GLASSY BEHAVIOR OF DISORDERED FRACTIONAL EXCLUSION STATISTICS SYSTEMS*

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Received October 29, 2014

We employ a general method based on fractional exclusion statistics and Monte Carlo simulations to study the low temperature dynamics of systems with quenched disorder. Depending on the degree of disorder and type of the interacting potentials, different dynamical behaviors are observed at low temperatures. In particular, the systems with a random mixture of repulsive and atractive interactions exhibit a slow typical glassy dynamics. Analyzing the spatial and autocorrelation functions we point out aging effects which are enhanced in the presence of interactions.

Key words: Fractional exclusion statistics, Monte Carlo simulation, Glassy dynamics.

In Ref. [1] Haldane introduced the concept of fractional exclusion statistics (FES) as a generalization of the Pauli exclusion principle. This concept was further developed by many authors (see Ref. [2] for a review). More recently, in Refs. [3–8], FES has been applied to general systems of (quantum and classical) particles with short or long-range interaction and in general external potentials. By this method the interacting particle systems are transformed into ideal FES gases by merging the interaction and the density of states into statistics parameters. Conversely, ideal FES systems may be transformed *back* into Bose or Fermi systems of interacting particles [9], realizing by this a two-way correspondence between the two descriptions. Making use of this correspondence we propose here an unitary interpretation of different types of glasses (*e.g.* Fermi, Coulomb, Bose glasses) in terms of what we shall call *FES glasses* and we present the tools for their investigation.

Our method is based on Monte Carlo simulations of disordered FES systems with localized states, developed in Refs. [8, 10]. The examples of FES systems studied are obtained by the above mentioned transformation into FES description of Fermi (FG) and Coulomb (CG) glasses with different degrees of disorder in the spatial distribution of sites and different types of (attractive and/or repulsive) interaction potentials [6, 7]. The slow dynamics and aging effects found in our models are prototypical for glassy systems.

*Paper presented at the conference "Advanced many-body and statistical methods in mesoscopic systems II", September 1-5, 2014, Brasov, Romania.

Rom. Journ. Phys., Vol. 60, Nos. 5-6, P. 691–695, Bucharest, 2015

The disordered systems studied here by FES consist of particles which interact through the two-particle potential $V(|\mathbf{r}_1 - \mathbf{r}_2|)$ and occupy localized single-particle states of density $\sigma(\mathbf{r}, E)$–$\mathbf{r}$ or $\mathbf{r}_i$ are position vectors and E denotes the single-particle energy; the external potential is zero. By the procedure of Refs. [6–8] we construct the species by coarse-graining the system's volume and the energy axis into small volumes and segments, respectively, building up the cells $\delta\mathbf{r}_\xi \times \delta E_i$, called species. For the simplicity of the calculations we assume that $\sigma(\mathbf{r}, E) \equiv \sigma(\mathbf{r})$ depends only on $\mathbf{r}$ and the interaction potential is short range, so that the interaction is relevant only within the same volume grain δr_ξ. The energy independent density of states within one species is $\sigma_\xi = \sigma(\mathbf{r}_\xi)\delta r_\xi$. The systems investigated are two-dimensional and boundary conditions are imposed in both spatial directions, x and y.

We consider four types of systems: the uniform non-interacting Fermi system (UN), a typical Fermi glass (FG), a Coulomb glass (CG) and a generic glass system with random interaction potential (FESG). The UN system has a constant density of states $\sigma_\xi = \sigma_0$ and $V(|\mathbf{r}|) = 0$ for all species. The FG-type system is also non-interacting, but has a randomly distributed two-value density of states $\sigma_\xi = \sigma_l$ or σ_h, with $\sigma_l + \sigma_h = 2\sigma_0$. The CG-type system is a FG-type system with short-range repulsive interactions $V_\xi = V_0 > 0$, where V_ξ represent the average particle-particle interaction energy within the same volume grain, ξ. As mentioned above, the interaction between particles in different grains is neglected. Finally, the FESG-type system is similar to the CG, but with disorder introduced also in the interaction potential, V_ξ, which takes two randomly distributed values, $\pm V_0$.

