are off-shell first order reducible

$$\left(\partial^{[i_1}\delta^{i_2}_{j_1}\cdots\delta^{i_{k-1}]}_{j_{k-2}}\right)\gamma^{(1)}_{i_1\cdots i_{k-1}} + \left(-\delta^{[i_1}_{j_1}\cdots\delta^{i_{k-2}}_{j_{k-2}}\sigma^{i_{k-1}]j}\right)G^{(2)}_{i_1\cdots i_{k-1}\|j} = 0, \tag{46}$$

and finally the constraints (10) and (40b) are irreducible. The aforementioned reducibilities of the first-class constraints implies that the number of degrees of freedom is

$$N^{(III)}_{DOF} = D\begin{pmatrix} D-2 \\ k \end{pmatrix} - \begin{pmatrix} D \\ k+1 \end{pmatrix} + \begin{pmatrix} D-2 \\ k-3 \end{pmatrix}. \tag{47}$$

By returning to the Lagrangian formulation of the analyzed model (40) we infer the generating set of gauge transformations

$$\delta^{(III)}_{\epsilon,\xi} A_{\mu_1\cdots\mu_k\|\alpha} = \sigma_{\alpha[\mu_1}\epsilon_{\mu_2\cdots\mu_k]} + \partial^{\sigma}\epsilon_{\sigma\mu_1\cdots\mu_k\alpha} + \partial_{[\mu_1}\epsilon_{\mu_2\cdots\mu_k]\alpha} + \partial_{[\mu_1}\xi_{\mu_2\cdots\mu_k]|\alpha}. \tag{48}$$

In the generating set of gauge transformations (48) the gauge parameters of ϵ-type are completely antisymmetric and $\xi_{\mu_1\cdots\mu_{k-1}|\alpha}$ displays the symmetries (39).

2.4. CASE IV

From the dynamical point of view, this situation is quite similar to the previous one as we shall see in the following. With the choice (17), the Lagrangian density (2) becomes

$$\begin{aligned}\mathcal{L}^{(IV)}_0 \equiv \tfrac{(-)^{k+1}}{2(k+1)(k+1)!}\Big[&\tfrac{1}{k+1}\left(F_{\mu_1\cdots\mu_{k+1}\|\alpha}\right)^2 + \tfrac{1}{k}F_{\mu_1\cdots\mu_k\beta\|\alpha}F^{\mu_1\cdots\mu_k\alpha\|\beta}\\ &-(-)^k a\left(F_{\mu_1\cdots\mu_k}\right)^2\Big],\end{aligned} \tag{49}$$

where a is a real constant with the range given in (17).

The definitions (9) of the canonical momenta put into evidence the primary constraints (10) and (40b) [that are Abelian] and also lead to the canonical Hamiltonian density

$$\begin{aligned}\mathcal{H}_0^{(IV)} &= -kA^{0i_1\cdots i_{k-1}\|\mu}\left(\partial^l \pi_{li_1\cdots i_{k-1}\|\mu}\right) + \tfrac{(-)^k}{2(k+1)^2(k+1)!}\left(F_{i_1\cdots i_{k+1}\|\mu}\right)^2 \\ &- (-)^k \tfrac{k(k+1)!}{2}\left(\tfrac{k+1}{k+1-(-)^k ka}\left(\pi_{i_1\cdots i_k\|0}\right)^2 + \left(\pi_{i_1\cdots i_k\|j}\right)^2\right) \\ &- \tfrac{ak^3(k+1)!}{2\left[k+1-(-)^k ak(D-k)\right]}\left(\pi'_{i_1\cdots i_{k-1}}\right)^2 + \tfrac{ka}{k+1-(-)^k ka}\pi_{i_1\cdots i_k\|0}F'^{i_1\cdots i_k} \\ &- \tfrac{a}{2\left(k+1-(-)^k ka\right)(k+1)!}\left(F'_{i_1\cdots i_k}\right)^2 + \tfrac{(-)^k}{2k(k+1)(k+1)!}F^{i_1\cdots i_k j\|l}F_{i_1\cdots i_k l\|j} \\ &- \tfrac{(-)^k}{2(k+1)}\pi_{i_1\cdots i_k\|j}F^{i_1\cdots i_k j\|0}.\end{aligned} \tag{50}$$

