COHERENCE-BASED METHOD TO DETECT TIME SHIFTS SMALLER THAN THE SAMPLING RATE OF TIME SERIES*

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Received August 5, 2014

The paper presents a method to evaluating the average time shift between two time series by exploiting both components of the complex-valued cross coherence function: the magnitude and the relative phase. What is new is there are detected temporal shifts smaller than the sampling interval of the series which could be useful to the timing of macro-economic indicators according to process evolution, not to the calendar day of publication. Depending on the key quantities grouped in the descriptor the method is able to detect time gaps as small as one tenth of the sampling rate. The method applies on non stationary series that are lacking long run correlations. In the paper is presented the theoretical justification of the technique, the calibration procedure, a validation test using aggregate replicas as well as how the technique could be applied onto the series of exchange rates, aggregate index BET of the Bucharest stock exchange market, and macro-economic indicators. When used to ordering the series inside groups of coherent clusters, the results should be cautiously concatenated to establishing the relative time succession among more than two series.

Key words: time series, cross spectrum, magnitude coherence function, magnitude coherence index, relative phase shift, time gap below the sampling rate, Hurst exponent, descriptor, coherent cluster, econophysics.

1. INTRODUCTION

Coherence is defined as the existence of correlation properties between optical fields at certain frequency [1]. The extension of the coherence measures to time series belonging to other domains than optics is in line with the explosive development of computational techniques and measuring capabilities. Detecting of time shifts among processes evidenced by synchronously sampled quantities in the form of time series is of ultimate importance to measuring propagation delays in optical self-focusing media [2], to discriminate among distinct species based on their specific dynamic [3], to disclose deterministic relationships in flocking clusters [4], or to predicting the evolution of economic and financial phenomena

* Paper presented at the 14th International Balkan Workshop on Applied Physics, July 2-4, 2014, Constanta, Romania.

[5]. Particularly in econo-physics there is large effort to insert the financial quantities into physics models in order to find the taking decision pattern and thus to get advantage over the competitors [6].

Unlike the general case, this paper proposes a method that can estimate time shifts smaller than the sampling rate. Prediction below the sampling threshold is useful for economic forecasting of macro-economic indicators before they become publicly available – the case of *bridge* and *factor* models for gross domestic product (GDP) estimates [7] – or in the case of speculative trades on the stock exchange market, where every trader tries to anticipate the competitors' sell/buy actions by looking for the logical fingerprint of their decision taking – the case of chaotic models with missing states in the phase space, subsequently filled in by using interpolation or filtering procedures [8]. Moreover, the method is suitable for the estimation of distinct time shifts in separate frequency bands, allowing for the disentangling of long-term analysis of business cycles from the short-term speculative transactions as prerequisites for clustering techniques [9].

Here the method is based on exploiting both the modulus and the phase of the complex cross coherence function. The measure of the time shift – if any – is given by the dependence on frequency of the phase shift provided that the existence of meaningful values of the coherence coefficients and the statistical significance are fulfilled [10].

The paper is presenting the theoretical justification of the technique, the calibration procedure, and a validation test analysis using aggregate replicas. Finally, the method is applied onto the series of exchange rates involving the Romanian currency (ROL) and the aggregate index BET of the Bucharest stock exchange market (BVB), as well as of macro-economic indicator GDP. The presence of relevant time shifts was seldom found among the exchange rates. The influence of the financial shock that started in August 2007 is mentioned when the case. The picture is richer in the case of quarterly GDP where the underlying dynamics of the series are endorsing the grouping in correlated clusters.

The organization of the work is as follows: section 2 explains the principles of the method; section 3 presents examples of how method works and its limitations; finally, section 4 summarizes the conclusions.

2. METHOD

2.1. COMPLEX COHERENCE FUNCTION

The method is using the analogy between electromagnetic waves and dynamic processes (economic, financial, other) put in the form of time series. By denoting x and y two time series and X and Y their Fourier images, the spectrum of their sum at frequency f is $S(f)$:

$$S_{x,y}(f) \propto S_x(f) + S_y(f) + < 2\,\mathrm{Re}\{X(f) \cdot Y^*(f)\} >_\tau, \quad (1)$$

where the window length τ depends on the spectral resolution of the detector, and the asterisk denotes the complex conjugate. Here the estimator "$< >_\tau$" is taken over the full length of the series using the sliding window technique [11].

