

# ESTIMATION OF GLOBAL SOLAR IRRADIATION BY USING TAKAGI-SUGENO FUZZY SYSTEMS

R. BOATA<sup>1</sup>, N. POP<sup>2\*</sup>

<sup>1</sup>Faculty of Physics, West University of Timisoara, V. Parvan 4, 300223, Timisoara, Romania,  
E-mail: rboata@physics.uvt.ro

<sup>2</sup>Department of Physical Foundations of Engineering, Politehnica University of Timisoara,  
V. Parvan 2, 300223, Timisoara, Romania, E-mail: nicolina.pop@upt.ro;

\*Corresponding author: nicolina.pop@upt.ro; nicolinapop@gmail.com

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Artificial intelligence, as an alternative to the conventional statistical methods, has the ability to track complicated dependencies between different variables, where traditional methods have their limits. The fuzzy sets theory replaces the classical bivalent logic with a multi-valued logic. In this paper is developed a new model for estimation of daily global solar irradiation using Takagi-Sugeno fuzzy algorithms. Model construction procedure is presented in detail and a comparison with other models from literature is carried out. A critical assessment of the model performance is presented. Even if the model was built with data measured from the meteorological station of Timisoara, its application can be extended to other locations with similar meteorological regime. This conclusion is supported by the results obtained from testing the model in several locations.

*Key words:* Fuzzy systems, global solar irradiation, model accuracy.

## 1. INTRODUCTION

The quantity of electricity generated by the photovoltaic systems is expected to grow in the next future. Because the solar resource has a fluctuating character and it depends on the weather conditions, an accurate forecast of the solar energy is necessary for a balanced operation of the power grid [1].

In the world the number of meteorological stations equipped for measuring solar radiation is very small [2]. In this situation the numerical methods based on measured meteorological data can be used with success for estimation of solar radiation.

An important weather parameter is the air temperature measured by all meteorological stations but still it is not a frequent parameter for solar irradiation models. However, there are some empirical models based on air temperature at input [3].

At present most of the models for solar radiation are based on traditional statistics but in recent years the models build on fuzzy algorithms [4] were developed. These models basically replace the Boolean logic with a multi-valued logic and an improvement of prediction accuracy it is expected by using them. A model for computing the solar irradiance on arbitrarily oriented surfaces based on fuzzy logic procedures is developed in [5]. An algorithm for estimating the solar irradiation from sunshine duration measurements based on fuzzy technique is reported in [6]. In [7] was developed a fuzzy logic model for estimate the global solar irradiation based on daily amplitude of air temperature.

In the following rows is described in a few words a fuzzy systems. A fuzzy system [4] is a map between input (antecedent part) and output (consequent part) represented by if - then rules. Depending on the structure of the consequent part there are two important classes of fuzzy systems. The fuzzy logic models are the first class at which the antecedent and the consequent parts are linguistic expressions. Takagi-Sugeno fuzzy models [8] are the second class and here the consequent part is a mathematical function, while the antecedent part is a linguistic expression.

In this work we apply a Takagi-Sugeno fuzzy algorithm to estimate daily global solar radiation in the five locations from Europe: Budapest, Galati, Innsbruck, Sofia and Timisoara.

## 2. TAKAGI-SUGENO (TS) FUZZY SYSTEMS

TS fuzzy systems form a special class of the fuzzy systems since the conclusion of each rule is crisp. Let,  $M = \{x_i, y_j\}$ ,  $i = 1, 2, \dots, N_i$ ,  $j = 1, \dots, N_j$  to be the set of the available experimental data. For a model with 3 inputs and a single output,  $N_i = 3$  and  $N_j = 1$ , the TS model consists of a collection of  $r$  rules, expressed as:

$$\#R_k : \text{if } x_1 \text{ is } A_{k1} \text{ and } x_2 \text{ is } B_{k2} \text{ and } x_3 \text{ is } C_{k3} \text{ then } y_k = f_k(x_1, x_2, x_3) \quad (1)$$