The consistency of the primary constraints displays the same secondary constraints as in the previous situation (44). This is due to the Poisson brackets

$$\begin{aligned}\left[G^{(1)}_{i_1\cdots i_{k-1}}, H_0^{(IV)}\right] &\equiv G^{(2)}_{i_1\cdots i_{k-1}}, \\ \left[G^{(1)}_{i_1\cdots i_{k-1}\|j}, H_0^{(IV)}\right] &\equiv G^{(2)}_{i_1\cdots i_{k-1}\|j}, \\ \left[\bar{\gamma}^{(1)}_{i_1\cdots i_{k+1}}, H_0^{(IV)}\right] &\equiv -(-)^k\,\bar{\gamma}^{(2)}_{i_1\cdots i_{k+1}}.\end{aligned} \tag{51}$$

Concerning the consistency of the secondary constraints (44) this does not imply new constraints because firstly, the constraints (10), (40b) and (44) are Abelian and secondly, the Poisson brackets between the canonical Hamiltonian (50) and secondary constraints (44) weakly vanish.

Invoking again the results derived in the previous situation, we conclude that: the constraints (10) and (40b) are irreducible, the constraints $G^{(2)}_{i_1\cdots i_{k-1}} \approx 0$ and $G^{(2)}_{i_1\cdots i_{k-1}\|j} \approx 0$ are off-shell reducible of order $L = k-1$ with the reducibility functions given in (36a)–(36b) and finally the constraints $\bar{\gamma}^{(2)}_{i_1\cdots i_{k+1}} \approx 0$ are off-shell reducible of order $(D-k-2)$ with the reducibility functions expressed in (45). Based on these results we compute the number of independent degrees of freedom for the system under study

$$N_{DOF}^{(IV)} = (D-1)\begin{pmatrix} D-2 \\ k \end{pmatrix} - \begin{pmatrix} D-1 \\ k+1 \end{pmatrix}. \tag{52}$$

Finally, on behalf of the Dirac's conjecture, if we pass again to the Lagrangian formulation [*via* extended action], we derive for the model (49) the generating set of gauge transformations

$$\delta^{(IV)}_{\epsilon,\xi} A_{\mu_1\cdots\mu_k\|\alpha} = \partial^\sigma \epsilon_{\sigma\mu_1\cdots\mu_k\alpha} + \partial_{[\mu_1}\epsilon_{\mu_2\cdots\mu_k]\alpha} + \partial_{[\mu_1}\xi_{\mu_2\cdots\mu_k]|\alpha}. \tag{53}$$

The gauge parameters in (53) possess the same symmetries as the corresponding ones in (48).

2.5. CASE V

Here, the constants that label (2) take the values (18). For this choice, the Lagrangian density (2) has the expression

$$\mathcal{L}_0^{(V)} = \frac{(-)^{k+1}}{2(k+1)(k+1)!}\left[\frac{1}{k+1}\left(F_{\mu_1\cdots\mu_{k+1}\|\alpha}\right)^2 - (-)^k \bar{a} F_{\mu_1\cdots\mu_k\beta\|\alpha} F^{\mu_1\cdots\mu_k\alpha\|\beta} + \frac{(-)^k\bar{a}-1}{D-k}\left(F_{\mu_1\cdots\mu_k}\right)^2\right]. \tag{54}$$

where the range of the real parameter $\bar{a}$ is given in (18). In the specified context, the definitions of the canonical momenta (9) furnish the primary constraints set consisting in (10) and (40a) [that is Abelian] and also display the canonical Hamiltonian density