In the case of computational approach the ratio of the estimated cross-spectrum to the product of the square roots of the auto-spectra measures the normalized complex-valued coherence $\gamma_{x,y}(f)$ [12]:

$$\gamma_{x,y}(f) = \frac{< X(f) \cdot Y^*(f) >_\tau}{\sqrt{< X(f) \cdot X^*(f) >_\tau} \cdot \sqrt{< Y(f) \cdot Y^*(f) >_\tau}} = \left|\gamma_{x,y}(f)\right| \cdot e^{\mathbf{i}\alpha_{x,y}(f)}, \quad (2)$$

where $\left|\gamma_{x,y}(f)\right| \leq 1$ is the magnitude coherence function (MCF) and $\alpha_{x,y}(f)$ is the relative phase function (RPF). More precisely, the relative phase is:

$$\alpha_{x,y}(f) = \mathrm{Atan}\frac{\mathrm{Im}\{\gamma_{x,y}(f)\}}{\mathrm{Re}\{\gamma_{x,y}(f)\}}. \quad (3)$$

According to the time shift theorem [13] one has $y(\Delta t) \leftrightarrow Y(f) \cdot e^{\mathbf{i}(2\pi f \cdot \Delta t)}$, where Δt is a constant time shift $t \rightarrow t + \Delta t$. Hence $x(0)$ is a non shifted series, and $x(\Delta t)$ is the shifted version. By denoting $\gamma_{x(0),y(\Delta t)}(f)$ the normalized complex-valued coherence of the pair consisting in the genuine and the artificially shifted series, Eq.(2) becomes

$$\gamma_{x(0),y(\Delta t)}(f) = \left|\gamma_{x(0),y(0)}(f)\right| \cdot e^{\mathbf{i}(\alpha_{x,y}(f) - 2\pi f \cdot \Delta t)}, \quad (4)$$

where the RPF is

$$\alpha_{x(0),y(\Delta t)}(f) = \alpha_{x,y}(f) - 2\pi f \cdot \Delta t \quad (5)$$

and includes the variation of RPF generated by the artificial delay

$$\Delta\alpha_{x,y}(f) = -2\pi f \cdot \Delta t. \quad (5')$$

Such an artificial time shift produces the same phase shift irrespective the frequency.

Conversely, the RPF of any pair of series could be checked for the existence of a linear term in coordinates $(f, \alpha_{x,y})$:

$$\alpha_{x,y}(f) \cong \hat{a} + \hat{b} \cdot f + \sum_{n>1}^{G} \hat{c}_n f^n, \quad f \in (f_{\min}, f_{\max}), \quad (5'')$$

where $\hat{a}$ and $\hat{b}$ are the linear, and $\hat{c}_n$ the non-linear estimates respectively, and G is the degree of the approximating polynomial. If such a linear term does exist, then its slope could be a measure of the time gap δt between the series in the couple:

$$\delta t \propto \hat{b}\,, \tag{6}$$

This conclusion should be cautiously drawn because the reciprocal of the time shift theorem is not generally valid. For this reason a practical calibration procedure is used in order to find the correct relationship between the time gap and the slope.

Hereafter we convene on the following notations: Δt_s – the sampling step, Δt – the artificial time shifts (always $\Delta t = q\cdot\Delta t_s$, q integer), δt – the time gap subjected to investigation, and $\delta\hat{t}$ - the computed time gap. The time gaps δt and $\delta\hat{t}$ could be greater or smaller than the sampling step as it will be demonstrated in Sec. 2.3. Obviously it is desirable the computed time gap to be as close as possible to the real time gap $\delta\hat{t} \to \delta t$.

Since the values of detectable cycles T_f (in physical units of time) are related to the real frequency f by the formula $f = (T_f)^{-1}$, and the number of time samples is $N=\tau/\Delta t_s$, the dimensionless numeric frequency k is related to the real frequency f by

$$k = \tau\cdot f\,,\qquad k=1,\ldots, N/2. \tag{7}$$

The main correspondences among the main quantities are given in table 1 for even number of samples N. The values of the window width τ is in arbitrary physical time units (a.u.).

Table 1

Time-frequency correspondences

Time cycle T_f (a.u.)	τ	$\tau/2$	$\tau/3$	...	$\tau/(N/2-1)$	$2\cdot\Delta t_s$
Real frequency f (a.u)$^{-1}$	$1/\tau$	$2/\tau$	$3/\tau$	…	$(N/2-1)/\tau$	$1/(2\cdot\Delta t_s)$
Numeric frequency k	$k_{\min}=1$	2	3	…	$(N/2)-1$	$k_{\max}=N/2$

2.2. THE PROPERTIES OF RELATIVE PHASE FUNCTION

The properties below hold for artificial time shifts multiple integer q of the sampling step $\Delta t=q\Delta t_s$. Provided that any artificial time shift Δt is computationally simulated by rotating the samples in series – advance by rotating to the left $x(\Delta t)$, and lag by rotating to the right $x(-\Delta t)$ – the RPF behaves anti-symmetrically. For different, not shifted series $x(0)$ and $y(0)$ one has

$$\alpha_{x(0),y(0)}(f) = -\alpha_{y(0),x(0)}(f)\,, \tag{8}$$

such as for identical series $x(0)\equiv y(0)$ RPF is zero irrespective the frequency

$$\alpha_{x(0),x(0)}(f) = 0\,. \tag{8′}$$

A particular case is obtained when replacing the second series with the shifted version of the first one $y(0) = x(\Delta t)$:

$$\alpha_{x(0),x(\Delta t)}(f) = \alpha_{x(-\Delta t),x(0)}\,, \tag{9}$$

meaning that the advance (lag) of the shifted version relative to the genuine one is equivalent with the lag (advance) of the genuine series with respect to the shifted version, with the same time interval.