where  $A_{k1}$ ,  $B_{k2}$  and  $C_{k3}$  are fuzzy sets corresponding to the partitioned domains of the input variables  $x_1$ ,  $x_2$  and  $x_3$ , respectively. Note that an attribute is associated to each fuzzy set.  $y_k(x_1, x_2, x_3)$  stand for the output of the rule  $\#R_k$ . In the standard TS model the output  $y_k$  is calculated as a linear combination of the input variables:

$$y_k = f_k(x_1, x_2) = a_k x_1 + b_k x_2 + c_k x_3 + d_k \quad (2)$$

but can be considered any other function  $f_k(x_1, x_2)$ . The weight  $m_k$  of a rule is computed in the inference step, by intersecting the individual premises [4]:

$$m_{A_{k1}} \wedge m_{B_{k2}} \wedge m_{C_{k3}} = \min(m_{A_{k1}}, m_{B_{k2}}, m_{C_{k3}}) \quad (3)$$

where  $m_{A_{k1}}$  and  $m_{B_{k2}}$  are the membership functions related to the fuzzy sets  $A_{k1}$  and  $B_{k2}$ , respectively. If several rules drive to the same conclusion than the individual confidence levels of the rules are combined by applying the fuzzy operator OR:

$$m_k = m_{A_{k1}} \vee m_{B_{k2}} \vee m_{C_{k3}} = \max(m_{A_{k1}}, m_{B_{k2}}, m_{C_{k3}}) \quad (4)$$

Defuzzification is a decoding operation of the information encapsulated into the results of the fuzzification and inference processes. In the TS model the defuzzification procedure is simply performed by taking the weighted average  $y_k$  outputs of the active rules; the output crisp value is extracted with the relation:

$$y = \frac{\sum_{k=1}^n m_k y_k}{\sum_{k=1}^n m_k} \quad (5)$$

where  $n$  is the total number of the active rules.

### 3. DATA BASE AND MODEL DESCRIPTION

Data measured in Timisoara (45.76° N; 21.15° E; 85 m) during three years, 1997–1999, have been used to build the fuzzy model. To test the model we used data measured in year 2000. The locations listed in Table 5 are used to test the model. The source of data is WRDC [9] for daily solar irradiation and NCDC [10] for daily maximum and minimum air temperature.

Daily amplitude of air temperature,  $\Delta t_j = t_{max,j} - t_{min,j}$ , has been assumed as the first input variable because it is as a measure of the state of the sky [11]. Julian day,  $j$ , is the second input variable and its role is to enhance the estimation quality in the cold season [7]. The third input variable is  $\Delta tm5_j = tm_j - t5_j$ , where  $tm_j = (t_{max,j} + t_{min,j})/2$  and  $t5_j = (tm_{j-2} + tm_{j-1} + tm_j + tm_{j+1} + tm_{j+2})/5$ ; this variable is another measure for the state of the sky, but it takes into account the state of the sky in the two previous days and the next two days. Introduction of the third variable,  $\Delta tm5_j$ , represents a novelty of this model. In order to introduce the algorithm dependence to the geographical coordinates and day length, clearness index,  $k_t = H/H_{ext}$ , has been considered as the output linguistic variable. Here  $H$  represents the daily global solar irradiation while  $H_{ext}$  represents the extraterrestrial solar irradiation.

The membership functions of the input variables attributes,  $\Delta t$  and  $\Delta tm5$ , have been calculated with a fuzzy clustering procedure [4]. Thus, these membership functions are not artificially constructed and the outcomes are given by the  $c$ -mean clustering algorithm.

We denoted L1 (LOW), M1 (MEDIUM), H1 (HIGH) the attributes of variable  $\Delta t$ . The membership functions of  $\Delta t$  attributes are given by the equations (see Fig. 1a where the notations are indicated):

$$m_{\Delta t,i}(\Delta t) = \frac{a_{1,i} + a_{3,i} \cdot \Delta t + a_{5,i} \cdot \Delta t^2 + a_{7,i} \cdot \Delta t^3 + a_{9,i} \cdot \Delta t^4}{1 + a_{2,i} \cdot \Delta t + a_{4,i} \cdot \Delta t^2 + a_{6,i} \cdot \Delta t^3 + a_{8,i} \cdot \Delta t^4}, \quad i = 1 \dots 3 \quad (6)$$

The coefficients  $a_{j,i}, j = 1 \dots 9, i = 1 \dots 3$ , are listed in Table 1.

