PAIRING, QUARTET CONDENSATION AND WIGNER ENERGY IN NUCLEI*

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In this paper we summarize the study we have recently performed on the effect of isovector pairing correlations upon the symmetry and Wigner energies. First, we review the basic assumptions of the quartet condensation model (QCM) and show how this model can be used in Hartree-Fock (HF) mean field calculations in order to take into account the isovector pairing correlations. Then, within the HF+QCM approach we discuss the influence of proton-neutron pairing on symmetry and Wigner energies for the isobaric chains of even-even nuclei with $24 < A < 100$.

Key words: Proton-neutron pairing, Wigner energy.

The experimental masses indicate that the nuclei with $N = Z$ have an additional binding energy compared to the neighboring nuclei. The origin of this additional binding energy, which in phenomenological mass formulas [1] is usually called Wigner energy, is not yet clear. In some studies it is supposed that the Wigner energy originates from the proton-neutron pairing correlations, which become stronger in $N = Z$ nuclei. Thus, it was recently argued that the isovector proton-neutron pairing can describe most of the extra binding associated to the Wigner energy, provided the isovector pairing is treated beyond the BCS approximation [2].

One of the reasons why the BCS-like models are not appropriate for calculating the contribution of the isovector pairing correlations to Wigner energy is related to the fact that in BCS are not conserved exactly the particle number and the isospin. However, restoring exactly these two broken symmetries it is not enough for obtaining precise correlation energies for isovector pairing [3, 4], which demonstrates the need of going beyond the BCS-type models. In Refs. [5, 6] it was proved that an approach based on quartets formed by two neutrons and two protons coupled to the total isospin $T = 0$ is able to predict with very high accuracy the isovector pairing correlations in the ground state of both $N = Z$ and $N > Z$ nuclei. This approach was recently applied for analyzing the contribution of isovector proton-neutron pairing correlations to symmetry and Wigner energies [7]. The results of this study will be summarized in the present proceedings paper.

We start by presenting briefly the quartet model introduced in Refs. [5, 6]. This model describes the ground state of a system formed by N neutrons and Z protons

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moving outside a self-conjugate core and interacting *via* an isovector pairing force. The Hamiltonian describing this system is

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i\tau} N_{i\tau} - g \sum_{i,j,t=-1,0,1} P^{+}_{i,t} P_{j,t}, \tag{1}$$

where the isovector interaction is expressed in terms of the isovector pair operators $P^{+}_{i,1} = \nu^{+}_{i} \nu^{+}_{\bar{i}}$, $P^{+}_{i,-1} = \pi^{+}_{i} \pi^{+}_{\bar{i}}$ and $P^{+}_{i,0} = (\nu^{+}_{i} \pi^{+}_{\bar{i}} + \pi^{+}_{i} \nu^{+}_{\bar{i}})/\sqrt{2}$; the operators ν^{+}_{i} and π^{+}_{i} create, respectively, a neutron and a proton in the state i while $\bar{i}$ denotes the time conjugate of the state i.

Following Ref. [5], the ground state of Hamiltonian (1) for a system with $N = Z$ is described by the state

$$|\Psi\rangle = (A^{+})^{n_q} |0\rangle, \tag{2}$$

where $n_q = (N + Z)/4$ and A^{+} is the collective quartet built by two isovector pairs coupled to the total isospin $T = 0$ defined by

$$A^{+} = \sum_{i,j} \bar{x}_{ij} [P^{+}_{i} P^{+}_{j}]^{T=0} = \sum_{ij} x_{ij} (P^{+}_{i,1} P^{+}_{j,-1} + P^{+}_{i,-1} P^{+}_{j,1} - P^{+}_{i,0} P^{+}_{j,0}). \tag{3}$$

Supposing that the amplitudes x_{ij} are separable, *i.e.*, $x_{ij} = x_i x_j$, the collective quartet operator can be written as

$$A^{+} = 2\Gamma^{+}_{1} \Gamma^{+}_{-1} - (\Gamma^{+}_{0})^2, \tag{4}$$

where $\Gamma^{+}_{t} = \sum_i x_i P^{+}_{i,t}$ denote, for $t = 0, 1, -1$, the collective Cooper pair operators for the proton-neutron (pn), neutron-neutron (nn) and proton-proton (pp) pairs. Thus, it can be seen that in this approximation the quartet condensate is a particular superposition of condensates of nn, pp and pn pairs.

