PROTON-NEUTRON PAIRING IN SELF-CONJUGATE NUCLEI IN A FORMALISM OF QUARTETS*

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The proton-neutron pairing in self-conjugate nuclei is discussed in a formalism of quartets. Quartets are four-body correlated structures built from two neutrons and two protons coupled to total isospin $T = 0$ and total angular momentum $J = 0$. The pairing ground state is described as a product of quartets. We review both the case in which the Hamiltonian has a pure isovector character and the case in which, in addition, it contains isoscalar components. The approach is tested by making comparisons with exact shell model calculations for $N = Z$ nuclei with valence nucleons outside the ^{16}O, ^{40}Ca and ^{100}Sn cores. The quartet formalism is seen to reproduce with great accuracy the exact ground states energies.

Key words: Self-conjugate nuclei, isovector-isoscalar pairing, quartet formalism.

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1. INTRODUCTION

Proton-neutron (pn) pairing is a subject of long-standing interest in nuclear physics (for a review of some early work see Ref. [1]; for a recent analysis see Ref. [2]). In self-conjugate nuclei owing to the charge independence of the nuclear interaction and to the large spatial overlap between proton and neutron single-particle wave functions, pn pairing correlations are expected to come into significant play.

Extending traditional approaches to pairing like BCS or projected-BCS to pn systems has not led to a satisfactory description of the pairing ground state [3–6]. An important limitation of these approaches is that they violate isospin symmetry. However, restoring this symmetry has not significantly improved the quality of the

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results even in the presence of particle number conservation [7]. This fact points to the need for a more general isospin conserving formalism which goes beyond the BCS-based approximations.

Recently [8], an approach for the treatment of the isovector pn Hamiltonian has been proposed which is based on quartets. Quartets are four-body correlated structures formed by two protons and two neutrons coupled to total isospin $T = 0$ and total angular momentum $J = 0$. In this approach, named Quartet Condensation Model (QCM), the ground state of a self-conjugate nucleus is assumed to be a product of identical quartets. The structure of these quartets is determined by minimizing the ground state energy. This approach has been afterwards generalized by allowing the quartets to be all distinct from one another [9]. In this case, quartets are constructed by means of an iterative variational procedure. In the following, this approach will be referred to as Quartet Model (QM). Both QCM and QM have been tested in the case of an isovector pairing Hamiltonian by making comparisons with exact shell model calculations for $N = Z$ nuclei with valence nucleons outside the ^{16}O, ^{40}Ca and ^{100}Sn cores. These comparisons will be illustrated in Section 2 together with a presentation of the quartet formalism. Quite recently [10], this formalism has been extended to the isoscalar pairing as well and, in Section 3, we will illustrate some of the results that have been obtained in this case. Finally, in Section 4, we will give some conclusions.

2. THE QUARTET FORMALISM IN THE CASE OF ISOVECTOR PAIRING

We will introduce the quartet formalism in the case of pn isovector pairing. The isovector pairing Hamiltonian in a spherically symmetric mean field has the form

$$H^{(iv)} = \sum_i \epsilon_i (N_i^\pi + N_i^\nu) + \sum_{i,j} V_{J=0}^{T=1}(i,j) \sum_{T_z} P_{i,T_z}^+ P_{j,T_z} \tag{1}$$

In the first term, ϵ_i and N_i^π (N_i^ν) are, respectively, the energy and the proton (neutron) particle number operator relative to the single-particle state i. The symbol i is a short cut notation for $\{n_i, l_i, j_i, \tau_i\}$, with $\{n_i, l_i, j_i\}$ being the standard orbital quantum numbers and τ_i denoting the isospin projection. The Coulomb interaction between the protons is not taken into account so that the single-particle energies of protons and neutrons are assumed to be equal. The second term in Eq. (1) is the isovector pairing interaction. This is formulated in terms of the non-collective pair operators

$$P_{i,T_z}^+ = \frac{1}{\sqrt{2}} [a_i^+ a_i^+]_{T_z}^{T=1,J=0}, \tag{2}$$

where T_z denotes the three projections of the isospin $T = 1$ corresponding to neutron-neutron ($T_z = 1$), proton-proton ($T_z = -1$) and proton-neutron ($T_z = 0$) pairs.