Table 1

The coefficients of the membership functions (Eq. 6)

| $a_{j,i}$ |   | $i$       |           |           |
|-----------|---|-----------|-----------|-----------|
|           |   | 1         | 2         | 3         |
| $j$       | 1 | 0.670244  | 0.214072  | 0.115662  |
|           | 2 | -0.364973 | -0.364940 | -0.364939 |
|           | 3 | -0.210173 | -0.096900 | -0.057890 |
|           | 4 | 0.050623  | 0.050612  | 0.050612  |
|           | 5 | 0.024522  | 0.015513  | 0.010585  |
|           | 6 | -0.003127 | -0.003125 | -0.003126 |
|           | 7 | -0.001261 | -0.001029 | -0.000836 |
|           | 8 | 0.000072  | 0.000072  | 0.000072  |
|           | 9 | 0.000024  | 0.000024  | 0.000024  |

The Julian day is assigned as second linguistic variable with two attributes: the winter (W) and the summer (S). Fig. 1b shows the trapezoidal membership functions of the Julian day attributes:

$$m_{j,s} = \begin{cases} \max\left(0, 1 - \frac{j-c_1}{b_1-c_1}, \frac{j-c_3}{b_3-c_3}\right) & \text{if } c_1 < j < b \\ 1 & \text{otherwise} \end{cases} \quad (7a)$$

$$m_{j,w} = \begin{cases} \max\left(0, \frac{j-a_2}{c_2-a_2}\right) & \text{if } j < c_2 \\ \max\left(0, 1 - \frac{j-c_3}{b_3-c_3}\right) & \text{if } j > c_3 \\ 1 & \text{otherwise} \end{cases} \quad (7b)$$

The numerical values of the coefficients from Eqs. (7a–7b) are:  $c_1 = a_2 = 45$ ,  $b_1 = c_2 = 120$ ,  $c_3 = 240$  and  $b_3 = 320$ .

The third input variable  $\Delta tm5_j$  has three attributes: L2, M2 and H2. The membership functions of  $\Delta tm5$  attributes are given by the equations (see Fig. 1c where the notations are indicated):

$$m_{\Delta tm5,i}(\Delta tm5) = \frac{b_{1,i} + b_{3,i} \cdot \Delta tm5 + b_{5,i} \cdot \Delta tm5^2 + b_{7,i} \cdot \Delta tm5^3 + b_{9,i} \cdot \Delta tm5^4}{1 + b_{2,i} \cdot \Delta tm5 + b_{4,i} \cdot \Delta tm5^2 + b_{6,i} \cdot \Delta tm5^3 + b_{8,i} \cdot \Delta tm5^4 + b_{10,i} \cdot \Delta tm5^5} \quad (8)$$

with  $i = 1 \dots 3$ .