In Ref. [6] the quartet condensate model was extended to nuclei with $N > Z$. For these nuclei it is supposed that the neutrons in excess form a pair condensate which is appended to the quartet condensate. Thus, the ground state of $N > Z$ nuclei is approximated by

$$|\Psi\rangle = (\tilde{\Gamma}^{+}_{1})^{n_N} (A^{+})^{n_q} |0\rangle = (\tilde{\Gamma}^{+}_{1})^{n_N} (2\Gamma^{+}_{1} \Gamma^{+}_{-1} - \Gamma^{+2}_{0})^{n_q} |0\rangle, \tag{5}$$

where $n_N = (N - Z)/2$ is the number of neutron pairs in excess and $n_q = (N - 2n_N + Z)/4$ is the maximum number of alpha-like quartets which can be formed by the neutrons and protons. Since the quartets A^{+} have zero isospin, the state (6) has a well-defined total isospin given by the excess neutrons, *i.e.*, $T = n_N$. The neutron pairs in excess are described by the collective pair operator $\tilde{\Gamma}^{+}_{1} = \sum_i y_i P^{+}_{i1}$, which has a different structure from the collective neutron pair entering in the collective quartet. The mixing amplitudes x_i and y_i which define the ground state (6) are determind from the minimization of $\langle \Psi | H | \Psi \rangle$ under the normalization condition

$\langle\Psi|\Psi\rangle = 1$. To calculate the average of the Hamiltonian and the norm it is used the recurrence relations method [5, 6].

Recently the quartet condensation model (QCM) presented above was employed to treat the isovector pairing correlations in Hartree-Fock (HF) mean field calculations [7]. The HF mean field is generated with a zero range Skyrme functional and the HF calculations are performed in a single-particle basis generated by an axially deformed harmonic oscillator, as described in Ref. [8]. After the HF calculations are converged, we select a set of neutron and proton single-particle states with the energies located around the HF chemical potentials. The energies of these states are considered in the Hamiltonian (1) for performing the QCM calculations. Then, from the QCM calculations we extract the occupation probabilities of the pairing active orbits which are further used to redefine the HF densities. For example, the particle density for neutrons and protons ($\tau = n, p$) are defined by

$$\rho_\tau(r,z) = \sum_i v_{\tau,i}^2 \|\psi_{\tau,i}(r,z)\|^2, \tag{6}$$

where $v_{\tau,i}^2$ are the occupation probabilities for the single-particle states $\psi_{\tau,i}(r,z)$. They are taken equal to $1(0)$ for the occupied (unoccupied) HF states which are not considered active in pairing calculations and equal to the QCM values otherwise. The HF and QCM calculations are iterated together until the convergence. Finally, the pairing energy is calculated by averaging the isovector pairing force on the QCM state and is added to the mean-field energy.

The HF+QCM calculation scheme outlined above was applied for studying the influence of isovector pairing correlations on symmetry and Wigner energies. In the phenomenological mass formulas these energies are parametrized by a quartic and, respectively, a linear term in $N-Z$. Thus, for an isobaric chain with $A = N+Z$ the ground state energy relative to the nucleus with $N = Z$ can be written as

$$E(N,Z) = E(N=Z) + a_A \frac{|N-Z|^2}{A} + a_W \frac{|N-Z|}{A} + \delta E_{shell}(N,Z) + \delta E_P(N,Z). \tag{7}$$

In the equation above it is not considered the contribution of the Coulomb energy, which is supposed to be extracted from all the isotopes of the isobaric chain, and it is also implicitly assumed that for all nuclei with $A = N+Z$ the volume and the surface energies are the same and therefore included in the term $E(N = Z)$. The last two terms in Eq.(7) are the corrections associated to the shell structure and pairing measured relative to the nucleus with $N = Z$. Supposing that these two energy corrections can be also described by a linear and a quartic term in $|N-Z|$ and taking $T = |T_z|$, which is the case for the ground state of even-even nuclei, Eq.(7)

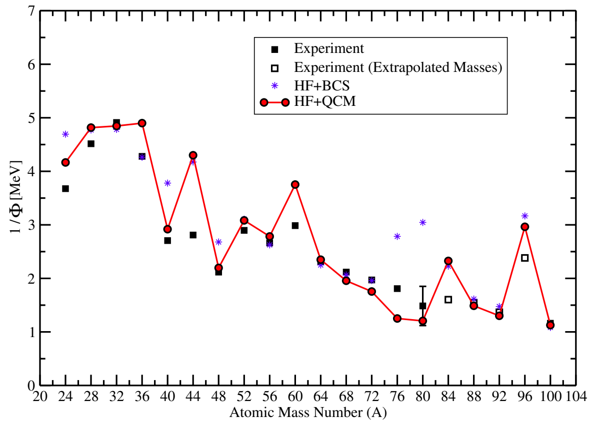

Fig. 1 – The quantity $1/\Phi$ (see Eq.(8)), expressing the strength of the symmetry energy term proportional to T^2, as a function of mass number. The experimental values, obtained by removing the contribution of Coulomb energy, are from Ref. [2].

can be written as

$$E(T) = E(T=0) + \frac{T(T+X)}{2\Phi} \tag{8}$$

In the equation above X quantifies the contribution of the linear term in isospin to the ground state energy and takes into account all the possible effects, including the ones from the shell structure. The fit of Eq.(7) with experimental data shows that for many nuclei $a_A \approx a_W$. Thus, when the contribution of the last two terms of Eq.(7) are negligible, $X \approx 1$. In this case the ground state energies of the isobaric chain relative to the nucleus with $N = Z$ depend on $T(T+1)$, as the eigenvalues of the total isospin T^2. However, a systematic survey based on experimental masses fitted with Eq.(8) [2] shows that X is fluctuating quite strongly around $X = 1$ (see Fig.2 below).

