

NEW HINTS FROM THEORY FOR PUMPING SPIN CURRENTS IN QUANTUM CIRCUITS*

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In the ballistic regime, polygonal quantum rings connected with an external circuit can be used to inject into it charge and/or spin, with no need for an external bias. This is a ‘one-parameter’ form of quantum pumping; it requires a time-dependent magnetic field and a proper selection of the geometry. With the field in the plane of a ring having one electron per site and an even number of sites a pure spin current, driven by the spin-orbit interaction, is excited in both wires. One can prove that the charge current vanishes identically if the whole system is a bipartite lattice. The pure spin current can be called a *magnetic current* since magnetization propagates while the system behaves like a gapless insulator. One can use storage units to concentrate and save this magnetization, in much the same way as capacitors store the electric charge. The stored magnetization is long lived and can be used later. By connecting up and down-polarized reservoirs one produces spin currents in another circuit. These currents show a dynamics. They oscillate while the storage units exchange their polarizations. The intensity and amplitude of the oscillations can be controlled by tuning the conductance of the wire used to connect the units.

Key words: Pumping, spin current, quantum transport. PACS: 05.60.Gg Quantum transport.

1. INTRODUCTION

Classically an external bias is necessary to create a current in a circuit. Charge pumping is any process which produces a current in a ballistic circuit without a bias; it is a purely quantum effect. The first process of this sort was introduced by Thouless [1] who considered the effects of a time-periodic and space-periodic potential in a 1d wire. Since then the literature on this intriguing kind of phenomena has been growing, and some connections to the Berry phase [2] have been brought to light. Some general properties of charge pumping in linear response theory are well established. P. Brower [3] has shown that in linear systems one gets two-parameter or several parameter pumping. D. Cohen [4] has demonstrated that the charge is quantized and that the quantization occurs in terms

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of Chern numbers. Later it was found that nonlinear systems can give rise to one-parameter forms of pumping and in this case the quantization of the pumped charge does not occur. L. Foa Torres [5] predicted the one-parameter pumping from a non-linear system consisting of a ring pierced by a magnetic field and symmetrically connected to a wire; the pumping of current into the wire is allowed provided that one of the arms contains a quantum dot acted upon by an external time-dependent potential. Romeo and Citro [6] also considered a nonlinear system involving a ring. Since there is no one-parameter Berry phase, these findings show that pumping cannot be reduced to the Berry phase concept in general, when nonlinear phenomena play a crucial role.

Quantum effects are important for the magnetic properties of rings in the ballistic regime. The magnetic moment of the ring is not obtained [7] by inserting the quantum mechanical current into the classical formula [8]. An even more dramatic departure from classical expectations occurs if one uses the rings as sources of charge and/or spin pumping in an external circuit [9]. It is remarkable that pumping from rings cannot exist in the adiabatic case [10]. It is clear from simple topological considerations (Fig. 1) that asymmetrically connected rings are needed if one wants to excite pumping by magnetic fields. The field selects a chirality and in turn the chirality determines a direction in the external circuit only if the two arms of the rings are different. So, here we concentrate on laterally connected rings, tangent to the external circuit, having a single bond in common with it. It has been shown [8] that indeed one can pump current by threading a magnetic flux in the ring. Already the limit in which one neglects spin is interesting, because for each fluxon inserted in the ring one pumps an amount of charge of the order of one electron, while the ring remains neutral and un-excited. I stress here that this is a purely quantum effect; moreover it is an example of one-parameter pumping (the only parameter which depends on the time is the magnetic flux) and according to the above arguments this is possible because the ring-field interaction is nonlinear and non-adiabatic. When spin is included and spin-orbit effects are considered, several interesting possibilities arise for pumping not only charge but also spin. It is hardly necessary to stress the focus of interest on spin-polarized currents, also for possible applications in spintronics. Alternative arrangements for producing spin polarized currents have been proposed elsewhere [11–13] using rotating rings or rotating magnetic fields. For reasons of space, I concentrate here on the production of a pure spin current. This means that the spin-up current is exactly opposite to the spin-down current. One can easily see that such a statement is independent of the choice of the spin quantization direction. Indeed, the pure spin currents transfer no electricity but only magnetization. Therefore the pure spin currents deserve to be called *magnetic currents*. Some electron-electron interaction effects have been considered in Ref. [14–16] within the Luttinger model but will not be included here. In Sections 2 and 3 I review the

model, state the reasons why at half filling the charge current vanishes exactly and include new results about the production of pure spin currents using a magnetic field in the plane of the ring. In Section 4 I propose a new scheme for the storage and delayed release of such currents. Conclusions are presented in Section 5.