For the FES description of the systems we adopt Wu's perspective [11]. If in the absence of any particles in the system the number of available single-particle states in the species (ξ, i) is $G_{\xi i} = \sigma_\xi \delta E_i$ and we denote by $\alpha_{\xi i,\eta j}$ the FES parameters, then in the presence of the particles the number of available states becomes $G_{\xi i} - \sum_{\eta,j} \alpha_{\xi i,\eta j} N_{\eta j}$. For the ideal Fermi system (UN and FG), $\alpha_{\xi i,\eta j} = \delta_{\xi i,\eta j}$, whereas for the interacting Fermi gases (CG and FESG) the FES parameters $\alpha_{\xi i,\eta j} = \delta_{\xi i,\eta j}(1 + V_\xi \sigma_\xi)$ [8] have random distributions. We describe the dynamics using Monte Carlo simulations. We define a Markov chain in the phase space of the system, where each state is represented by the set $\{N_{\xi i}, G_{\xi i}\}_{\xi i}$. The move class is set by one particle jumps from a species (ξ, i) to a species (η, j), *i.e.* the states $\Upsilon \equiv \left\{N_{\xi i}, G_{\xi i}\ ;\ N_{\eta j}, G_{\eta j}\ ;\ \{N_{\zeta k}, G_{\zeta k}\}\right\}$ and $\Phi \equiv \left\{N_{\xi i} - 1, G'_{\xi i}\ ;\ N_{\eta j} + 1, G'_{\eta j}\ ;\ \{N_{\zeta k}, G'_{\zeta k}\}\right\}$ are neighbors. In the present study only nearest neighbor hopping in the real space is allowed. The transition rate for the $\Upsilon \to \Phi$ process is calculated using the statistical parameters $\alpha_{\xi i,\eta j}$ as [10] $\Gamma_{\Phi\Upsilon} \sim n_{\xi i}(1 + n_{\eta j} - A_{\eta j}) \prod_{(\zeta,k)} [1 + n_{\zeta k} - A_{\zeta k}]^{-\alpha_{\eta j,\zeta k}} \prod_{(\zeta,k)} [1 - A_{\zeta k}]^{\alpha_{\eta j,\zeta k}}$, where $n_{\xi i} \equiv N_{\xi i}/G_{\xi i}$ is the population of species ξi and $A_{\xi i} = \sum_{\eta,j} \alpha_{\xi i,\eta j} n_{\eta j}$. Using the FES parameters specified above we

write the transition rates as

$$\Gamma_{\Phi\Upsilon}(\alpha_\eta) \sim n_{\xi i}(1-\alpha_\eta n_{\eta j})^{\alpha_\eta}[1+(1-\alpha_\eta)n_{\eta j}]^{1-\alpha_\eta}, \tag{1}$$

where $\alpha_\eta \equiv 1+\sigma_\eta V_\eta$.

The aforementioned systems present as a hallmark a typically slow glassy dynamics when they are quenched from a high temperature state. We investigate the dynamics at low temperatures by analyzing the time dependence of the accumulations and depletions of particles in the real space with respect to equilibrium configuration $\{N_\xi^{\rm eq}\}$. For this we define two types of domains, generically denoted by $\mathcal{D}_\xi = \pm 1$, representing species with $N_\xi/N_\xi^{\rm eq} > 1+\delta_{\rm eq}$ and $N_\xi/N_\xi^{\rm eq} < 1-\delta_{\rm eq}$, respectively, where typically $\delta_{\rm eq} \ll 1$. A third domain type, $\mathcal{D}_\xi = 0$, defined by $1-\delta_{\rm eq} \leq N_\xi/N_\xi^{\rm eq} \leq 1+\delta_{\rm eq}$, represents a group of species which are close to equilibrium. The time evolution of the $\mathcal{D}_\xi$ domains is described by the spatial correlation function and the autocorrelation function [12]:

$$\bar{C}(r,t_w) = \frac{1}{N_s}\sum_\xi [\langle \mathcal{D}_\xi \mathcal{D}_\eta\rangle - \langle \mathcal{D}_\xi\rangle\langle \mathcal{D}_\eta\rangle]_{av}, \quad r=|\mathbf{r}_\xi - \mathbf{r}_\eta| \tag{2}$$

$$\tilde{C}(t,t_w) = \frac{1}{N_s}\sum_\xi [\langle \mathcal{D}_\xi(t+t_w)\mathcal{D}_\xi(t_w)\rangle]_{av}, \tag{3}$$

where N_s is the total number of sub-volumes in the real space, t_w is the waiting time and $\langle\cdot\rangle$, $[\cdot]_{av}$ are the thermal and disorder averages, respectively. Using the correlation functions typical glassy phenomena like domain growth and aging can be analyzed.