$$\begin{aligned}\mathcal{H}_0^{(V)} = & -kA^{0i_1\cdots i_{k-1}\|\mu}\left(\partial^l \pi_{li_1\cdots i_{k-1}\|\mu}\right) + \frac{(-)^k}{2(k+1)^2(k+1)!}\left(F_{i_1\cdots i_{k+1}\|\mu}\right)^2 \\ & + \frac{(k+1)(k+1)!}{2\left(\bar{a}-(-)^k\right)}\left(\frac{D-k}{D-k-1}\left(\pi_{i_1\cdots i_k\|0}\right)^2 + \left(\pi_{i_1\cdots i_k\|j}\right)^2 \right. \\ & \left. - \frac{\bar{a}}{k\bar{a}+(-)^k}\pi_{i_1\cdots i_k\|j}\pi^{[i_1\cdots i_k\|j]}\right) + \frac{(-)^k\bar{a}}{k\bar{a}+(-)^k}\pi_{i_1\cdots i_k\|j}F^{i_1\cdots i_k j\|0} \\ & + \frac{(-)^k}{D-k-1}\pi_{i_1\cdots i_k\|0}F'^{i_1\cdots i_k} - \frac{\bar{a}^2}{2(k+1)\left(k\bar{a}+(-)^k\right)(k+1)!}\left(F_{i_1\cdots i_{k+1}\|0}\right)^2 \\ & + \frac{1}{2(k+1)(k+1)!}\left(\frac{\bar{a}-(-)^k}{D-k-1}\left(F'_{i_1\cdots i_k}\right)^2 - \bar{a}F^{i_1\cdots i_k j\|l}F_{i_1\cdots i_k l\|j}\right).\end{aligned} \tag{55}$$

The next step in the canonical analysis is represented by the requirement of preservation in time for the primary constraints (10) and (40a). At this stage one obtains the secondary constraints $G^{(2)}_{i_1\cdots i_{k-1}} \approx 0$ and $G^{(2)}_{i_1\cdots i_{k-1}\|j} \approx 0$ respectively defined in (42a) and (42b).

By direct computations we infer the Abelian character of the constraints (10), (40a), $G^{(2)}_{i_1\cdots i_{k-1}} \approx 0$ and $G^{(2)}_{i_1\cdots i_{k-1}\|j} \approx 0$. Moreover, the consistency of the secondary constraints $G^{(2)}_{i_1\cdots i_{k-1}} \approx 0$ and $G^{(2)}_{i_1\cdots i_{k-1}\|j} \approx 0$ no longer produces tertiary constraints as

$$\left[G^{(2)}_{i_1\cdots i_{k-1}}, H_0^{(V)}\right] = 0 = \left[G^{(2)}_{i_1\cdots i_{k-1}\|j}, H_0^{(V)}\right]. \tag{56}$$

The irreducible character of the first-class constraints (10), together with the first-order reducibilities (46) of the constraints (40a) and $G^{(2)}_{i_1\cdots i_{k-1}\|j}$ supplemented with the $L = k-1$ off-shell reducibilities of the constraints $G^{(2)}_{i_1\cdots i_{k-1}} \approx 0$ as well as $G^{(2)}_{i_1\cdots i_{k-1}\|j} \approx 0$ [the reducibility functions are given in (36a)–(36b)] allow us to compute the number of independent degrees of freedom

$$N_{DOF}^{(V)} = D\begin{pmatrix} D-2 \\ k \end{pmatrix} - \begin{pmatrix} D-1 \\ k-1 \end{pmatrix} + \begin{pmatrix} D-1 \\ k-2 \end{pmatrix}. \tag{57}$$

Finally, by passing to the Lagrangian formulation, we get for the model (54) the generating set of gauge transformations

$$\delta_{\epsilon,\xi}^{(V)} A_{\mu_1\cdots\mu_k\|\alpha} = \sigma_{\alpha[\mu_1}\epsilon_{\mu_2\cdots\mu_k]} + \partial_{[\mu_1}\epsilon_{\mu_2\cdots\mu_k]\alpha} + \partial_{[\mu_1}\xi_{\mu_2\cdots\mu_k]|\alpha}, \tag{58}$$

where the gauge parameters possess the same symmetries as the corresponding ones in (48).