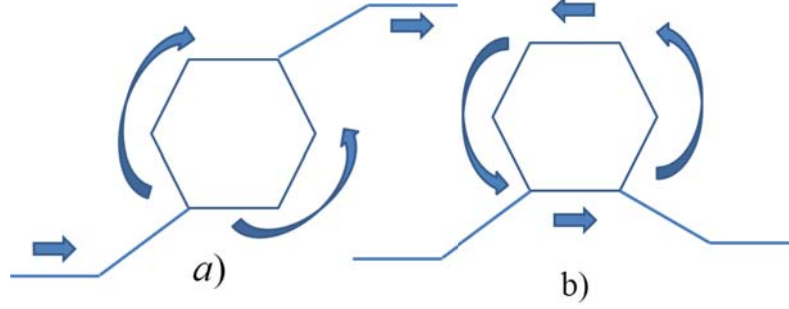


Fig. 1 – A symmetrically bonded ring a) in a current carrying circuit does not have a preferred chirality. A laterally bonded ring b) does.

2. MODEL

From now on, I consider a geometry with the ring in the xy plane, the wires along the x axis and the magnetic field along the y axis. The spin quantization axis is along z . The model Hamiltonian which leads to the spin current may be written as:

$$H_{prod} = H_{Lwire} + H_{Rwire} + H_{ring} + H_{ring-wires} + H_{magn} \quad (1.1)$$

where the left wire Hamiltonian

$$H_{Lwire} = t_h \sum_{n,\sigma} c_{n,\sigma}^\dagger c_{n+1,\sigma} \quad (1.2)$$

is a simple tight-binding model ($\sigma = \pm$ denotes spin), the hopping term t_h is taken as the unit of energy. (Below we shall measure time in units of $\tau = \frac{\hbar}{t_h}$).

The right-wire term is of the same form, and both wires are so long that the numerical results are length-independent. The term $H_{ring-wires}$ represents a nearest-neighbor connection between both wires and the ring. The ring is laterally connected as in Figure 1b); this means the following. The left wire is connected to site A and the right wire to site B where AB is one of the sides of the polygonal shaped ring. N_{ring} is the number of sides. The ring Hamiltonian is

$$H_{ring} = \sum_a^{N_{ring}} (t_h \sum_{\sigma=\pm 1} \exp[i\sigma\alpha_{SO}] c_{a+1,\sigma}^\dagger c_{a,\sigma} + h.c.) + \mu_B B (c_{a,\uparrow}^\dagger c_{a,\downarrow} + c_{a,\downarrow}^\dagger c_{a,\uparrow}) \quad (1.3)$$

where α_{SO} is a spin-orbit constant and $\mu_B = 5.79 \cdot 10^{-5}$ eV/T is the Bohr-Procopiu magneton. Reasonable values of $|\alpha_{SO}|$ can vary from 0.01, say, to about 1.5, depending on the atomic number.

The magnetic field B is taken to be in the y direction, that is, in the plane of the ring, and treats up and down (along z) spins on equal footing. The spin symmetry is broken by the spin-orbit term [17] proportional to α_{SO} which provides the driving force for the spin current. The current operator is

$$J_{m,\sigma} = \frac{it_h}{\hbar} (c_{m+1,\sigma}^\dagger c_{m,\sigma} - c_{m,\sigma}^\dagger c_{m+1,\sigma}). \quad (1.4)$$

I use my own partition-free approach [18] to evaluate the current. The code is based on a time-slicing technique that allows to evolve the many-electron wave function in each time slice as if the Hamiltonian were constant and to calculate the matrix elements of the lesser Green's function. Taking τ as the unit of time and assuming that t_h is 1 eV, the unit of current corresponds [12] to $3.87 \cdot 10^{-5}$ A.