When dealing with shifted versions of distinct series one needs to consider only the *variation* of RPF instead of the whole RPF such as:

$$\Delta\alpha_{x(0),y(\Delta t)}(f) = -\Delta\alpha_{x(\Delta t),y(0)}\,. \tag{10}$$

As in the case of Eq.(9) the advance (lag) of the second series relative to the first one is equivalent with the lag (advance) of the first relative to the second:

$$\Delta\alpha_{x(0),y(\Delta t)}(f) = \Delta\alpha_{x(-\Delta t),y(0)}\,. \tag{10′}$$

In the case of effective displacements among series, which are the scope of this work, the property of additivity does not work strictly. Be three distinct series *x, y*, and *z* and suppose there are significant time gaps between them, namely $\delta t_{x,y}$ between *x* and *y*, and $\delta t_{y,z}$ between *y* and *z*. The apparent additivity $\delta t_{x,z}=\delta t_{x,y}+\delta t_{y,z}$ does not hold because the time gaps are computed as estimates and therefore they are affected by errors in terms of standard errors. Moreover the time gap gets the proper sense only when comparing the series with itself when MCF is unity at all frequencies. Whenever *different* series are investigated one must first examine whether MCF has significant values. At a given frequency a significant MCF value indicates that the estimated terms of RPF are indeed concentrated around the mean and therefore they are meaningful.

To conclude, despite there is no strictly property of additivity among the computed time gaps, the method could provide useful information about the time shifts among more than two series.

2.3. THE MEANINGFULNESS AND THE STATISTICAL SIGNIFICANCE OF THE TIME GAP

The estimated time gap $\delta\hat{t}$ could be interpreted as effective displacement between cycles of same period in series *X* and *Y* if the value is the same irrespective the frequency over investigated band, *i.e.* the RPF $\alpha_{x,y}(f)$ should behave perfectly linear with *f*. Quantitatively that means $\hat{c}_n = 0$ for all higher degrees $n > 1$ in Eq.(5″). Since all estimates must be statistical significant, the previous conditions are quite complex and time consuming. For the sake of simplifying the computing, the conditions to assessing the linearity are replaced here with a practical one, the well known coefficient R^2. Perfect linearity is achieved when $R^2 = 1$. The lower threshold should depend on the particular scope of the study and on the characteristics of the time series as well (length, window width etc.).

Another constraint on the meaningfulness of the time gap is the existence of coherence between processes. Coherence at frequency f shows to what extent the cycles of $1/f$ length are correlated in the pair of series subjected to analysis. For example, when shuffling the amplitudes of a certain series by preserving their phases, one obtain a new series with zero time gap but totally incoherent with the genuine one. In this case MCF is approaching zero values at all frequencies. This case would be of no interest. Consequently MCF is an additional necessary measure, and, again, for the sake of simplifying the computing, here we define the magnitude coherence index (MCI). If the investigated band is delimited by the frequency f_0 (*e.g.* f_0 separates the long/short runs), MCI could be defined as:

$$\Gamma_S = \frac{1}{f_{max} - f_0} \cdot \sum_{f=f_0}^{f=f_{max}} |\gamma(f)|; \qquad \Gamma_L = \frac{1}{f_0 - f_{min}} \cdot \sum_{f=f_{min}}^{f=f_0} |\gamma(f)|. \tag{11}$$

The lower threshold for the statistical significance of MCF and MCI is given by noise considerations [12]. 2000 shuffles were operated onto the phases of the Fourier image of the series so that their spectra remained unchanged. The effect was the lowering of the coherence level in the whole band. The statistic results do not change when shuffling the first series only, the second series only, or both. Since any phase shuffling modifies the distribution of the time samples in the series, it followed the type of distribution in the series does not matter. The same parameters and computational rules were operated onto synthesized Gaussian series of random walk type with Hurst exponent of 0.5. The effect was the same irrespective the shuffled series or the synthesized Gaussian: noise is imprinting in the MCI a level of uncertainty normally distributed across the whole frequency band with zero mean and standard deviation of 0.1. Therefore the lower threshold for the meaningfulness of MCI is as small as 0.1:

$$\Gamma > 0.1. \tag{12}$$

Apart the meaningfulness, here the statistical significance of the estimates is considering the p-value of the associated t-statistics. To conclude, in order to find a meaningful time gap, one has to compute the estimate of the slope $\hat{b}$, MCI and the coefficient R^2. However, a calibration procedure is mandatory.

