

# HALF-LIVES OF THIRTEEN DOUBLE $\beta^-$ -DECAY CANDIDATES WITH TWO NEUTRINOS\*

YUEJIAO REN<sup>1</sup>, ZHONGZHOU REN<sup>1,2,3,4,a</sup>

<sup>1</sup>Department of Physics, Nanjing University, Nanjing 210093, China

<sup>2</sup>Center of Theoretical Nuclear Physics, National Laboratory of Heavy-Ion Accelerator,  
Lanzhou 730000, China

*E-mail*<sup>a</sup>: zren@nju.edu.cn

<sup>3</sup>Joint Center of Nuclear Science and Technology, Nanjing University, Nanjing 210093, China

<sup>4</sup>Kavli Institute for Theoretical Physics China, Beijing 100190, China

*Received October 30, 2014*

Double  $\beta^-$ -decay with two neutrinos is one of the important decay modes for an unstable nucleus which can spontaneously change into a stable nucleus. It is also a very complex decay process where three kinds of interactions, the strong interaction, the Coulomb interaction, and the weak interaction, are involved. Recently we have made a systematic analysis on the experimental data of nuclear double  $\beta^-$ -decay and proposed that there is a law between the logarithm of double  $\beta^-$ -decay half-lives and the reciprocal of the decay energy. This is a simple and accurate law for half-lives of the double  $\beta^-$ -decay. We use this law to predict the half-lives of the double  $\beta^-$ -decay candidates where the mass number of parent nuclei ranges from  $A=70$  ( $^{70}\text{Zn}$ ) to  $A=204$  ( $^{204}\text{Hg}$ ). Numerical results of the half-lives show that  $^{142}\text{Ce}$  is a very interesting candidate for future experiments to search new emitters of the double  $\beta^-$ -decay. We also compare the half-lives of double  $\beta^-$ -decay and  $\alpha$ -decay for  $^{142}\text{Ce}$  where its decay energies of two modes are close and find that double  $\beta^-$ -decay dominates in this case although double  $\beta^-$ -decay is a second order process of the weak interaction. The branching ratios between double  $\beta^-$ -decay and  $\alpha$ -decay are listed for the candidates of double  $\beta^-$ -decay.

*Key words:* Double- $\beta$  decay, new candidates of emitters, systematics law of half-lives.

*PACS:* 23.40.-s, 23.40.Bw.

## 1. INTRODUCTION

Up to date we know that there are several thousands of atomic nuclei and among them the number of stable nuclei is less than three hundred. Many nuclei are unstable and have different half-lives and decay modes. By various ways of decay, the unstable parent nuclei become more stable daughter nuclei and researches on these decay processes lead to the discovery of many new phenomena and new laws of nuclear physics. For example, researches on  $\beta$  decay lead to the assumption

\*Paper presented at the conference “Advanced many-body and statistical methods in mesoscopic systems II”, September 1-5, 2014, Brasov, Romania.

of the existence of the neutrino and this plays an important role in the basic formulation of the weak interaction. The confirmation of parity violation in the  $\beta$  decay of the polarized  $^{60}\text{Co}$  nucleus clearly shows that parity can not conserve in the weak interaction [1, 2]. For some even-even nuclei they are stable for a single  $\beta$  decay but they are unstable for double  $\beta$  decay, based on the difference of the nuclear masses between parent nuclei and daughter nuclei [3–12]. Therefore the double  $\beta$  decay can be observed for these nuclei although their half-lives are very long [13–18].

Up to now the double  $\beta^-$ -decay half-lives of eleven nuclei have been measured with definite values and new experiments to search more emitters of the double  $\beta^-$ -decay are planned. For example, it is planned to measure the half-lives of some double  $\beta^-$ -decay nuclei in the underground laboratory of Jinping mountain in the southwest of China where experimental researches on dark matter are being carried out. In this case it is useful to predict new emitters of double  $\beta^-$ -decay for future experiments. We have recently analyzed the experimental data of both double  $\beta^-$ -decay half-lives and decay energies for eleven nuclei [14–17] and proposed a systematic law between the half-lives and decay energies [12]. In this manuscript the law will be used to calculate the half-lives of thirteen double  $\beta^-$ -decay candidates. Some of these candidates may have  $\alpha$ -decay. So both double  $\beta^-$ -decay half-lives and  $\alpha$ -decay half-lives are calculated for some nuclei and the branching ratios are obtained.