3. PURE SPIN CURRENT

An analytic theory of the purely spin current was presented in [12] and deepened in [13]. The system must be half filled, that is, the number of electrons equals the number of sites. Another crucial ingredient is topological: is the lattice must be bipartite (*i.e.* it must consist of two kinds of sites, say blue and red sites, such that the nearest neighbors of any site are of the opposite color). This holds true for even N_{ring} . In a bipartite lattice at half filling each site has exactly 1 electron in the ground state, independently of its position. Finally, the hopping matrix elements for spin-down electrons must be the complex conjugates of those of up-spin electrons. Therefore, below these conditions are understood in all examples. Under the above conditions, the transport problem is exactly invariant under a canonical transformation that exchanges electrons with holes, spin up with spin down and changes sign to the wave functions of the blue sites. From this property one can prove that any current is purely magnetic, that is, no charge current can occur. The proof given in Ref. [12, 13] extends directly to finite temperature.

Instead, the results are sensitive to the filling, and for concentrations of the order of 0.51 one gets a spin current with a small charge current while the ring gets charged. The mechanism [13] which produces the spin current works basically in

second-order in B . In summary it works as follows. Opposite spins receive opposite phases by the spin-orbit interactions; B flips spin and allows an electron to visit the opposite spin sector for a while; thus the electrons of each spin see a fluctuating phase which acts like a bias. For any time dependence of $V(t) = \mu_B B(t)$ the charge current vanishes identically at half filling by symmetry, since the populations of each spin are 0.5; thus a pure spin current flows, for any time dependence. Remarkably, the geometrical symmetry plays no role in that. The charge is totally pinned and each site remains with 1 electron all the time. This is a gapless insulator of topological origin, so essentially it is a topological insulator. Within the present model, the adjacency graph for the Hamiltonian in the spin-orbital basis is also bipartite, and this implies that in the adiabatic limit each site has 0.5 electrons of each spin at all times, and so there is no spin current in the adiabatic limit, in agreement with Ref. [10]. The above results can be illustrated by numerical solutions. In Figure 2 lower panel I report the spin current response to the field dependence of the form $B(t) = B_{\max} (1 - \exp(-\frac{t}{60}))$, $B_{\max} = 50T$ (upper panel). The spin current is the same in both wires, while the electron current density is 1 electron/site everywhere. The time unit is $\tau = \frac{\hbar}{t_h}$.

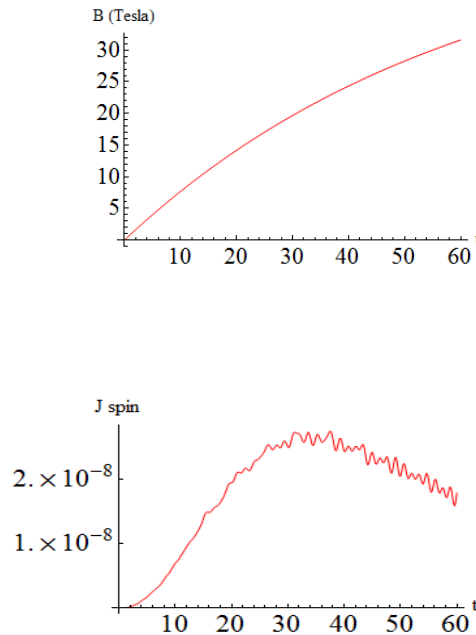


Fig. 2 – Numerical results of the model with B in the plane of the ring (with 30 sites) and 100 sites in each lead. The currents are excited by the field (upper panel).

As another example I report in Figure 3 the response to an oscillatory field. This example shows quite clearly that the response clearly not adiabatic, since the current does not follow the same pattern as the field. Rather it has approximately a double frequency, in agreement with its basically second-order nature.