2.6. CASE VI

In this part, the real constants that label the local function (2) reads as in (19). For this setting, the Lagrangian density (2) becomes

$$\begin{aligned} \mathcal{L}_0^{(VI)} = \tfrac{1}{2(k+1)(k+1)!}\Big[& \tfrac{(-)^{k+1}}{k+1}\left(F_{\mu_1\cdots\mu_{k+1}\|\alpha}\right)^2 + \tilde{a}F_{\mu_1\cdots\mu_k\beta\|\alpha}F^{\mu_1\cdots\mu_k\alpha\|\beta} \\ & + \left((-)^k - \tilde{a}\right)\left(F_{\mu_1\cdots\mu_k}\right)^2\Big]. \end{aligned} \tag{59}$$

Replacing the choice (19) in the definitions (9), the lasts lead to the set of primary constraints constituted by (10) and

$$\tilde{\gamma}_{i_1\cdots i_k}^{(1)} \equiv \pi_{i_1\cdots i_k\|0} + \frac{(-)^k\tilde{a}-1}{(k+1)(k+1)!}F'_{i_1\cdots i_k} \approx 0. \tag{60}$$

Expressing from the equations (9) [corresponding to the choice (19)] some of the generalized velocities, we get the canonical Hamiltonian density

$$\begin{aligned} \mathcal{H}_0^{(VI)} = & -kA^{0i_1\cdots i_{k-1}\|\mu}\left(\partial^l\pi_{li_1\cdots i_{k-1}\|\mu}\right) + \tfrac{(-)^k}{2(k+1)^2(k+1)!}\left(F_{i_1\cdots i_{k+1}\|j}\right)^2 \\ & + \tfrac{(k+1)(k+1)!}{2\left(\tilde{a}-(-)^k\right)}\Bigg(\left(\pi_{i_1\cdots i_k\|j}\right)^2 - \tfrac{k}{D-k-1}\left(\pi'_{i_1\cdots i_{k-1}}\right)^2 \\ & - \tfrac{\tilde{a}}{k\tilde{a}+(-)^k}\pi_{i_1\cdots i_k\|j}\pi^{[i_1\cdots i_k\|j]}\Bigg) + \tfrac{(-)^k\tilde{a}}{k\tilde{a}+(-)^k}\pi_{i_1\cdots i_k\|j}F^{i_1\cdots i_kj\|0} \\ & + \tfrac{\tilde{a}-(-)^k}{2(k+1)(k+1)!}\left(F'_{i_1\cdots i_k}\right)^2 + \tfrac{1+(-)^kk\tilde{a}-(k+1)\tilde{a}^2}{2(k+1)^2(k+1)!\left(k\tilde{a}+(-)^k\right)}\left(F_{i_1\cdots i_{k+1}\|0}\right)^2 \\ & - \tfrac{\tilde{a}}{2(k+1)(k+1)!}F^{i_1\cdots i_kj\|l}F_{i_1\cdots i_kl\|j}. \end{aligned} \tag{61}$$

It simply verifies that the primary constraints (10) and (60) are Abelian so that their consistency reduces only to the computation of the Poisson brackets between them and the canonical Hamiltonian (61). By direct computation one obtains

$$\left[G^{(1)}_{i_1\cdots i_{k-1}}, H_0^{(VI)}\right] = G^{(2)}_{i_1\cdots i_{k-1}}, \quad \left[G^{(1)}_{i_1\cdots i_{k-1}\|j}, H_0^{(VI)}\right] = G^{(2)}_{i_1\cdots i_{k-1}\|j}, \tag{62a}$$