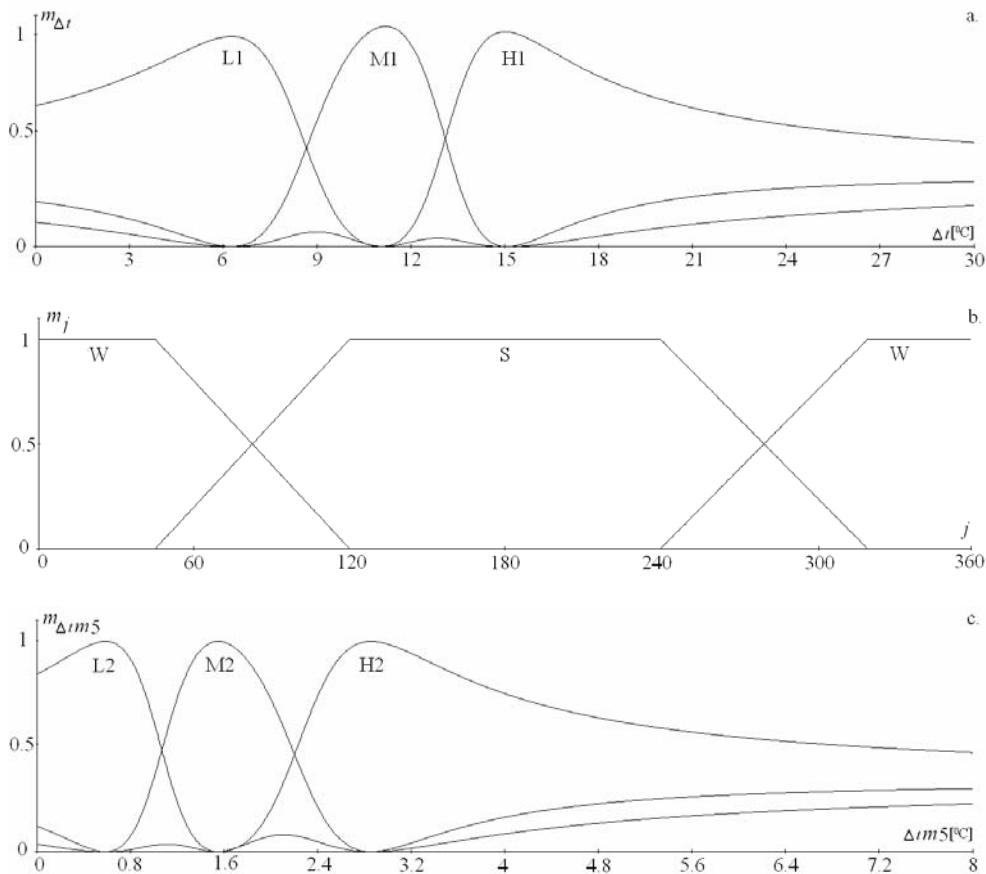


Fig. 1 – Membership functions associated to the input variables: (a)  $\Delta t$ , (b)  $j$ , (c)  $\Delta tm5$ .

The coefficients  $b_{j,i}$ ,  $j = 1 \dots 10$ ,  $i = 1 \dots 3$ , are listed in Table 2.

Table 2

The coefficients of the membership functions (Eq. 8)

| $b_{j,i}$ |    | $i$          |              |              |
|-----------|----|--------------|--------------|--------------|
|           |    | 1            | 2            | 3            |
| $j$       | 1  | 0.843779360  | 0.120729607  | 0.035494963  |
|           | 2  | -2.345287292 | -2.345288419 | -2.345263100 |
|           | 3  | -1.681274941 | -0.496942035 | -0.167100160 |
|           | 4  | 2.150091125  | 2.150110300  | 2.150087869  |
|           | 5  | 1.219363409  | 0.655813320  | 0.274986401  |
|           | 6  | -0.861991115 | -0.862022727 | -0.862011711 |
|           | 7  | -0.380434159 | -0.297268577 | -0.184357832 |
|           | 8  | 0.129582815  | 0.129598880  | 0.129594206  |
|           | 9  | 0.043202633  | 0.043201566  | 0.043205283  |
|           | 10 | 0.000003153  | 0.000000723  | 0.000001800  |

The output functions  $y_i$  has been choused linearly in  $\Delta t, j$  and  $\Delta tm5$ :

$$y_i(\Delta t, j, \Delta tm5) = a_i \Delta t + b_j + c_i \Delta tm5 + d_i, i = 1 \dots 18. \quad (9)$$

The coefficients of the  $y_i$  output functions have been identified using the multivariate least square regression and they are listed in Table 3.

Table 3

The coefficients of the membership functions (Eq. 9)