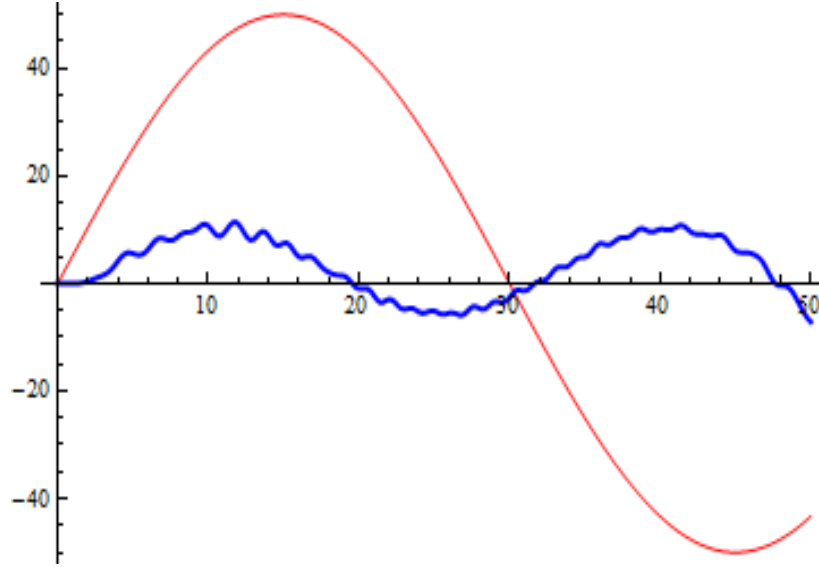


Fig. 3 – A field $B = B_{\max} \sin(\frac{2\pi t}{3\tau})$, $B_{\max} = 50T$ produces the spin current (here multiplied by 10^8), thick curve. Here, $N_{\text{ring}} = 6$.

4. STORAGE AND DELAYED RELEASE OF THE SPIN CURRENT

The possibility of exciting a pure spin current suggests that a magnetization transfer without charging of the kind outlined in the Introduction should be feasible using a laterally connected ring and a magnetic field in the plane of the ring. It should be possible to store and possibly concentrate the spin current in the form of a relatively long-lived static polarization of an electron gas for later use. Actually the polarization would be permanent in the absence of the spin-orbit interaction. In low atomic number materials the polarization could be long lived enough to make interesting experiments on it and maybe technological applications. If the current were partially polarized one should spend energy to charge a condenser, while for a magnetic current the process would be fast and very cheap energetically.

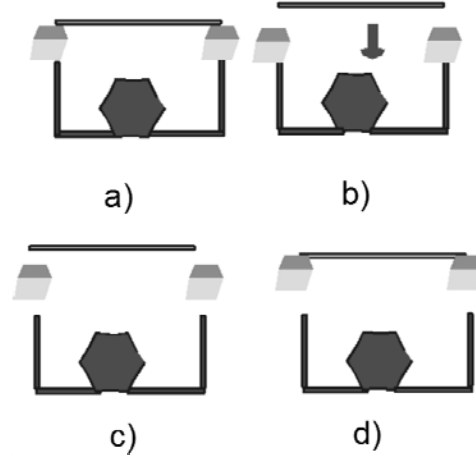


Fig. 4 – The 4 phases of a thought experiment in which a pure spin current is stored in two initially unpolarized cubes and released on command. In a) the storage cubes are linked to a laterally connected ring and directly among themselves. In phase b) the cube-cube connection is removed and a magnetic field produces a spin current. In c) the oppositely polarized cubes are isolated. In d) the cube-cube connection is established and a spin current flows.

To this end, I wish to present the computer experiment outlined in Figure 4. Let the total Hamiltonian become:

$$H = H_{prod} + H_{ext} + H_{L,store}(t) + H_{R,store}(t) \quad (1.5)$$

This is the sum of the above described H_{prod} plus the terms describing the left and right storage units $H_{L,store}(t), H_{R,store}(t)$, and the external wire H_{ext} of the same form as Equation (1.2) connecting the storage units. Here $H_{L,store}(t), H_{R,store}(t)$ also contain the connections to the external wire and to the respective wires to the ring, and these depend on time according to the pattern of Figure 4. One must choose $H_{L,store}(t), H_{R,store}(t)$ such that the whole graph is bipartite, and the storage units will be half filled initially and for all times.