$$\left[\tilde{\gamma}^{(1)}_{i_1\cdots i_k}, H_0^{(VI)}\right] = -\tilde{\gamma}^{(2)}_{i_1\cdots i_k} \equiv -\partial^j \left(\pi_{i_1\cdots i_k\|j} + \tfrac{(-)^k-\tilde{a}}{(k+1)(k+1)!} F_{ji_1\cdots i_k\|0}\right), \tag{62b}$$

where the functions in the right-hand sides of (62a) are given in formulas (42a), (42b). At this stage we identify the secondary constraints possessed by the model under study

$$G^{(2)}_{i_1\cdots i_{k-1}} \approx 0, \quad G^{(2)}_{i_1\cdots i_{k-1}\|j} \approx 0, \quad \tilde{\gamma}^{(2)}_{i_1\cdots i_k} \approx 0. \tag{63}$$

Proceeding again by pedestrian way one infers the Abelian character of the constraints set consisting in (10), (60) and (63). Using this remark, the requirement of preservation in time for the secondary constraints reduces to the computation of the Poisson brackets between them and the canonical Hamiltonian (61)

$$\left[G^{(2)}_{i_1\cdots i_{k-1}}, H_0^{(VI)}\right] = 0, \quad \left[G^{(2)}_{i_1\cdots i_{k-1}\|j}, H_0^{(VI)}\right] = 0 = \left[\tilde{\gamma}^{(2)}_{i_1\cdots i_k}, H_0^{(VI)}\right]. \tag{64}$$

The outputs (64) allow us to conclude that the model under study possesses no tertiary constraints and, in addition, the canonical Hamiltonian (63) coincides with the first-class Hamiltonian.

In view of the counting of number of independent degrees of freedom, one observes that: i) the first-class constraints (10) and (60) are irreducible and ii) the secondary constraints (63) are off-shell reducible of order $L = k-1$. Putting together the previous results we determine the number of physical degrees of freedom

$$N^{(VI)}_{DOF} = (D-1)\begin{pmatrix} D-2 \\ k \end{pmatrix} - \begin{pmatrix} D-1 \\ k \end{pmatrix}. \tag{65}$$

In the end, by employing the same procedure as in the previous subsections, we get for the model under study (59) the generating set of gauge transformations

$$\delta^{(VI)}_{\epsilon,\xi} A_{\mu_1\cdots\mu_k\|\alpha} = \partial_{[\mu_1}\epsilon_{\mu_2\cdots\mu_k]\alpha} + \partial_{[\mu_1}\xi_{\mu_2\cdots\mu_k]|\alpha} + \partial_\alpha \bar{\epsilon}_{\mu_1\cdots\mu_k}, \tag{66}$$

where the bosonic gauge parameters display possess the same symmetries as the corresponding ones in (48).

2.7. CASE VII

In the last situation, the real parameters a_1 and a_2 are restriced to fulfill (20). The Lagrangian density is the original one (2). Here, the definitions (9) of the canonical momenta lead to the Abelian primary constraints (10) and produce the canonical

Hamiltonian density

$$\mathcal{H}_0^{(VII)} = -kA^{0i_1\cdots i_{k-1}\|\mu}\left(\partial^l \pi_{li_1\cdots i_{k-1}\|\mu}\right) + \tfrac{(-)^k}{2(k+1)^2(k+1)!}\left(F_{i_1\cdots i_{k+1}\|j}\right)^2$$
$$+ \tfrac{(k+1)(k+1)!}{2\left(a_1-(-)^k\right)}\pi^{i_1\cdots i_k\|j}\left(\pi_{i_1\cdots i_k\|j} - \tfrac{a_1}{ka_1+(-)^k}\pi_{[i_1\cdots i_k\|j]}\right.$$
$$\left. - \tfrac{a_2}{a_1+a_2(D-k)-(-)^k}\pi'_{[i_1\cdots i_{k-1}}\sigma_{i_k]j}\right)$$
$$+ \tfrac{(k+1)(k+1)!}{2\left(a_1+a_2+(-)^k\right)}\left(\pi_{i_1\cdots i_k\|0}\right)^2 + \tfrac{(-)^k a_1}{ka_1+(-)^k}\pi_{i_1\cdots i_k\|j}F^{i_1\cdots i_k j\|0}$$
$$- \tfrac{(-)^k a_2}{a_1+a_2+(-)^k}F'_{i_1\cdots i_k}\left(\pi^{i_1\cdots i_k\|0} + \tfrac{1+(-)^k a_1}{2(k+1)(k+1)!}F'^{i_1\cdots i_k}\right)$$
$$+ \tfrac{1+(-)^k ka_1-(k+1)a_1^2}{2(k+1)^2(k+1)!\left(ka_1+(-)^k\right)}\left(F_{i_1\cdots i_{k+1}\|0}\right)^2 - \tfrac{a_1}{2(k+1)(k+1)!}F^{i_1\cdots i_k j\|l}F_{i_1\cdots i_k l\|j}. \tag{67}$$

