PYGMY DIPOLE RESONANCE IN A SCHEMATIC MODEL*

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We present a model which extends the approach introduced by D. Brink to evidence the collective nature of Giant Dipole Resonance. For neutron-rich nuclei the emergence of an additional low energy mode that can be associated to the Pygmy Dipole Resonance (PDR) is predicted. We explore the role of a separable dipole-dipole interaction where the condition to have a unique coupling constant was relaxed in order to account for the density dependence of the symmetry energy. The values of the coupling constants are not affecting too much the position of energy centroid of the pygmy state but are strongly influencing the Energy Weighted Sum Rule (EWSR) exhausted by it.

Key words: Nuclear shell model, Giant Dipole Resonance, Schematic model.

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1. INTRODUCTION

After its discovery the Giant Dipole Resonance becomes one of the most studied collective state of the atomic nuclei. Its properties as the energy centroid E_{GDR}, the width, the fine structure, provide informations about the structure of nuclear systems, the features of the nuclear interactions as well as about the damping mechanisms in finite fermionic systems [1].

One of the first microscopic investigations based on the shell model showing that in nuclear photo-effect the protons vibrate against the neutrons was proposed by David Brink in 1957 [2]. Indeed, starting from a Harmonic Oscillator Shell Model (HOSM) Hamiltonian for N neutrons and Z protons

$$H_{SM} = \sum_{i=1}^{A} \frac{\vec{p_i}^2}{2m} + \frac{m\omega_0^2}{2} \sum_{i=1}^{A} \vec{r_i}^2 \tag{1}$$

a separation into four commuting terms:

$$H_{SM} = H_{n,int} + H_{p,int} + H_{GDR} + H_{CM} \tag{2}$$

was proposed. In this decomposition $H_{n,int}$ and $H_{p,int}$ depend only on the neutron-neutron and proton-proton relative coordinates and characterize the internal motion

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of the two species, H_{CM} describe the center of mass motion, while H_{GDR}

$$H_{GDR} = \frac{\vec{P}^2}{2M_D} + \frac{M_D\omega_0^2}{2}\vec{X}^2 \tag{3}$$

determine a Goldhaber-Teller motion with the frequency ω_0 of the protons against neutrons. Here $M_D = \frac{mNZ}{A}$ is the collective mass associated to the dipolar motion, $\vec{X} = \vec{R}_p - \vec{R}_n$ define the distance between the centers of mass of protons and of neutrons respectively, $\vec{P}$ is the canonically conjugate momentum to $\vec{X}$, m is the nucleon mass. The frequency ω_0 is derived from the requirement to reproduce the nuclear size and is obtained $\hbar\omega_0 = 40A^{-1/3}$ MeV. This value is almost half the observed value of the GDR energy centroid which is well described by the parametrization $E_{GDR} = 80A^{-1/3}$ MeV. However by adding to H_{GDR} a separable dipole-dipole residual interaction [3, 4] $V_D = \frac{1}{2}\chi D^2$ the experimental value of the GDR energy can be reproduced when the coupling constant χ is related to the value of the potential symmetry energy at saturation density [5], *i.e.* $\chi = \chi(\rho_0)$.

2. PYGMY DIPOLE RESONANCE WITHIN A HARMONIC OSCILLATOR SHELL MODEL WITH SEPARABLE RESIDUAL INTERACTION

For a neutron rich nucleus the weaker coupled neutrons can be treated as a distinct system. We call them neutrons in excess and denote their number by N_e. The remain neutrons $N_c = N - N_e$ and all protons define a second system, namely the core which is expected to be more stable. Also in this case it is possible a separation of the shell model Hamiltonian into six commuting terms [6]:

$$H_{SM} = H_{n_c,int} + H_{n_e,int} + H_{p,int} + H_{CM} + H_c + H_y \tag{4}$$

with the first three terms describing the internal motion of protons, core neutrons and neutrons in excess respectively. The fourth term, $H_{CM} = \frac{1}{2mA}\vec{P}^2_{CM} + \frac{m\omega_0^2 A}{2}\vec{R}^2_{CM}$, characterizes the center of mass motion. Here $\vec{R}_{CM}$ define the center of mass position while $\vec{P}_{CM}$ is the corresponding linear momentum. H_c determines the dynamics of the core coordinate $\vec{X}_c = \vec{R}_p - \vec{R}_{n,c}$, defined as the distance between the core neutrons and core protons, while H_y describes a Goldhaber-Teller type vibration of the neutrons in excess against the core. The corresponding coordinate for the latter motion is the distance between the core center of mass and neutrons in excess center of mass, $\vec{Y} = \frac{N_c\vec{R}_{n,c} + Z_c\vec{R}_p}{N_c + Z} - \vec{R}_{n,e}$. Then:

$$H_c = \frac{\vec{P}_c^2}{2M_c} + \frac{M_c}{2}\omega_0^2\vec{X}_c^2 \quad ; \quad H_y = \frac{\vec{P}_y^2}{2M_y} + \frac{M_y}{2}\omega_0^2\vec{Y}^2 \tag{5}$$

Here $M_c = m\dfrac{N_c Z}{A_c}$, $M_y = m\dfrac{N_e A_c}{A}$ are the collective masses while P_c and P_y are the canonically conjugated momenta to X_c and Y respectively. As in the case of Brink model both degrees of freedom are oscillating with the same frequency ω_0.