## 2. NUMERICAL RESULTS AND DISCUSSIONS

Current experimental data show that there are double  $\beta^-$ -decays for eleven even-even nuclei and that there is double electron-capture (ECEC) for a single nucleus [14–18]. Very recently a systematic analysis on double  $\beta^-$ -decay data of ground-state transitions is made by our group and a new systematic law between the decay half-lives and the decay energy is proposed [12]. This law is an analytical equation [12]. It is very simple and accurate for the ground state transition of double  $\beta^-$ -decay between parent nuclei and daughter nuclei [12]. Without introducing any extra adjustments, the law is in good agreement with the data of the double  $\beta^-$ -decay between the ground state of parent nuclei and the first excited  $0^+$  state of daughter nuclei [12] although the law is proposed according to the data of ground-state transitions. As the law is proposed based on the experimental data of ground-state transitions of eleven even-even nuclei from  $^{48}\text{Ca}$  to  $^{238}\text{U}$ , the effect of the weak interaction, the correction of the Coulomb interaction and the leading term of nuclear structure have been taken into account in the law [12]. The expression of the systematic law of double  $\beta$ -decay half-lives with two neutrinos is given as follows [12],

$$\lg T_{1/2}(Ey) = [a - 2\lg(2\pi Z/137) + S]/Q_{2\beta}(MeV) \quad (1)$$

where  $T_{1/2}$  represents the double  $\beta^-$ -decay half-life and its unit is Ey (1 Ey =  $10^{18}$  years) [12].  $Q_{2\beta}$  (MeV) is the double  $\beta^-$ -decay energy and  $Z$  is the charge number of the parent nucleus [12]. The first term of this equation corresponds to the effect of the weak interaction and the value of the constant  $a$  is 5.843 which is obtained by fitting the experimental double  $\beta^-$ -decay data of ground state transitions for eleven even-even nuclei. The second term of equation (1) corresponds to the influence of the Coulomb potential for double  $\beta^-$ -decay half-lives. The third term is directly related to the nuclear shell effect [12] and  $S$  is chosen to be  $S = 2$  when the neutron number of the parent nucleus is a magic number;  $S = 0$  when the neutron number of the parent nucleus is not a magic number.

We use equation (1) to calculate the half-lives of thirteen candidates of double  $\beta^-$ -decay and numerical results are presented in Fig. 1 and in Table 1. Firstly we focus on Fig. 1. The variation of the logarithms of double  $\beta^-$ -decay half-lives with the decay energies is drawn in Fig. 1 for thirteen even-even nuclei from  $^{70}\text{Zn}$  ( $Z=30$ ) to  $^{204}\text{Hg}$  ( $Z=80$ ). In Fig. 1, the X-axis denotes the decay energy of ground-state transitions and the Y-axis represents the logarithm of double  $\beta^-$ -decay half-lives. Although the decay energies vary in a very narrow range from  $Q_{2\beta} = 0.07$  MeV of  $^{146}\text{Nd}$  to  $Q_{2\beta} = 1.417$  MeV of  $^{142}\text{Ce}$ , the half-lives vary in a very wide range from  $8.26 \times 10^{70}$  Ey of  $^{146}\text{Nd}$  to  $3.34 \times 10^6$  Ey of  $^{142}\text{Ce}$ . This clearly shows the very important effect of decay energies on double  $\beta^-$ -decay half-lives. The large decay energies can lead to a short half-life for the double  $\beta^-$ -decay of nuclei. Because  $^{142}\text{Ce}$  has the shortest half-life in the thirteen nuclei of Fig. 1, we strongly recommend it as a very ideal candidate to observe the double  $\beta^-$ -decay in future. The nuclei  $^{176}\text{Yb}$ ,  $^{70}\text{Zn}$ ,  $^{134}\text{Xe}$ , and  $^{170}\text{Er}$  are also good candidates as they have shorter double  $\beta^-$ -decay half-lives in Fig. 1. Of course, the isotope abundance (IS) is also crucial for experimental observation of the double  $\beta^-$ -decay. So we list more numerical results in Table 1 where the isotope abundance (IS) and  $\alpha$ -decay results of nuclei are also included.