Using the HF+QCM calculations scheme presented above we have calculated the quantities Φ and X of Eq.(8) for isobaric chains of even-even nuclei with $24 < A < 100$. For each isobaric chain the values of Φ and X are extracted from the binding energies of three nuclei with $T = |Tz| = 0, 2, 4$, *i.e.*, nuclei with $N - Z = 0, 4, 8$. The Skyrme-HF calculations are done with the Skyrme functional SLy4 [9] and neglecting the contribution of the Coulomb interaction. The deformation is calculated self-consistently in axial symmetry using an harmonic oscillator basis [8]. From the HF spectrum we considered in the QCM calculations 10 single-particles states, both for protons and neutrons, above a self-conjugate core. For the isovector force we use the strength $g = 12/A^{3/4}$[MeV] which gives a reasonable description of the odd-even mass difference calculated along $N = Z$ line [7].

Figs.1-2 display the results of HF+QCM calculations for $1/\Phi$ and X in com-

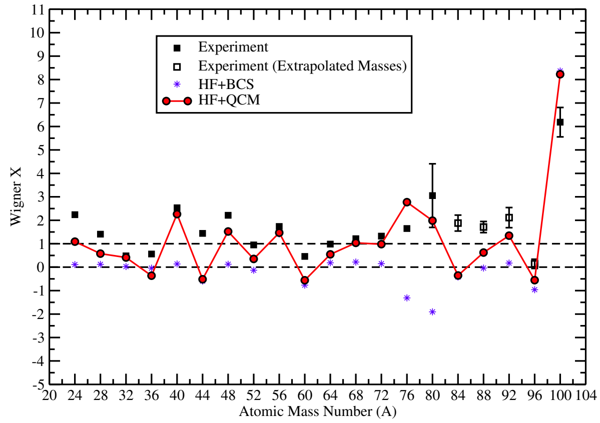

Fig. 2 – The quantity X (see Eq.(8)), which gives the contribution of Wigner energy relative to the standard symmetry energy, as a function of mass number. The experimental values, obtained by removing the contribution of Coulomb energy, are from Ref. [2].

parison with the experimental values [2]. The latter are obtained employing in Eq. (8) the experimental masses of Ref. [10] from which the Coulomb energy was removed (for details, see [2]). In these figures are shown also the results of HF+BCS calculations. In the BCS calculations, performed with the same model space and cores as in the QCM calculations, the pairing correlations for protons and neutrons are treated independently and the proton-neutron pairing is not taken into account.

From Fig.1 one can notice that the HF+QCM calculations describe very well the mass dependence of the quantity $1/\Phi$ associated to the standard symmetry energy proportional to T^2. The largest deviations appear for the isotopic chains which cross a magic number at $T = 2$, *i.e.*, for the nuclei with $N - Z = 4$. The discrepancies are related to the inaccuracy of the deformations predicted by the mean field calculations for nuclei with two particles or two holes above/below a magic or semi-magic number.

The predictions for the quantity X are shown in Fig. 2. One can now notice that the HF+BCS calculations fail to describe the linear term in T associated to Wigner energy (see also Ref. [12]). In fact, as seen in Fig. 2, for the majority of chains the HF+BCS calculations predict for X values close to zero. On the other hand we observe that the HF+QCM results are following well the large fluctuations of X with the mass number. The largest deviations from experimental values appear again for the isobaric chains which cross a magic number for $T = 2$. It can be thus seen that for these chains the calculated X values are underestimated (overestimated) when $1/\Phi$ are overestimated (underestimated). This fact can be simply traced back to the expression $X = (3r - 1)/(r - 1)$, where $r = (E(4) - E(2))/(E(2) - E(0))$. For example, the underestimation of X for the chain A=44 is due to the overestimation of the ratio r, which reflects the overestimation of $1/\Phi$ discussed above. Thus, as in the case of $1/\Phi$, the largest discrepancies of X seen in Fig. 2 are related to the inaccuracy of level densities predicted by mean field model for nuclei with two neutron or two holes above/below a magic number.

In conclusion, we have shown how the isovector pairing interaction can be treated in the mean-field models by conserving exactly the particle number and the isospin. To treat the isovector pairing correlations we use a condensate of alpha-type quartets to which it is appended, in the case of nuclei with $N > Z$, a condensate of neutron pairs. This formalism is applied to analyze the effect of isovector pairing on symmetry and Wigner energies. The results show that the isovector pairing acting on a self-consistent mean field can explain reasonably well the mass dependence of Wigner energy.

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