In Figure 4a), the whole system is in equilibrium in the absence of external fields. Then the direct connection is removed and in 4b) the external field in the plane of the ring excites a spin current that will tend to polarize the 8-site cubes in opposite way. In 4c) the cubes are isolated and the spin current is stored in them as permanent magnetization. In this phase it is possible in principle to measure the magnetic dipole moments; this is also a way to measure the spin current in the previous phase. Finally, a direct cube-cube connection is established in 4d) and a current is allowed, driven by the opposite spin polarization of the cubes.

4.1. STORAGE CUBES AND A NEW WAY TO EXCITE SPIN CURRENTS

I performed a numerical experiment in which both storage units were 8-atom cubes. The field along the y axis was taken to be $B(t) = B_{\max} \sin(\frac{\pi t}{125})$ where $B_{\max} = \frac{B_{\phi}}{100}$ which is about 58 Tesla for $a = 10^{-10}$ m and a $N_{\text{ring}} = 60$ atom ring. The spin-orbit constant was taken to be $\alpha_{\text{SO}} = 1$. The leads from the ring to the cubes are 70 sites long while the two cubes are at direct contact with each other (that is, H_{ext} does not exist and there is direct hopping between the polarization reservoirs in this run). A magnetic current flows in the wires from the ring to the cubes, as expected from the above theorems, and its polarization axis is in the y-z plane; it is more intense when computed with the spin quantization axis along y than along z. The contacts to the ring are broken at $t = t_1 = 60 \tau$ while the hopping between cubes is allowed after $t = t_1 = 70 \tau$ (with $\tau = \frac{\hbar}{t_h}$).

In Figure 5 the populations of the two cubes are shown, and they are opposite for up and down spins and also opposite for the two cubes. This is polarization transfer without charging, and is quite substantial, since the spike corresponds to a shift of one per cent of the cube population. The populations start from half filling and begin to evolve after a time 40τ which is needed for the perturbation to cover the 70 site distance from the ring. The speed of the disturbance is about 2 sites in time τ . Then the left cube receives a up polarization and the right ring polarization is opposite. While this behavior had been expected in the design of the thought experiment, there were surprises, too. After the peak polarization is achieved, most of it goes away in a rather short time. One could engineer the timing of the process and the size of the storage units if the aim is to maximize the polarization. From t_1 to t_2 the rings are isolated and polarization is constant, as it should. The cubes remain strictly neutral like the rest of the system. At time t_2 the polarized cubes are set in direct contact (in this example H_{ext} is replaced by a direct hopping between the cubes) and the populations start to oscillate. This is a new way to produce a purely spin current, driven by the kinetic energy that was stored in the cubes in order to build an excess population of one spin direction.

In Figure 6 one can observe the immediate, rapidly oscillating purely spin current which is obtained in this way. These oscillations are interesting by their own right and it is likely that one can engineer the process, by varying the number of sites in the storage units and many available parameters, in order to modify both frequency and amplitude of the oscillations, which must produce electromagnetic

waves with a different pattern than any charge current. The oscillations are complex and fast, and will require further investigations. It should be interesting to observe the spectrum and polarization of electromagnetic waves emitted by these oscillating magnetic currents. That would provide a new way to detect the spin current.