To describe the ground state of the Hamiltonian (1) for systems with $N = Z$ we shall use as building blocks not collective pairs, as done in BCS-type models, but collective quartets formed by two neutrons and two protons. These are defined as

$$Q_\nu^{+(iv)} = \sum_{i,j} x_{ij}^{(\nu)} [P_i^+ P_j^+]^{T=0}. \tag{3}$$

By construction, each quartet has a total isospin $T = 0$ and total angular momentum $J = 0$.

For a self-conjugate nucleus with $N = Z \equiv 2n$, the QCM assumes that the ground state of the isovector pairing Hamiltonian (1) has the form [8]

$$|\Psi^{(QCM)}\rangle = (Q^{+(iv)})^n |0\rangle, \tag{4}$$

where $|0\rangle$ denotes a self-conjugate core of nucleons not affected by the pairing interaction. The mixing amplitudes defining the quartet $Q^{+(iv)}$ are determined by minimizing the expectation value of H in the state $|\Psi^{(QCM)}\rangle$ properly normalized. Assuming that the coefficients x_{ij} of Q^+ have a separable form, *i.e.* $x_{ij} \equiv x_i x_j$, allows the use of recurrence relations which greatly facilitate the evaluation of this expectation value [8]. With this approximation the collective quartet can be written as

$$Q^{+(iv)} = 2\Gamma_1^+ \Gamma_{-1}^+ - (\Gamma_0^+)^2, \tag{5}$$

where $\Gamma_\tau^+ = \sum_i x_i P_{i,\tau}^+$. Due to the isospin invariance, all the collective pairs have the same mixing amplitudes x_i.

According to QM, instead, the isovector pairing ground state has the form [9]

$$|\Psi^{(QM)}\rangle = \prod_{\nu=1}^{n} Q_\nu^{+(iv)} |0\rangle, \tag{6}$$

namely it is a product of distinct quartets of the type (3). In order to search for the most appropriate $x_{ij}^{(\nu)}$'s we make use of an iterative variational procedure which is derived from an analogous treatment of like-particle pairing in terms of independent pairs [11]. No approximation is introduced in this case for the mixing amplitudes $x_{ij}^{(\nu)}$.

To test the accuracy of the quartet approaches just described we have performed calculations for three sets of $N = Z$ nuclei with valence nucleons outside the cores ^{16}O, ^{40}Ca and ^{100}Sn. Isovector pairing forces have been extracted from the (T = 1, J = 0) part of standard shell model interactions. Details about the single-particle energies employed in the calculations are given in Refs. [8, 9]. The results for the pairing correlation energy, defined as the difference between the ground state energies obtained without and with the pairing force, are given in Table 1.

In this table, the correlation energies predicted by QM and QCM are compared to the exact results. We notice that for the systems with one quartet outside the

Table 1

Ground state correlation energies (in MeV) calculated for the isovector pairing Hamiltonian (1) with strengths extracted from standard shell model interactions (for details see Ref. [8, 9]). The results are shown for the exact diagonalizings, the QM and the QCM. In brackets we give the errors relative to the exact results.

	Exact	QM	QCM
^{20}Ne	9.174	9.174 (-)	9.170 (0.04%)
^{24}Mg	14.461	14.458 (0.02%)	14.436 (0.17%)
^{28}Si	15.787	15.780 (0.04%)	15.728 (0.37%)
^{32}S	15.844	15.844 (-)	15.795 (0.31%)
^{44}Ti	5.965	5.965 (-)	5.964 (0.02%)
^{48}Cr	9.579	9.573 (0.06%)	9.569 (0.10%)
^{52}Fe	10.750	10.725 (0.23%)	10.710 (0.37%)
^{104}Te	3.832	3.832 (-)	3.829 (0.08%)
^{108}Xe	6.752	6.752 (-)	6.696 (0.83%)
^{112}Ba	8.680	8.678 (0.02%)	8.593 (1.00%)

closed core the state (6) is by construction exact. This is not the case for the quartet condensate (4) because of the factorization approximation $x_{ij} = x_i x_j$. Both QM and QCM results are characterized by very small errors, with those of QM being always smaller. This reflects the gain in correlation energy obtained in QM by allowing the quartets to be different.