2.4. CALIBRATION

Eq.(5′) suggests the theoretical relationship between the estimated time gap and the slope

$$\delta\hat{t} = -\frac{1}{2\pi}\hat{b} \tag{13}$$

in (f,α) coordinates, or, alternatively

$$\delta\hat{t} = -\frac{N}{2\pi}\hat{b} \tag{13′}$$

in (k,α) coordinates. Since there is no reciprocal of the time shift theorem a practical calibration procedure is mandatory to verify the applicability of the theoretical relations (13–13′). A corrective factor is assumed to compensate the influence of long run correlations in the series:

$$\delta\hat{t} = -\frac{N}{2\pi}\cdot P(H)\cdot\hat{b}\,, \tag{13″}$$

where $P(H)$ is a non-linear corrective polynomial that depends on what extent the long run correlation are present as measured by the Hurst exponent H.

The calibration consists in finding the polynomial $P(H)$ and the domain it gives useful estimates. The calibration was performed onto pair of series with the same Hurst exponent synthesized *via* Mathematica software [14]. To accomplish the task the following sequence was performed: artificially shifting of the second series with one sampling step $\Delta t = \pm\Delta t_s$, then estimating the corresponding slope $\hat{b}\big|_{\Delta t=\pm\Delta t_s}$ of RPF, followed by the computing the estimated time gap according to Eq.(13′), and finally comparing it to the shifted value. The results are given in Fig.1.

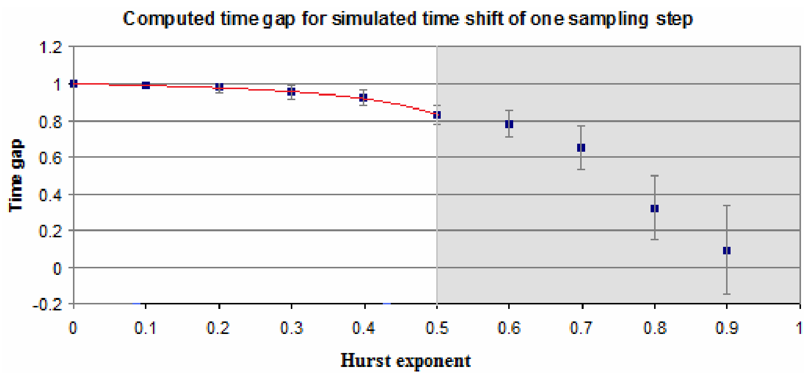

Fig. 1 – Computed time gap (in units of sampling steps) *vs*. Hurst exponent for simulated time shift of one sampling step. The light area marks the region 0≤H≤0.5 the method works. The red line corresponds to the corrective polynomial.

The general remark is the computed results diverge from the theoretical relation (13′) as H increases. It holds rigorously true for series characterized by H=0, *i.e.* series of white noise type. For the scope of this work the calibration is limited to the series that are lacking the long run correlations from one sample to another, that means $H \le 0.5$. Such series can be always obtained from the genuine ones by nonlinear filtering, for example by taking the returns in the case of the financial series [15]. The results are symmetrically for positive and negative time shifts (not shown).

The degree of the calibration polynomial $P(H)$ that fits the computed time gap to the artificial time shift was chosen to be as low as of the simplest interpolating polynomial

$$P(H) = 1 + \sum_{q=1}^{5} \varepsilon_q H^q \,, \quad 0 \le H \le 0.5, \tag{14}$$

where coefficients ε_q do not depend on the number of samples N (equivalently, on the window width τ). The best corrector (according to the ordinary less square method) is:

$$P(H) = 1 + 0.363 \cdot H - 5.792 \cdot H^2 + 33.125 \cdot H^3 - 70.833 \cdot H^4 + 54.167 \cdot H^5 \,. \tag{14′}$$

The typical RPF variation in the case of an artificial time shift of one sampling step is given in Fig.2. Any slope smaller (in absolute values) than the one corresponding to the time shift of one sampling step indicates a time gap *smaller* than the sampling step $0 \le |\delta t| \le \Delta t_s$ (the dark zone).