However it was noticed above that a separable dipole-dipole interaction changes the frequency of the GDR and places it closer to the experimentally observed values. Here, in order to explore the role of the residual interaction on the two collective modes evidenced in HOSM a generalized separable dipole-dipole interaction is introduced. Since the neutrons in excess are in a lower density nuclear environment and because the coupling constants are determined by potential symmetry energy, which is density dependent, we relax the condition to have a unique coupling constant for all particle-hole pairs in the residual interaction. We consider a more general structure

$$V_{int} = \frac{1}{2}\chi_1 D_c^2 + \frac{1}{2}\chi_2 D_y^2 + \chi_3 D_c D_y \tag{6}$$

with $D_c = \dfrac{N_c Z}{A_c} X_c$, $D_y = \dfrac{N_e Z}{A} Y$ and $\chi_1 > \chi_3 > \chi_2$. We assume that $\chi_1 = \chi(\rho_0)$. Here χ_3 determine the coupling between the two subsystems and therefore will be defined by a symmetry energy corresponding to a density intermediate between that of the skin and that of the core respectively. The part of the Hamiltonian describing the two collective motions becomes

$$H = \frac{P_c^2}{2M_c} + \frac{M_c}{2}\left(\omega_0^2 + \omega_c^2\right) X_c^2 + \frac{P_y^2}{2M_y} + \frac{M_y}{2}\left(\omega_0^2 + \omega_y^2\right) Y^2 + C X_c Y. \tag{7}$$

where:

$$\omega_c^2 = \frac{\chi_1}{m}\frac{N_c Z}{A_c} \quad ; \quad \omega_y^2 = \frac{\chi_2}{m}\frac{N_e Z^2}{A A_c} \quad ; \quad C = \frac{N_c Z^2 N_e}{A A_c}\chi_3 = \frac{\chi_3}{\sqrt{\chi_1 \chi_2}}\sqrt{M_c M_y \omega_c^2 \omega_y^2}. \tag{8}$$

With the definitions $\omega_1^2 = \omega_0^2 + \omega_c^2$, $\omega_2^2 = \omega_0^2 + \omega_y^2$ from the Hamilton equations the following system of coupled equations for X_c and Y is obtained:

$$\begin{cases} M_c \ddot{X}_c + M_c \omega_1^2 X_c + C Y = 0 \\ M_y \ddot{Y} + M_y \omega_2^2 Y + C X_c = 0, \end{cases} \tag{9}$$

The frequencies of the two normal modes are:

$$\omega_{\alpha,\beta}^2 = \omega_0^2 + \frac{\omega_c^2 + \omega_y^2}{2} \pm \frac{1}{2}\sqrt{(\omega_c^2 - \omega_y^2)^2 + 4\frac{\chi_3^2}{\chi_1 \chi_2}\omega_c^2 \omega_y^2}, \tag{10}$$

In terms of the normal coordinates and the associated momenta defined as:

$$\begin{aligned} X_1 &= R(X_c - \frac{M_y}{C}(\omega_\beta^2 - \omega_2^2)Y) \quad ; \quad X_2 = R(X_c - \frac{M_y}{C}(\omega_\alpha^2 - \omega_1^2)Y) \\ P_1 &= P_c + \frac{M_c}{C}((\omega_\alpha^2 - \omega_1^2))P_y \quad ; \quad P_2 = \frac{M_y}{C}((\omega_\beta^2 - \omega_2^2))P_c + P_y \end{aligned} \tag{11}$$

the Hamiltonian splits into two independent terms:

$$H = \frac{P_1^2}{2M_1} + \frac{M_1\omega_\alpha^2}{2}X_1^2 + \frac{P_2^2}{2M_2} + \frac{M_2\omega_\beta^2}{2}X_2^2 \tag{12}$$

In these expressions the factor R and the masses M_1 and M_2 are:

$$R^{-1} = 1 - \frac{M_y M_c}{C^2}(\omega_\alpha^2 - \omega_1^2)(\omega_\beta^2 - \omega_2^2) \;\; ; \;\; M_1 = \frac{M_c}{R} \;\; ; \;\; M_2 = \frac{M_y}{R} \tag{13}$$

Summarizing the analysis presented above we conclude that in the presence of the separable dipole-dipole interaction (6) the HOSM models predicts the existence of two collective states with energies $E_1 = \hbar\omega_\alpha$ and $E_2 = \hbar\omega_\beta$ respectively.