|       | $i$       |           |           |           |           |           |
|-------|-----------|-----------|-----------|-----------|-----------|-----------|
|       | 1         | 2         | 3         | 4         | 5         | 6         |
| $a_i$ | 0.03389   | 0.03027   | 0.0251    | 0.01075   | 0.01349   | -0.05738  |
| $b_i$ | -0.000004 | -0.000283 | -0.000108 | 0.000087  | -0.000214 | -0.00138  |
| $c_i$ | 0.000784  | 0.009094  | -0.01222  | -0.000355 | -0.08166  | 0.3868    |
| $d_i$ | 0.07671   | 0.1009    | 0.1252    | 0.2961    | 0.434     | 0.1388    |
|       | $i$       |           |           |           |           |           |
|       | 7         | 8         | 9         | 10        | 11        | 12        |
| $a_i$ | 0.01063   | -0.008479 | 0         | 0.02116   | -0.1685   | 0.0326    |
| $b_i$ | -0.000186 | -0.000254 | 0         | 0.000898  | 0.004188  | -0.000068 |
| $c_i$ | -0.01702  | -0.02111  | -0.125    | 0.02016   | -0.4286   | 0.008256  |
| $d_i$ | 0.3739    | 0.613     | 1         | -0.0227   | 0.75      | 0.06172   |
|       | $i$       |           |           |           |           |           |
|       | 13        | 14        | 15        | 16        | 17        | 18        |
| $a_i$ | 0.04329   | 0.03436   | 0.0342    | 0.0154    | 0.02589   | -0.002078 |
| $b_i$ | 0.000723  | 0.000236  | 0.00006   | 0.000056  | 0.000411  | 0.000126  |
| $c_i$ | -0.006129 | -0.03123  | 0.03983   | -0.004617 | -0.06747  | 0.01907   |
| $d_i$ | -0.1496   | 0.1577    | -0.03316  | 0.335     | 0.2256    | 0.5077    |

In Table 4 is presented the mapping of the inputs to the output of our fuzzy system, materialized in the rules-base. It has been elaborated in a heuristic way from measured data at the station of Timisoara during the years 1997–1999. Due to the form of the membership functions, it is evident that for any  $\Delta t, j$  and  $\Delta tm5$  all the rules are active.

Once the rule-base is set, the fuzzy algorithm becomes operational and for every input  $\Delta t, j$  and  $\Delta tm5$  the output crisp value  $k_i$  can be computed. The result of the inference process is translated into a crisp output using Eq. (5).

TS fuzzy model developed in this article and its testing has been implemented in a MathCAD14 application [12].

Table 4

Matrix of system rule-based. Each rule is an implication in the sense of Eq. (1)

| The rule base<br>$j$ has S attribute |    | $\Delta t$ |          |          |
|--------------------------------------|----|------------|----------|----------|
|                                      |    | L1         | M1       | H1       |
| $\Delta tm5$                         | L2 | $y_1$      | $y_4$    | $y_7$    |
|                                      | M2 | $y_2$      | $y_5$    | $y_8$    |
|                                      | H2 | $y_3$      | $y_6$    | $y_9$    |
| The rule base<br>$j$ has W attribute |    | $\Delta t$ |          |          |
|                                      |    | L1         | M1       | H1       |
| $\Delta tm5$                         | L2 | $y_{10}$   | $y_{13}$ | $y_{16}$ |
|                                      | M2 | $y_{11}$   | $y_{14}$ | $y_{17}$ |
|                                      | H2 | $y_{12}$   | $y_{15}$ | $y_{18}$ |

#### 4. TS MODEL PERFORMANCE ASSESSMENT

To assess the model performance we use a data series recorded in the year 2000 at five locations: Budapest, Galati, Innsbruck, Sofia and Timisoara.

For each station used in this study the local climate type according to Köppen climate classification are listed in Table 5. This classification is based on the digital Köppen-Geiger world map [13] on climate classification. In accord with was said above the stations sites climate particularities are described in follows rows. Stations Timisoara and Budapesta have a warm temperate climate, fully humid (Köppen climate classification Cfb) with warm summer, typical for the Pannonia Basin. The locations Innsbruck and Sofia (Köppen climate classification Dfb) are characterized by snow climate, fully humid with warm summer. Station Galati has a warm temperate climate, fully humid with hot summer (Köppen climate classification Cfa).

Figure 2 shows the measured and calculated daily global solar irradiation. Data recorded in 2000 at the station Timisoara and Sofia are used. A good agreement is observed.