In Table 1, the first column represents the parent nuclei of double  $\beta^-$ -decay or the parent nuclei of  $\alpha$ -decay when it is available in energy. The second column of Table 1 denotes the experimental double  $\beta^-$ -decay energies for the ground-state transitions between the parent nuclei and the daughter nuclei where  $Q_{2\beta} = M(A, Z) - M(A, Z - 2)$  [16, 17]. The calculated double  $\beta^-$ -decay half-lives ( $T_{1/2}(\text{theor})$ ) are given in the third column of the table and the units of double  $\beta^-$ -decay half-lives are Ey (1 Ey =  $10^{18}$  years). We also list the experimental  $\alpha$ -decay energy of some nuclei when the decay energy is positive. As  $\alpha$ -decay is possible for some nuclei, it may compete with double  $\beta^-$ -decay in these nuclei. So it is interesting to calculate the  $\alpha$ -decay half-lives ( $T_{1/2}^\alpha$ ). We use the new Geiger-Nuttall law to calculate the  $\alpha$ -decay half-lives [19, 20] where the input  $\alpha$ -decay energies (column 4) are from references [16, 17]. The expression of the new Geiger-Nuttall law is given in next paragraph. The fifth column is the calculated  $\alpha$ -decay half-lives with new Geiger-Nuttall law

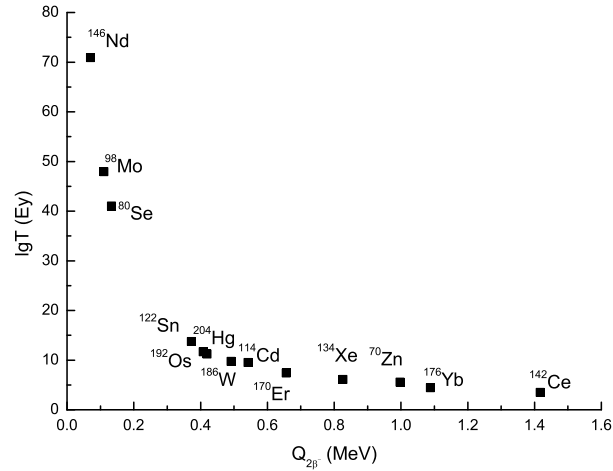


Fig. 1 – Variation of the logarithms of calculated double  $\beta$ -decay half-lives with the decay energies for thirteen even-even nuclei from  $^{70}\text{Zn}$  ( $Z = 30$ ) to  $^{204}\text{Hg}$  ( $Z = 80$ ). It is interesting to note that the half-lives also depend on the charge number from Eq. (1). It is seen that the half-lives are very sensitive to the decay energies. The nucleus  $^{142}\text{Ce}$  ( $Z = 58$ ) is very interesting for future double  $\beta$ -decay experiments due to its shortest double  $\beta$ -decay half-life in thirteen nuclei:  $T_{1/2} = 3.34 \times 10^3$  Ey for  $^{142}\text{Ce}$ . The nuclei  $^{176}\text{Yb}$ ,  $^{70}\text{Zn}$  and  $^{134}\text{Xe}$  are also interesting for future double  $\beta$ -decay experiments for their shorter half-lives.