4.2. STORAGE RINGS

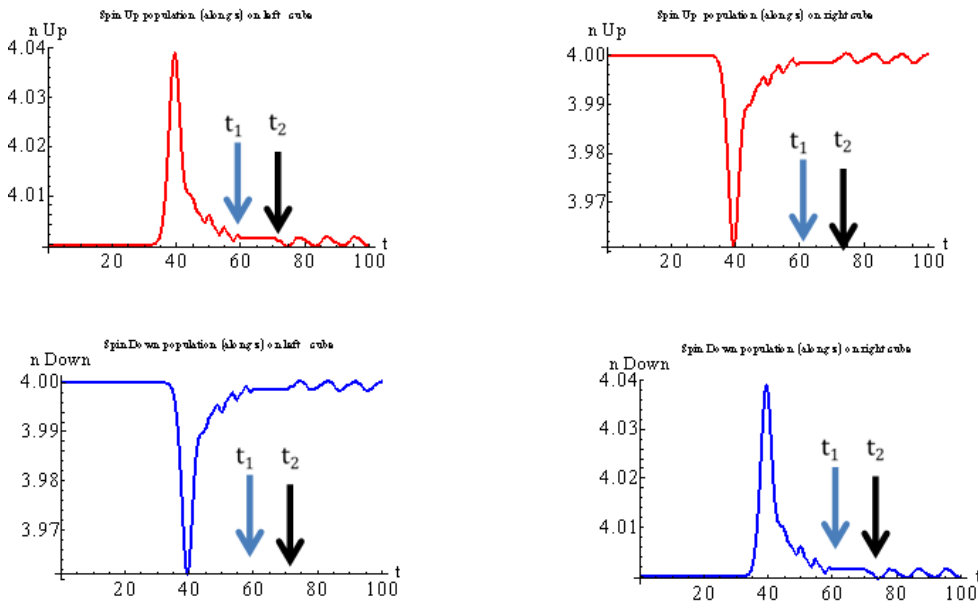


Fig. 5 – Electron populations on the two storage cubes during the thought experiment. Top left: spin up (along z) on left cube. Top Right: spin up on right cube. Bottom left: spin down on left cube. Bottom right: spin down on right cube. The arrows mark the disconnection from the ring at $t = t_1$ and the direct connection of the cubes at $t = t_2$. If the spin polarization axis is taken parallel to the magnetic field, the trend is similar, but the magnitude of the polarization is about twice as large.

In the last computer experiment reported in the present work the storage cubes are replaced by 4-atom storage rings; the connection H_{ext} between them at the beginning and at the end of the experiment is obtained with a wire composed by $N_{exter} = 200$ sites, with hopping t_h while the ring is $N_{ring} = 30$ sites large (Figure 7). The field grows linearly up to $t = 50\tau$ (Fig. 7a). The spin current in Figure 7 b) is largely reduced, compared to the previous figure, and this is due to the smaller object being magnetized. In Figure 7c) the length of the wire is

shortened, and this produces a shorter delay before the spin current arrives. Moreover the wire connecting the storage rings now has matrix elements $\frac{t_h}{3}$ and this reduction of the band width produces a decrease of the overall intensity of the spin current and a slowing down of the oscillations.

5. CONCLUSIONS

Tight-binding models suggest that Quantum Mechanics allows to build devices for producing spin-polarized currents. The scheme involves planar rings with an even number of atoms and magnetic fields. The polygonal rings have a side in common with the external circuit. Several possibilities arise [11–13]. In this paper I have shown that a tangent time-dependent magnetic field in the plane of the ring at half filling can be used to pump a purely spin current, and the driving force is provided by the spin-orbit interaction. The mechanism producing the spin current and no charge current is explained by a theorem in which the bipartite lattice is essential; analytically one can explain the effect in second-order in the field. The effect is found to work at finite temperature while it is sensitive to deviations from half filling.

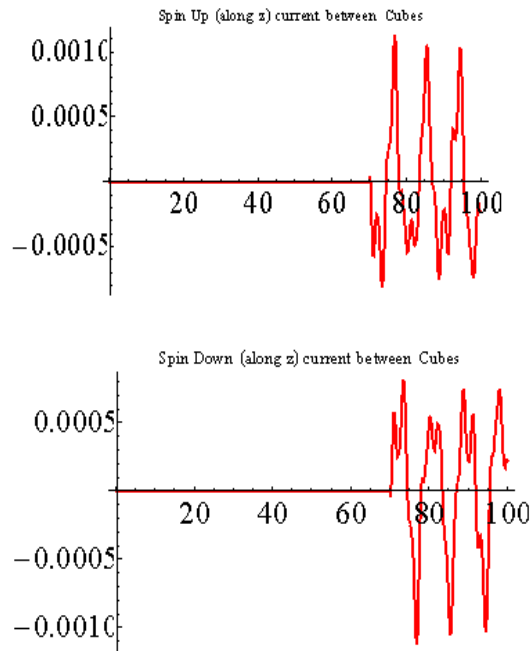


Fig. 6 – The current in the bond connecting the cubes during the thought experiment. Top: spin up. Bottom: spin down.