The dipole moment $D = \frac{NZ}{A}X = \frac{N_c Z}{A_c}X_c + \frac{N_e Z}{A}Y = D_c + D_y$ can be written as a sum of the dipole moments defined by the normal coordinates $D = D_1 + D_2 = d_1 X_1 + d_2 X_2$:

$$D_1 = (\frac{N_c Z}{A_c} + \frac{N_e Z}{A}\frac{M_c}{C}(\omega_\alpha^2 - \omega_1^2))X_1 \; ; \; D_2 = (\frac{N_e Z}{A} + \frac{N_e Z}{A_c}\frac{M_y}{C}(\omega_\beta^2 - \omega_2^2))X_2 \tag{14}$$

Consequently, the total EWSR which is proportional to $[D,[H,D]] = \frac{\hbar^2}{m}\frac{NZ}{A}$ will be distributed now among the two states:

$$[D,[H,D]] = [D_1,[H_1,D_1]] + [D_2,[H_2,D_2]] = \frac{\hbar^2}{M_1}d_1^2 + \frac{\hbar^2}{M_2}d_2^2 \tag{15}$$

In Fig. 1 we present the predictions of our model in the case of ^{68}Ni. We consider an intermediate value for the number of neutrons in excess, $N_e = 6$, assume a fixed value for χ_2, $\chi_2 = 0.2\chi_1$ and discuss the results as a function of χ_3, the strength which determine the coupling between the two subsystems, the core and the neutrons in excess. It is seen that one of the states has an energy close to the GDR energy and we interpret it as the usual GDR. The second state has an energy between 10.2 MeV and 9.5 MeV when χ_3/χ_1 varies from 0.2 to 1.0. We associate the latter state with PDR. The corresponding EWSR fraction exhausted by PDR changes from 4.2% to zero for the same values of χ_3. Recent experimental results [8] reported for PDR in ^{68}Ni an energy centroid at 9.55 MeV and a fraction of EWSR around 2.8%. In Fig. 1 is also represented the ratio of the variation of the two coordinates Y and X_c corresponding to the two states. We observe that these coordinates oscillates in phase in the case of GDR and out of phase in the case of PDR.

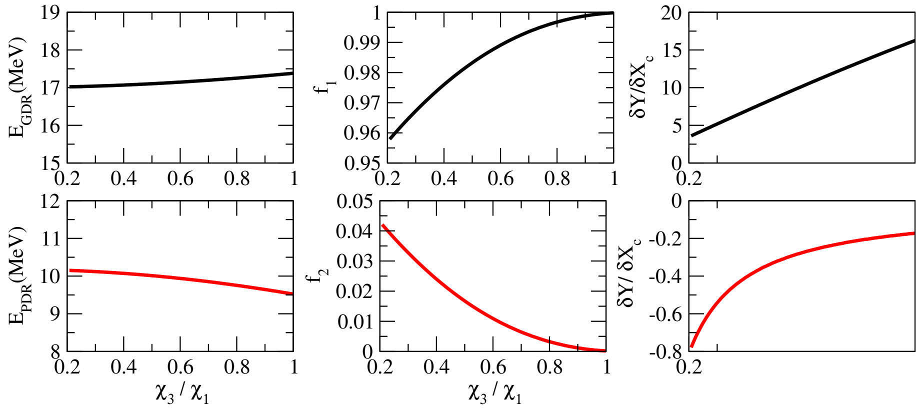

Fig. 1 – (color on-line) The GDR (top, black lines) and PDR (bottom, red lines) energies (E_{GDR} and E_{PDR}), the EWSR fractions exhausted by each state (f_1 and f_2) and the structure of the two normal modes in terms of the collective coordinates Y and X_c ($\delta Y/\delta X_c$).

3. CONCLUSIONS

In this paper we discussed a generalization of the Brink model based on the Harmonic Oscillator Shell Model in the presence of a separable dipole-dipole interaction where the condition to have a unique coupling constant was relaxed. Within this approach for ^{68}Ni we identified two dipolar collective states which are describing quite well the basic properties of GDR and PDR states observed experimentally.

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REFERENCES

1. M.N. Harakeh, A. van der Woude, *Giant Resonances*, (Clanderon Press, Oxford, 2001).
2. D.M. Brink, *Nucl. Phys.* A **4**, 215 (1957).
3. G.E. Brown, M. Bolsterli, *Phys. Rev. Lett.* **3**, 472 (1959).
4. V.V. Balashov, *Nucl. Phys.* A **40**, 93 (1963).
5. A. Bohr, B.R. Mottelson, *Nuclear Structure* vol II, p. 481 (World Scientific, Singapore, 1998).
6. V. Baran *et al.*, *Rom. J. Phys.* **57**, 36 (2012).
7. V. Baran, B. Frecus, M. Colonna, M. Di Toro, *Phys. Rev.* C **85**, 051601(R) (2012).
8. D.M. Rossi *et al.*, *Phys. Rev. Lett.* **111**, 242503 (2013).