(equation (2)). The sixth column represents the isotopic abundance (IS) of parent nuclei and the seventh column is the branching ratio between double  $\beta^-$ -decay and  $\alpha$ -decay ( $\approx T_{1/2}^\alpha/T_{1/2}(\text{theor})$ ).

A general expression of the new Geiger-Nuttall law can be derived from quantum tunneling theory of  $\alpha$ -decay and is written in the following way [19, 20]

$$\lg T_{1/2}^\alpha = a\sqrt{\mu}Z_cZ_d/\sqrt{Q} + b\sqrt{\mu}\sqrt{Z_cZ_d} + c + S + Pl(l+1), \quad (2)$$

$$S = 0 \quad \text{for } N \geq 127 \text{ and } S = 1 \quad \text{for } N \leq 126$$

Here the values of three parameters are  $a = 0.39961$ ,  $b = -1.31008$ ,  $c_{e-e} = -17.00698$  for even-even (e-e) nuclei [19–21].  $T_{1/2}^\alpha(\text{s})$  is the half-life of  $\alpha$ -decay and  $Q(\text{MeV})$  is the corresponding decay energy.  $Z_c$  and  $Z_d$  are the charge numbers of the cluster and daughter nucleus.  $\mu = A_cA_d/(A_c + A_d)$  is the reduced mass and  $A_c$ ,  $A_d$  are the mass numbers of the cluster and daughter nucleus, respectively. For  $\alpha$ -decay,  $Z_c = 2$  and  $A_c = 4$ . The three parameters,  $a$ ,  $b$ ,  $c$ , are obtained by fitting the data of even-even nuclei with  $Z \geq 84$  and  $N \geq 128$  [19, 21]. For the ground-state

transitions of even-even nuclei, there is no contribution from the last term of equation (2) as the angular momentum of  $\alpha$ -particle is zero  $l = 0$ .

Table 1

Theoretical double  $\beta^-$ -decay half-lives ( $T_{1/2}(\text{theor.})$ ) and the corresponding double  $\beta^-$ -decay energies ( $Q_{2\beta}$ ) for the thirteen double  $\beta^-$ -decay candidates. The calculated  $\alpha$ -decay half-lives ( $T_{1/2}^\alpha$ ) and the corresponding  $\alpha$ -decay energies ( $Q_\alpha$ ) are also listed for comparison when the  $\alpha$ -decay is possible. In the last two columns of the table, we also list the isotopic abundance (IS) of parent nuclei and the branching ratio (BR) between double  $\beta^-$ -decay and the  $\alpha$ -decay. The units of the double  $\beta^-$ -decay half-lives are Ey ( $10^{18}$  years). For  $^{142}\text{Ce}$ , the energy of  $\alpha$ -decay is 1.304 MeV. For  $^{146}\text{Nd}$ , the energy of  $\alpha$ -decay is 1.182 MeV. For  $^{170}\text{Er}$ , the energy of  $\alpha$ -decay is 0.050 MeV. For  $^{176}\text{Yb}$ , the energy of  $\alpha$ -decay is 0.569 MeV. For  $^{186}\text{W}$ , the energy of  $\alpha$ -decay is 1.116 MeV. For  $^{192}\text{Os}$ , the energy of  $\alpha$ -decay is 0.361 MeV. The experimental  $\alpha$ -decay and double  $\beta^-$ -decay energies (MeV) in the table are from references [16] and [17], respectively. It is seen from the table that  $^{142}\text{Ce}$  will be very interesting for future experimental observation of double  $\beta^-$ -decay due to its shortest double  $\beta^-$ -decay half-life in the thirteen nuclei.  $^{70}\text{Zn}$  and  $^{176}\text{Yb}$  are also interesting for future experiments of double  $\beta^-$ -decay due to their shorter double  $\beta^-$ -decay half-life.