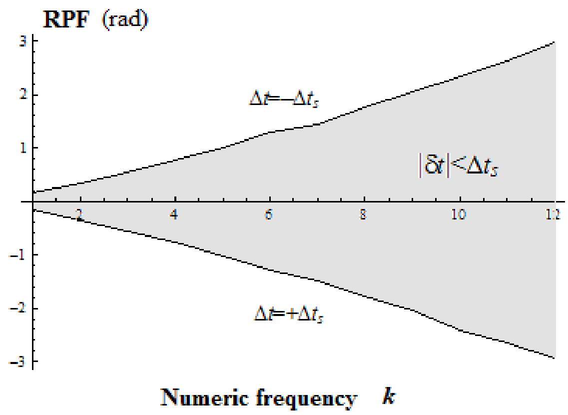

Fig. 2 – RPF *vs*. dimensionless frequency in the case of an artificial time shift of one sampling step (N = 24, H = 0.3).

According to Eq.(5′) the calibration domain can be extended for $\delta t > \Delta t_s$ to the limit of not exceeding the period 2π, *i.e.* $f_{max} \cdot \delta t_{max} = 2\pi$. Since $f_{max} = 1/(2 \cdot \Delta t_s)$ it follows the maximum domain allowed for measuring time gaps is $\delta t_{max} = 2\Delta t_s$. Consequently the validity domain the calibration works is

$$0 \leq |\delta t| \leq 2 \cdot \Delta t. \tag{15}$$

To conclude, for a given pair of series, the key quantities for computing the time gap are:

i/ Quantities depending on the scope of the study:

- The window width (or the averaging time) τ that establishes the frequency resolution;
- The investigated bands "band" given in the form of relevant frequencies (f_{min}, f_0) and/or (f_0, f_{max}), or in the form of cycle intervals.

ii/ Quantities that enable the use of the method:

- The Hurst exponents H_1, H_2 of each series in the couple; the method works if $H_{max} = \max\{ H_1, H_2 \} \leq 0.5$.

iii/ Quantities establishing the meaningfulness of the time gap:

- The corrective value $P(H_{max})$;
- The magnitude coherence index, $\Gamma > 0.1$.

iv/ Quantities for statistical significance of the slope $\hat{b}$:

- R^2 coefficient - the closer to one, the greater the significance to the disadvantage of lowering the number of candidates exhibiting meaningful time gaps;
- p-value, usually $p \leq 5\%$.

In condensed form the relevant quantities are grouped in the descriptor DES:

$$\text{DES: } [\tau/\Delta t_s, \text{Band (cycles)}, H_{max}, \Gamma, (p\text{-val}, R^2)] \tag{16}$$

2.5. VALIDITY TEST

The method was tested using aggregate replicas of synthesized series. The simulation used supra-sampled series (*e.g.* with a factor of five in Fig. 3, upper left) from which the corresponding number (accordingly five, see Fig. 3) of ordinary sampled replicas at rate Δt_s are aggregated by considering successively every first sample in the group of five (Fig. 3, upper right), the every second sample in the group (Fig. 3, lower left), and so on, the last being the every fifth sample in the group (Fig. 3, lower right).

The series obtained by aggregating the first value is operated successively with each of its time delayed replicas such that the method should be able to discriminate the time gaps as small as one fifth of the sampling interval. The descriptor is

DES: [$\tau/\Delta t_s$=12, Band = (cycles$\leq 4\Delta t_s$), $H_{\max}$ = 0.5, Γ > 0.1, (p-val $\leq$ 5%, $R^2 \geq 0.75$)].

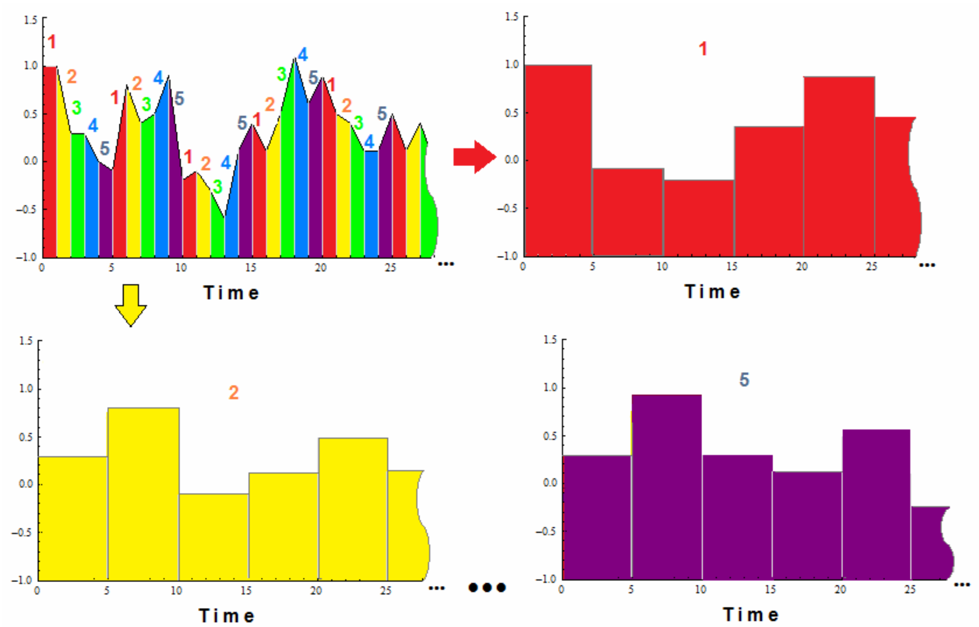

Fig. 3 – Supra-sampled genuine series ($H = 0.5$) and the aggregation procedure for time shifted replicas.