The consistency of the primary constraints (10) displays the secondary constraints $G^{(2)}_{i_1\cdots i_{k-1}}$ and $G^{(2)}_{i_1\cdots i_{k-1}\|j}$ [whose concrete expressions are respectively written in (42a) and (42b)]as the Poisson brackets hold

$$\left[G^{(1)}_{i_1\cdots i_{k-1}}, H_0^{(VII)}\right] = G^{(2)}_{i_1\cdots i_{k-1}}, \quad \left[G^{(1)}_{i_1\cdots i_{k-1}\|j}, H_0^{(VII)}\right] = G^{(2)}_{i_1\cdots i_{k-1}\|j}. \tag{68}$$

The Abelian character of the constraints (10), (42a) and (42b) supplemented with the Poisson brackets

$$\left[G^{(2)}_{i_1\cdots i_{k-1}}, H_0^{(VII)}\right] = 0 = \left[G^{(2)}_{i_1\cdots i_{k-1}\|j}, H_0^{(VII)}\right] \tag{69}$$

allow us to finalize the Dirac algorithm at this level.

The irreducible character of the first-class constraints (10) supplemented with the $L = k-1$ reducibility functions (36a)–(36b) of the secondary first-class constraints $G^{(2)}_{i_1\cdots i_{k-1}\|j}$ and $G^{(2)}_{i_1\cdots i_{k-1}\|j}$ allow us to count the number of independent degrees of freedom for the model under study

$$N_{DOF}^{(VII)} = D\begin{pmatrix} D-2 \\ k \end{pmatrix}. \tag{70}$$

Finally, if we return to the Lagrangian formulation [*via* extended action], we conclude that the initial set of gauge transformations (1) is a generating one.

3. CONCLUSIONS AND PERSPECTIVES

The present paper has furnished a usefull way for unitary treating two massless tensor gauge fields that transform under the 'irreducible' representations of the Lorentz group namely a $(k+1)$-form and a tensor with the mixed symmetry $(k,1)$. The procedure has involved a massless tensor gauge field of degree $(k+1)$, denoted by $A_{\mu_1\cdots\mu_k\|\alpha}$, that is completely antisymmetric in its first k Lorentz indices and with no symmetry in respect to the last. At the outset, for the objects $A_{\mu_1\cdots\mu_k\|\alpha}$ we have postulated a set of gauge transformations that allowed the construction of the most general second-order invariant Lagrangian density. The specified local function depends on two arbitrary real constants [denoted in the paper by a_1 and a_2]. The canonical analysis of the considered model revealed a partition of the plane (a_1, a_2) in seven subsets. In each of the distinct seven situations we have computed the number of independent degrees of freedom and also we have identified a generating set of gauge transformations. Some of the generating sets of gauge transformations revealed conformal-like [20] behaviour [(48) and (58)] and/or topologically BF-type [21] comportment [(48) and (53)].

The results obtained in the present paper are indispensable in building the frame-like formulations for the tensor gauge fields of degree $(k+1)$ and also in the derivation of consistent (self-)interactions of the analyzed tensor field.

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