Nuclei	$Q_{2\beta}$ (MeV)	$T_{1/2}(\text{theor.})$ (Ey)	$Q_\alpha$ (MeV)	$T_{1/2}^\alpha$ (Ey)	IS(%)	BR ( $\approx$ )
$^{70}\text{Zn}$	0.997	$3.83 \times 10^5$			0.61	100%
$^{80}\text{Se}$	0.133	$1.08 \times 10^{41}$			49.61	100%
$^{98}\text{Mo}$	0.11	$8.83 \times 10^{47}$			24.39	100%
$^{114}\text{Cd}$	0.542	$3.29 \times 10^9$			28.72	100%
$^{122}\text{Sn}$	0.372	$5.89 \times 10^{13}$			4.63	100%
$^{134}\text{Xe}$	0.826	$1.32 \times 10^6$			10.44	100%
$^{142}\text{Ce}$	1.417	$3.34 \times 10^3$	1.304	$2.70 \times 10^8$	11.11	100%
$^{146}\text{Nd}$	0.07	$8.26 \times 10^{70}$	1.182	$5.77 \times 10^{14}$	17.19	0%
$^{170}\text{Er}$	0.656	$2.52 \times 10^7$	0.05	$9.13 \times 10^{394}$	14.91	100%
$^{176}\text{Yb}$	1.088	$2.75 \times 10^4$	0.569	$5.39 \times 10^{70}$	13.00	100%
$^{186}\text{W}$	0.491	$5.48 \times 10^9$	1.116	$1.45 \times 10^{35}$	28.43	100%
$^{192}\text{Os}$	0.408	$4.61 \times 10^{11}$	0.361	$5.77 \times 10^{121}$	40.78	100%
$^{204}\text{Hg}$	0.419	$1.78 \times 10^{11}$			6.87	100%

It is seen again from Table 1 that the shortest double  $\beta^-$ -decay half-life is  $3.34 \times 10^3$  Ey for  $^{142}\text{Ce}$  and the longest is  $8.26 \times 10^{70}$  Ey for  $^{146}\text{Nd}$ . They correspond to the highest double  $\beta^-$ -decay energy and the lowest double  $\beta^-$ -decay energy, respectively. This shows that the decay half-life strongly depends on the decay energy. The isotopic abundance (IS) of  $^{142}\text{Ce}$  is 11.11% in nature and this is good for future double  $\beta^-$ -decay experiments. Although the  $\alpha$ -decay energy of  $^{142}\text{Ce}$   $Q_\alpha = 1.304$  MeV is approximately equal to the double  $\beta^-$ -decay energy of  $^{142}\text{Ce}$   $Q_{2\beta} = 1.417$  MeV, the  $\alpha$ -decay half-life is much longer than that of double  $\beta^-$ -decay. This is surprising because the  $\alpha$ -decay is governed by both the strong interaction and the Coulomb interaction

and the double  $\beta$ -decay is a second order process of the weak interaction in Fermi's theory. This surprising phenomenon of  $^{142}\text{Ce}$  is general for other double  $\beta$ -decay nuclei and it is very different from that of particle physics where the decay processes of both the strong interaction and the electromagnetic interaction are much faster than that of the weak interaction when the decay energies of elementary particles are close for different decay channels. This clearly demonstrates the novel properties of the nuclear many-body problem and more new phenomena of nuclear physics can be explored by researches of both double  $\beta$ -decay and  $\alpha$ -decay. In the last column, we list the approximate value of branching ratio for double  $\beta$ -decay and we see that double  $\beta$ -decay is dominant in the decay process of many nuclei except that of  $^{146}\text{Nd}$ . Therefore it is seen again from Table 1 that  $^{142}\text{Ce}$  is the most interesting case for future experiments due to its shortest half-life and the nuclei  $^{176}\text{Yb}$ ,  $^{70}\text{Zn}$ , and  $^{134}\text{Xe}$  are also interesting for future double  $\beta$ -decay experiments for their shorter half-lives.