The time gaps were computed using Eqs.(13″) and (14′), and the results are given in Fig.4 including the standard errors; the value of MCI is indicated in the vicinity of each figurative point. The straight line corresponds to the brute linear formula (13′). All results are meaningful. One can easily remark that excepting the series shifted with $\Delta t_s/5$ whose gap is reached to the limit, the other time shifts are discriminated according to the expectations. However, the expectations themselves are altered because since the series are not identical there is no powerful argument to claim the time gaps between the reference and its j^{th} shifted replica is indeed $j\cdot\Delta t_s/5, j = 0,\ldots, 4$.

Despite of the genuine series was of random walk type it is not surprisingly to have high values of MCI (and MCF) in the case of the delayed aggregate replicas since the aggregation procedure is equivalent with a strong nonlinear filtering.

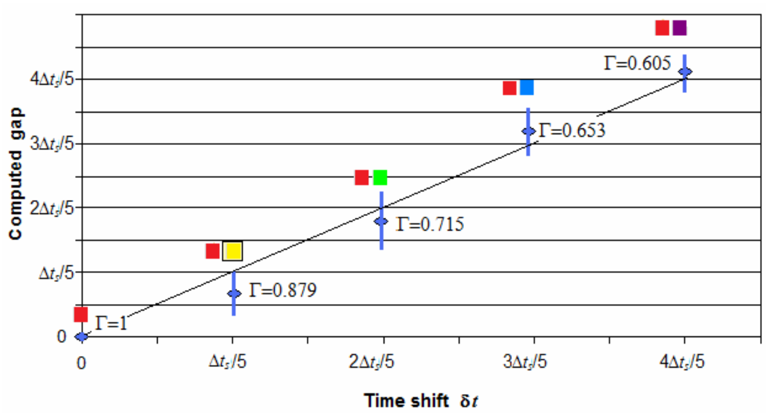

Fig. 4 – Computed gap with respect to the artificial time shift (Δt_s is one unit of sampling time, Γ is the value of MCI).

3. APPLICATIONS

Hereafter several applications on financial and economic time series are presented. One has to note that seldom the series are exhibiting significant time gaps across wide bands.

3.1. SIGNIFICANT TIME GAPS BETWEEN EXCHANGE RATE SERIES

The exchange rates values were acquired from the site OANDA Foreign trades database [16] and correspond to the interval 1 Jan. 1999 – 31 Dec. 2013, 5478 points/series, daily sampled. Ten exchange rates against Romanian Leu (denoted xxx/ROL) were investigated, where „xxx" is one of the codes according to ISO 4217 Currency Codes [17]: USD, EUR, CHF, JPY, PLN, HUF, SEK, NOK, KRW, TRY. The descriptor used in the analyses was:

$$\text{DES: } [\tau\text{=24days,Band=(cycles<4days)},H_{max}\leq 0.5,\Gamma>0.1,(p\text{-val}<5\%,R^2>0.75)] \quad (16')$$

The richest coherent cluster was found with respect to PLN/ROL. When taking PLN/ROL as reference the short run cycles (2-4 days period) of five exchange rates appeared to be in advance with fraction of day, as averaged values over 15 years (see Fig. 5a).

When considering separately the intervals before the hitting of the financial shock in August 2007 (Figs. 5b-5c) the coherence diminishes in aftershock, only CHF/ROL and HUF/ROL behaving stable *vs*. PLN/ROL in terms of preserving the relative advance. However CHF/ROL is the most advanced hence it could be used as a benchmark for PLN/ROL in speculative intra-day trades.

–0.6	–0.5	–0.4	–0.3	–0.2	–0.1	Time gap ←(days)→	0.1	0.2	0.3
					EUR/ROL				
				CHF/ROL		**xxx/ROL**			
					JPY/ROL	**vs.**			
					HUF/ROL	**PLN/ROL**			
					SEK/ROL	(reference)			

a). Whole interval, 1 Jan. 1999 – 31 Dec. 2013.