3. THE ISOVECTOR PLUS ISOSCALAR PAIRING

The isovector plus isoscalar pairing Hamiltonian in a spherically symmetric mean field has the form

$$H = H^{(iv)} + \sum_{i\leq j,k\leq l} V_{J=1}^{T=0}(ij,kl) \sum_{J_z} D^+_{ij,J_z} D_{kl,J_z}, \tag{7}$$

where $H^{(iv)}$ is the isovector Hamiltonian (1). The isoscalar pairing interaction is written in terms of the pair operators

$$D^+_{ij,J_z} = \frac{1}{\sqrt{1+\delta_{ij}}}[a_i^+ a_j^+]^{J=1,T=0}_{J_z}, \tag{8}$$

where J_z denotes the three projections of the angular momentum $J = 1$.

In addition to the isovector quartets (3), resulting from the coupling of two isovector pairs (2), one can form isoscalar quartets formed by two isoscalar pairs (8)

$$Q^{+(is)}_\nu = \sum_{ij,kl} y^{(\nu)}_{ij,kl}[D^+_{ij} D^+_{kl}]^{J=0}. \tag{9}$$

By summing up these quartets one constructs the generalized quartets

$$Q_\nu^+ = Q_\nu^{+(iv)} + Q_\nu^{+(is)}. \tag{10}$$

We approximate the ground state of the Hamiltonian (8) for an even-even $N = Z$ nucleus as a product of such quartets, namely

$$|\Psi_{gs}\rangle \equiv |QM\rangle = \prod_{\nu=1}^{N_Q} Q_\nu^\dagger |0\rangle. \tag{11}$$

The results for the correlation energy relative to the ground state of the Hamiltonian (7) are given in Table 2 ([10]). The errors with respect to the exact solution are very small (less than 1%). This shows that the ansatz (11) provides a very good approximation of the ground state of an isoscalar-isovector pairing Hamiltonian.

Table 2

Ground state correlation energies (in MeV) calculated for the isovector plus isoscalar pairing Hamiltonian (7) with strengths extracted from standard shell model interactions (for details see Ref. [10]). The results are shown for the exact diagonalizings and the QM. The errors relative to the exact results are given in brackets.

	Exact	QM
^{24}Mg	28.694	28.626 (0.24$\%$)
^{28}Si	35.600	35.396 (0.57$\%$)
^{32}S	38.965	38.865 (0.25$\%$)
^{48}Cr	11.649	11.624 (0.21$\%$)
^{52}Fe	13.887	13.828 (0.43$\%$)
^{108}Xe	5.505	5.495 (0.18$\%$)
^{112}Ba	7.059	7.035 (0.34$\%$)

4. CONCLUSIONS

We have described the ground state of a proton-neutron pairing Hamiltonian in even-even $N = Z$ nuclei in a formalism of alpha-like quartets. Quartets are built by two neutrons and two protons coupled to total isospin $T = 0$ and total angular momentum $J = 0$. We have first considered the case of an isovector pairing Hamiltonian and compared two different approximations for the ground state, one as a condensate of identical quartets and the other one as a product of distinct quartets. The latter formalism has therefore been applied to the treatment of an isovector plus isoscalar Hamiltonian. We have carried out a number of numerical tests for systems with valence nucleons outside the ^{16}O, ^{40}Ca and ^{100}Sn cores and with pairing interactions extracted from realistic shell model Hamiltonians. We have verified that ground state correlation energies are reproduced with high accuracy in the quartet formalism. These calculations therefore support the conclusion that this formalism is very appropriate for the treatment of proton-neutron pairing in self-conjugate nuclei.

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