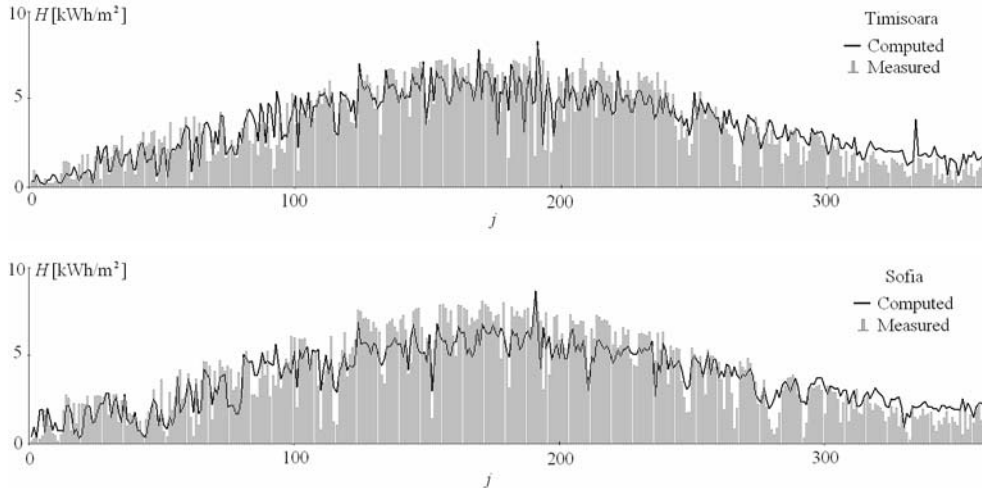


Fig. 2 – Comparison between calculated and measured daily global solar irradiation. Data recorded in 2000 at the station Timisoara and Sofia are used.

In the next step, the results were compared with measured data using two statistical indicators, the relative root mean square deviation and the relative mean bias error:

$$rrmse = \left[ n \cdot \sum_{i=1}^n (F_i - y_i)^2 \right]^{1/2} / \sum_{i=1}^n y_i \quad (10)$$

$$rmbe = \frac{\sum_{i=1}^N (F_i - y_i)}{\sum_{i=1}^N y_i} \quad (11)$$

where  $y_i$  and  $F_i$  are  $i$ -th measured and computed values of radiation, respectively, while  $n$  is the number of measurements taken into account.

Statistical indicators of the model accuracy calculated with Eqs. (10–11) are listed in Table 5. The results show that the accuracy for estimation solar irradiation is reasonable and compares well with the accuracy obtained using classical correlations and other fuzzy models. In [14] have been found for monthly mean daily global solar irradiation  $rmse$  ranging between 0.037... 0.260 and  $mbe$  ranging between  $-0.22$ ... 0.091, after the verification of five traditional solar models with cloudiness and sunshine duration. In Ref. [7]  $rmse$  and  $mbe$  for monthly mean daily



global solar irradiation ranging between 0.052... 0.305 and  $-0.253... 0.167$ , respectively, after the verification of a fuzzy model against data from 11 European stations.

Table 5

The statistical indicators *rrmse* and *rmbe* of the monthly mean of daily solar irradiation estimation

| Station        | Lat. (deg) | Long. (deg) | Alt. (m) | Climate | Test year | <i>rrmse</i> | <i>rmbe</i> |
|----------------|------------|-------------|----------|---------|-----------|--------------|-------------|
| Budapesta (H)  | 47°26'N    | 19°11'E     | 138      | Cfb     | 2000      | 0.245        | -0.15       |
| Galati (RO)    | 45°30'N    | 28°02'E     | 72       | Cfa     | 2000      | 0.256        | -0.196      |
| Innsbruck (A)  | 47°15'N    | 11°21'E     | 579      | Dfb     | 2000      | 0.148        | -0.087      |
| Sofia (BG)     | 42°39'N    | 23°23'E     | 586      | Dfb     | 2000      | 0.158        | -0.043      |
| Timisoara (RO) | 45°47'N    | 21°17'E     | 85       | Cfb     | 2000      | 0.153        | 0.015       |

## 5. CONCLUSIONS

A Takagi-Sugeno fuzzy procedure to estimate daily global solar irradiation using air temperature data is developed in this paper. The fuzzy model accuracy has been evaluated against data from five European stations. Following the results obtained from testing the model we can conclude that it performs with reasonable accuracy for most practical purposes, especially when it requires a balance between simplicity and accuracy. The model developed in this paper has the advantage that it uses only daily air temperature extremes at input, parameter widely available for most locations. Another advantage is that it in other regions with similar meteorological regime the TS model presented here can be successfully applied.

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