–0.6	–0.5	–0.4	–0.3	–0.2	–0.1	Time gap ←(days)→	0.1	0.2	0.3
				EUR/ROL					
				CHF/ROL		**xxx/ROL**			
					JPY/ROL	**vs.**			
					HUF/ROL	**PLN/ROL**			
					SEK/ROL	(reference)			

b). Interval before the hitting of the financial shock, 1 Jan. 1999 – 31 Jul. 2007.

–0.6	–0.5	–0.4	–0.3	–0.2	–0.1	Time gap ←(days)→	0.1	0.2	0.3
						xxx/ROL			
			CHF/ROL			**vs.**			
					HUF/ROL	**PLN/ROL**			
						(reference)			

c). Interval after the beginning of the financial shock, 1 Aug. 2007 – 31 Dec. 2013.

Fig.5 – Significant time gaps between short run cycles (2-4 days) of coherent exchange rates with respect to PLN/ROL.

3.2. TIME GAPS BETWEEN BET INDEX AND EXCHANGE RATES

A second investigation was undertaken considering the aggregate index BET of the Bucharest stock exchange market coupled successively with USD/ROL and EUR/ROL exchange rate. BET series was acquired from the BVB stock market database [18] and correspond to the same interval as to the exchange rates 1Jan.1999-31Dec.2013, 5478 points/series, daily sampled. The splitting because of the financial shock was also considered. The descriptor used in the analyses was the same as in the previous section:

$$\text{DES: } [\tau=24\text{days}, \text{Band}=(\text{cycles}<4\text{days}), H_{max}\leq 0.5, \Gamma>0.1, (p\text{-val}<5\%, R^2>0.75)] \quad (16'')$$

The results are given in Table 2. The symbol s stands for standard error. The shadowed zones indicate the sources of irrelevance. USD/ROL and EUR/ROL lag BET with approximately 0.8-0.9 days; this information could endorse the assumption the exchange rate – particularly the fluctuations of ROL currency – is largely determined by the condition of the stock market in the previous day.

It is also remarkable the financial shock enhance the coherence in the second period. This is similar to triggering signals in electronics that synchronize the processes initiated by the trigger.

Table 2

Time gaps between short run cycles (2-4 days) in pairs of series BET-USD/ROL and BET-EUR/ROL

	Time interval	Γ	Slope			Computed $\delta\hat{t}$ (days)
			$\hat{b}$	s	p-value	
BET-USD/ROL	1Jan.’99-31Jul.’07	0.113	–0.236	0.240	0.354	not relevant
	1Aug.’07-31Dec.’13	0.277	–0.259	0.043	0.031	0.9±0.2
	1Jan.’99-31Dec.’13	0.175	–0.241	0.040	0.035	0.9±0.2
BET-EUR/ROL	1Jan.’99-31Jul.’07	0.069	–0.223	0.175	0.239	not relevant
	1Aug.’07-31Dec.’13	0.231	–0.234	0.059	0.041	0.8±0.2
	1Jan.’99-31Dec.’13	0.109	–0.233	0.047	0.010	0.8±0.2

3.3. GDP CLUSTERS

Differing from the previous analyses, in this section are involved quarterly sampled economic indicators. 24 series of GDP variations were acquired from Eurostat database [19] and correspond to the interval 1Jan.2000-31Dec.2013, 56 points each, quarterly sampled. The codes of the countries correspond to ISO 3166 [20], plus Euro Zone 12 labeled as EZ12. Since the series are much shorter, the number of averaged samples in the sliding window of the estimator was shortened to N=12; however, in units of physical units of time, the averaging time is longer than in the previous cases: 3 years compared to 24 days.

The pairs were investigated in order to establish the time succession of the series inside groups of correlated clusters using the descriptor

$$\text{DES: } [\tau\text{=12Qs, Band=(cycles<4Qs)}, H_{\max}\leq 0.5, \Gamma>0.1, (p\text{-val}<5\%, R^2>0.75)]. \quad (16''')$$

Several clusters were found; the richest cluster was found when the Swiss’ GDP was taken as reference (see Fig. 6).

<table>
<tr><td>–10</td><td>Time gap
←(days)→</td><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td><td>100</td><td>110</td><td>120</td><td>130</td><td>140</td><td>150</td><td>160</td><td>170</td></tr>
<tr><td></td><td rowspan="6">CH
(reference)</td><td></td><td></td><td></td><td></td><td colspan="2">FI</td><td></td><td></td><td colspan="5">DE</td><td></td><td></td><td></td><td></td></tr>
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<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td colspan="2">GB</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td colspan="4">SE</td><td></td><td colspan="4">RO</td><td></td><td></td><td></td><td></td></tr>
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Fig. 6 – CH’s cluster: detected time gaps (days) of GDPs *vs*. reference Swiss’ GDP (significant results, cycles less than one year).