We perform Monte-Carlo simulations on two dimensional samples of $N_x \times N_y$ ($N_x = N_y = 16$) sub-volumes in the real space and $N_E = 750$ sub-volumes on the energy axis totalizing a number of $N_S = 192000$ species. The number of energy sub-volumes below the Fermi energy is $N_{E_F} = 50$, so that the energy range considered is $[0, 15E_F]$. The number of single-particle states in a species is $N_e = 120$. The number of particles is $N_p = N_{E_F} \times N_e \times N_x \times N_y$. The density of states for the uniform system is $\sigma_0 = 1$, while for the other three systems we take $\sigma_l = 0.5$ and $\sigma_h = 1.5$. The interacting potential is $V_0 = 1$. Here we use $\delta_{\rm eq} = 0$. For ensemble and thermal averages we use 10 disorder realizations, for each having 10 thermal runs. The initial state corresponds to a very high temperature state ($T \to \infty$).

Figure 1 shows the relaxation of the total energy for the systems of type UN, FG, CG and FESG, at two different temperatures, $T_1 = 0.5\ E_F/k_B$ and $T_2 = 0.05\ E_F/k_B$. For the glass type systems FG, CG and FESG the equilibration occurs at larger time scales than for UN. By construction, the non-interacting UN and FG systems have the same equilibration energy, whereas for the CG system the repulsive interaction leads to a higher energy and for the FESG system the mixed sign poten-

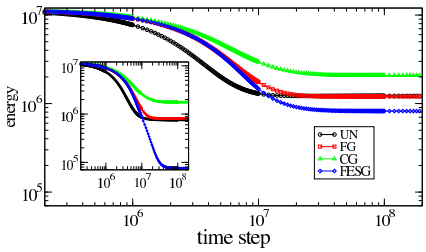

Fig. 1 – The relaxation of the total energy for the systems considered – UN, FG, CG and FESG – at two different temperatures, $T_1 = 0.5E_F/k_B$ (main plot) and $T_2 = 0.05E_F/k_B$ (inset).

tials lowers the energy.

Following a rapid quench to a low temperature, the particles loose mobility and regions with excess of particles as well as depleted regions with respect to the equilibrium distribution begin to appear. The initially scattered $\mathcal{D}_\xi$ domains begin to grow, as local equilibration takes place. However, the deviation from equilibrium in the particle density decreases with time, so that the domains decrease as the system approaches equilibration. Figure 2 presents a sequence of states for time interval where the decrease of the total energy is still noticeable but close to the equilibration point and the domain growth is visible.

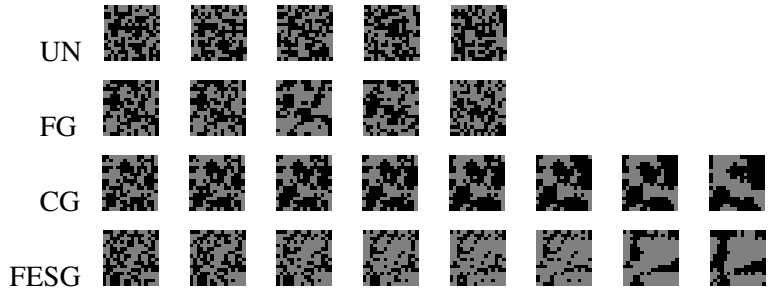

Fig. 2 – Typical domain growth, shown comparatively for UN, FG, CG and FESG at $T = 0.5$. The time scales t_w are the following: $6 \cdot 10^6 - 10^7$ (UN), $9 \cdot 10^6 - 4 \cdot 10^7$ (FG), $4 \cdot 10^6 - 2 \cdot 10^7$ (CG) and $10^6 - 5 \cdot 10^6$, $10^7 - 2 \cdot 10^7$ (FESG), in steps of 10^n for $m_1 10^n < t_w < m_2 10^n$.