### 3. CONCLUSIONS

We calculate double  $\beta$ -decay half-lives for thirteen candidates from  $^{70}\text{Zn}$  ( $Z = 30$ ) to  $^{204}\text{Hg}$  ( $Z = 80$ ) with the systematic law of double  $\beta$ -decay. Numerical results show that  $^{142}\text{Ce}$  is very interesting for future experiments to search new emitters of the double  $\beta^-$ -decay due to its shortest half-life in the thirteen nuclei. We compare the half-lives of double  $\beta^-$ -decay and  $\alpha$ -decay for  $^{142}\text{Ce}$  where its decay energies of two modes are close and find that double  $\beta^-$ -decay dominates in this case although double  $\beta^-$ -decay is a second order process of the weak interaction. This is very different from the decay process of elementary particles in particle physics and it demonstrates the novel properties of nuclear many-body problems for decay processes. The nuclei  $^{176}\text{Yb}$ ,  $^{70}\text{Zn}$  and  $^{134}\text{Xe}$  are also interesting for future double  $\beta$ -decay experiments for their shorter half-lives in thirteen nuclei. These can be useful for future double  $\beta^-$ -decay experiments.

#### *Acknowledgements.*

This work is supported by the National Natural Science Foundation of China (Grant Nos. 11035001, 10735010, 10975072, 11120101005, and 11375086), by 973 National Major State Basic Research and Development of China (Grants Nos. 2013CB834400 and 2010CB327803), by a Project Funded by the Priority Academic Programme Development of JiangSu Higher Education Institutions (PAPD), by the Research Fund of Doctoral Point (RFDP; Grant No. 20100091110028), by the Science and Technology Development Fund of Macau (Grant No. 068/2011/A).

## REFERENCES

1. C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes, R. P. Hudson, *Phys. Rev.* **105**, 1413 (1957).
2. T. D. Lee and C. N. Yang, *Phys. Rev.* **104**, 254 (1956).
3. M. Goeppert-Mayer, *Phys. Rev.* **48**, 512 (1935).
4. W. C. Haxton, G. J. Stephenson, *Prog. Part. Nucl. Phys.* **12**, 409 (1984).
5. A. Faessler, *Prog. Part. Nucl. Phys.* **21**, 183 (1988).
6. H. V. Klapdor and K. Grotz, *Phys. Lett. B* **142**, 323 (1984).
7. F. Simkovic, V. Rodin, A. Faessler, P. Vogel, *Phys. Rev. C* **87**, 045501 (2013).
8. H. Ejiri, *Phys. Rep.* **338**, 265 (2000).
9. J. Suhonen, *Phys. Rev. C* **87**, 034318 (2013).
10. J. Suhonen, O. Civitarese, *Phys. Rep.* **300**, 123 (1998).
11. A. A. Raduta, C. M. Raduta, *Phys. Lett. B* **647**, 265 (2007).
12. Yuejiao Ren and Zhongzhou Ren, *Phys. Rev. C* **89**, 064603 (2014).
13. S. R. Elliott, A. A. Hahn, and M. K. Moe, *Phys. Rev. Lett.* **59**, 2020 (1987).
14. A. S. Barabash, *Phys. Rev. C* **81**, 035501 (2010).
15. R. Saakyan, *Annu. Rev. Nucl. Part. Sci.* **63**, 503 (2013).
16. G. Audi, F. G. Kondev, M. Wang, B. Pfeiffer, X. Sun, J. Blachot, M. MacCormick, *Chin. Phys. C* **36**, 1157 (2012).
17. M. Wang, G. Audi, A. H. Wapstra, F. G. Kondev, M. MacCormick, X. Xu, B. Pfeiffer, *Chin. Phys. C* **36**, 1603 (2012).
18. K. A. Olive *et al.*, *Chin. Phys. C* **38**, 090001 (2014).
19. Yuejiao Ren and Zhongzhou Ren, *Phys. Rev. C* **85**, 044608 (2012).
20. Yuejiao Ren and Zhongzhou Ren, *Nuclear Science and Technology* **24**, 050518 (2013).
21. Dongdong Ni, Zhongzhou Ren, Tiekuan Dong, and Chang Xu, *Phys. Rev. C* **78**, 044310 (2008).