The picture reveals the timing of macro-economic indicators according to process evolution, not to the calendar day of computing or publication. The length of shadowed areas is the consequence of the standard errors. As in the case of the CHF/ROL exchange rate, the Swiss' GDP is leading in terms of the dynamics of underlying processes. Table 3 is exhibiting the details of the computations according to the chosen descriptor.

Table 3

Time gaps between short run cycles (less than one year) embedded in the series of Swiss' cluster

GDP series	Γ	Slope				Computed δt (days)
		$\hat{b}$	s	p-value	R^2	
CH - DE	0.645	−0.611	0.145	0.025	0.855	106±25
CH - EZ12	0.640	−0.604	0.125	0.017	0.887	105±22
CH - FI	0.634	−0.298	0.040	0.005	0.933	52±7
CH - BE	0.660	−0.291	0.067	0.022	0.865	51±12
CH - IT	0.526	−0.810	0.160	0.015	0.895	141±28
CH - GB	0.448	−0.587	0.057	0.002	0.972	102±10
CH - SE	0.448	−0.361	0.111	0.048	0.779	63±19
CH - RO	0.546	−0.634	0.094	0.007	0.938	110±16
CH - JP	0.538	−0.642	0.195	0.046	0.783	94±34

Other interesting remarks are the timing of the GDPs of Germany and EZ12 on one side, and of Finland and Belgium, on other side. In order to draw meaningful conclusion is necessary to further analyse the pairs DE-EZ12 and FI-BE. The results indicate indeed zero gaps to the drawback of not significant p-value and R^2 coefficient but with important differences of MCI values: extremely high Γ=0.957 in the case DE-EZ12, and lower coherence Γ=0.327 in the case of FI-BE. When taking Finland as reference Germany and EZ 12 are still in phase (Fig.6), but when taking Germany as reference, the Belgium's GDP is missing (is irrelevant according to DES (16'''), Fig.7). This is endorsing the conclusion Germany is really imprinting the timing of EZ12, but the relative dynamic of the series of Finland and Belgium has no relevance each other.

−120	−110	−100	−90	−80	−70	−60	−50	−40	−30	−20	−10	Time gap ← (days) →	10	20	30	40
HU						CH						**FI** (reference)			DE	
															EZ12	

Fig. 7 –Time gaps (days) of GDPs *vs*. reference FI's GDP
(significant results, cycles less than one year).

The method could provide information about the relative time succession among more than two series. As one can easily observe the relative succession between two specific entities may be preserved in different maps (*e.g*. FI-CH in

Figs. 7-8), but the time interval may differ (52 days in Fig. 7, and 70 days in Fig. 8) because of not being rigorously fulfilled the property of additivity among the time gaps (see Chp. 2.2). However, time relations could be extended among different clusters mainly if DES is imposing stricter conditions for significance (smaller p-value, greater R^2) and meaningfulness (larger N, larger band) to the disadvantage of detecting fewer gaps.

–130	–120	–110	–100	–90	–80	…	–40	–30	–20	–10	Time gap ← (days) →	10	20	…	80	90	100
CH							FI				**DE**				FR		
											(reference)		GB				

Fig. 8 –Time gaps (days) of GDPs *vs*. reference DE's GDP
(significant results, cycles less than one year).

4. CONCLUSIONS

A method to prediction below the sampling threshold was presented. The method works on non stationary series that lack long run correlations, *i.e*. with Hurst exponent ranging in the interval [0, 0.5].

The paper presents the theoretical justification of the technique, the calibration procedure, and a validation test. The theory relies on exploiting both the modulus and the phase of the complex cross coherence function. The measure of the time shift is given by the linear dependence on frequency of the phase shift provided that the statistical significance and the meaningfulness of the appropriate quantities are fulfilled. The calibration involves a corrective polynomial dependent on the Hurst exponent of the series in the interval [0, 0.5]. Testing the method with aggregate replicas of artificially generated series validates the capability to detecting time gaps smaller than the sampling rate.

Depending on the key quantities grouped in the descriptor the method allows to detect time gaps as small as one tenth of the sampling rate. In condensed form the relevant quantities grouped in the descriptor are DES: [$\tau/\Delta t_s$, Band (cycles), H_{max}, Γ, (p-val, R^2)]. The method could provide information about the time shifts among more than two series if DES is imposing stricter conditions for significance (smaller p-value, greater R^2) and meaningfulness (larger N, larger band) to the disadvantage of detecting fewer gaps. The series seldom admit meaningful time gaps.

The technique was applied onto the daily sampled series of exchange rates and aggregate index BET of the Bucharest stock exchange market, and quarterly sampled macro-economic indicators. Coherent clusters of series shifted before or behind the reference were found such as the method brings a powerful tool to reveal correlated dynamics among processes not only from economics but from any processes in the form of time series.

Acknowledgements. The author is grateful to Prof. Néda Zoltán for valuable suggestions.

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