However, the main results of the present paper concern possible schemes to store, gather, and then release magnetization. Suitable free-electron reservoirs have been shown to work with spin currents in analogy with condensers for the usual charge currents. The polarization can be stored and then used on command to produce spin currents in a wire. The use of polarized reservoirs to generate pure spin currents is discussed here for the first time. These currents show oscillations while the storage units exchange their polarizations and the intensity and amplitude of the oscillations can be modified by changing the conductivity of the link between the polarized units.

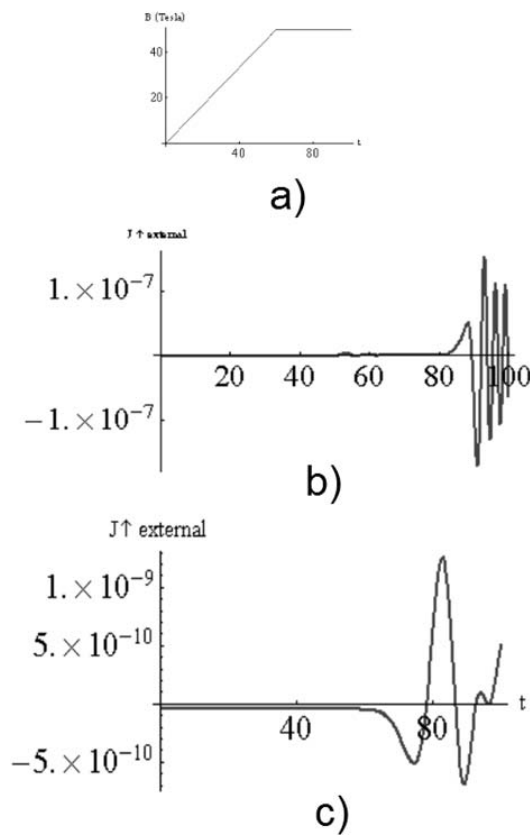


Fig. 7 – A thought experiment in which both storage cubes are replaced by storage rings. a) time dependent magnetic field. b) spin-up current in the 200 atom long external (storage ring-storage ring) connection, where the hopping matrix element is t_h ; the leads from the 30 atom central ring to the storage rings are 50 sites long. The down spin current is opposite. c) spin-up current in the 100 atoms long external (storage ring-storage ring) connection, where the hopping matrix element is $\frac{t_h}{3}$, the leads from the 30 atom central ring to the storage rings are 50 sites long. The down spin current is opposite.

As remarked above, the present model neglects electron-electron interactions, but it is physically reasonable that adding to the Hamiltonian a Hubbard-like correlation term proportional to $\hat{n}_\uparrow\hat{n}_\downarrow$ would tend to reinforce the charge confining effects described here; anyhow, a term in $\hat{n}_\uparrow\hat{n}_\downarrow$ is contained in $(\hat{n}_\uparrow + \hat{n}_\downarrow - 1)^2 = 1 - \hat{n}_\uparrow - \hat{n}_\downarrow + 2\hat{n}_\uparrow\hat{n}_\downarrow$; adding such a term to the Hamiltonian would produce no effect in the Hartree approximation, since its average at half filling vanishes strictly during the evolution of the system. This is a consequence of the theorem mentioned above.

This indicates that the above results are not too sensitive to interaction effects.

The above theoretical effort leads to several intriguing possibilities from the viewpoint of basic research and possible applications. The magnetic properties of laterally connected quantum rings are of great potential interest. I hope that this work will stimulate experiments, maybe using Graphene ribbons to realize the above arrangements.

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