To gain a more complete insight of the typical glassy properties of the considered systems, we plotted in Fig. 3 the spatial correlation and autocorrelation functions. The set of functions $\bar{C}(r, t_w)$ indicate for each system the growth rate of the $\mathcal{D}_\xi$ domains. Comparatively, the glass-type systems exhibit a larger domain growth than the uniform non-interacting system. Similarly, the auto-correlation functions point out a lower de-correlation with time and indication the samples present aging effects. Due to the fact that equilibration still occurs following the slow dynamics, the history-dependent phenomena is called interrupted aging [12, 13], *i.e.* it stops

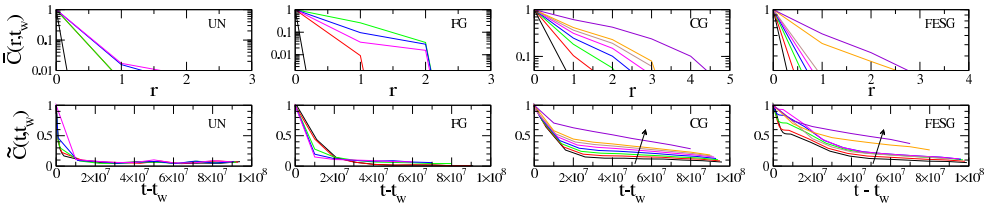

Fig. 3 – Spatial correlation functions $\bar{C}(r, t_w)$ and autocorrelation functions $\tilde{C}(t, t_w)$ for $T = 0.5$, at the time scales t_w indicated in Fig. 2. The arrows mark increasing t_w and the color ordering is kept for each sub-plot.

beyond the equilibration time. This can be seen, *e.g.* in the case of FG system at $t_w = 2 \cdot 10^7$ in Figs. 2 and 3, where the maximum domain size is reached.

To conclude, the low temperature dynamics was investigated in four types of systems: a uniform ideal Fermi gas (UN), a Fermi glass (FG), a Coulomb glass (CG) and a generic glass system with a random interaction potential (FESG). We showed that FES may be employed as an unitary method in the study of disordered systems, so that different types of disorders (*e.g.* in the density of states and interaction potential) in the original system are transferred into disordered FES parameters. We called these FES glasses and we studied their relaxation dynamics by Monte Carlo simulations, asserting the feasibility of the method.

Acknowledgements. The work was supported by ANCS under projects PN-II-ID-PCE-2011-3-0960 and PN 09 37 01 02/2011.

REFERENCES

1. F. D. M. Haldane, Phys. Rev. Lett. **67**, 937 (1991).
2. M. V. N. Murthy and R. Shankar, *Exclusion Statistics: From Pauli to Haldane.* Report of The Institute of Mathematical Sciences, Chennai, India, Mat. Sci. Rep. 120 (2013).
3. Dan Liu, J. Vanasse, G. Müller and M. Karbach, Phys. Rev. E **85**, 011144 (2012).
4. N. Gundlach, M. Karbach, D. Liu, and G. Müller, J. Stat. Mech.: Theory Exp. P04018 (2013).
5. B. Bakhti, M. Karbach, P. Maass, M. Mokim and G. Müller, Phys. Rev. E **89**, 012137 (2014).
6. G. A. Nemnes and D. V. Anghel, J. Phys.: Conf. Series **410**, 012120 (2013).
7. D. V. Anghel, G. A. Nemnes and F. Gulminelli, Phys. Rev. E **88**, 042150 (2013).
8. G. A. Nemnes and D. V. Anghel, Rom. Rep. Phys. **66**, 336 (2014).
9. D. V. Anghel, Phys. Lett. A **377**, 2922 (2013).
10. G. A. Nemnes and D. V. Anghel, J. Stat. Mech.: Theory Exp. P09011 (2010).
11. Y.-S. Wu, Phys. Rev. Lett. **73**, 922 (1994).
12. J. Kisker, L. Santen, M. Schreckenberg and H. Rieger, Phys. Rev. B **53**, 6418 (1996).
13. H. Rieger, B. Steckemetz and M. Schreckenberg, Europhys. Lett. **27**, 485 (1994).