

## SKYRME INTERACTION WITH VMC CALCULATIONS OF NUCLEAR MATTER

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The parameters of effective Skyrme interaction are determined by using *Variational Monte Carlo* (VMC) calculations that accurately reproduce the properties of symmetric and asymmetric nuclear matter. The realistic Urbana  $V_{14}$  nucleon–nucleon interaction potential of Lagaris and Pandharipande was used in the VMC calculations with addition of a phenomenological density-dependent term to simulate many-body interactions. A new Skyrme parameter set SKaan5 is found to consistently reproduce the characteristics of the nuclear matter obtained from VMC calculations. The properties of symmetric and asymmetric nuclear matter are calculated by the new Skyrme parameter set. It is shown that the new Skyrme parameter set SKaan5 satisfy the properties of symmetric nuclear matter, such the binding energy, saturation density and incompressibility. Also, the results obtained by using the new Skyrme parameter set are in good agreement with those obtained by other different Skyrme parameter sets in the literature.

*Key words:* Nuclear matter, asymmetric matter, equation of state, Skyrme interaction.

### 1. INTRODUCTION

It is important to obtain empirically known bulk properties, such as the binding energy, equilibrium density, incompressibility, etc. of nuclear matter starting from the underlying two-body interactions in nuclear physics. There are two major difficulties for describe all nuclear interactions from the basic interactions between nucleons which are called the microscopic approach of nuclear physics: (i) calculations in many-body systems are difficult and require advanced computation techniques; (ii) there is evidence that three-nucleon forces (and possibly higher multiparticle interactions) play a significant role. This means that three-nucleon potentials must be included into the model.

The properties of asymmetric nuclear matter (ASNM) have studied using various *Nucleon–Nucleon* (NN) potentials which are called “realistic” and

“phenomenological” [1]. The realistic ones are model-based but are constructed in order to fit experimental data for free NN scattering and properties of the deuteron [2]. The parameterization of the nuclear potential by an appropriate mathematical function is called phenomenological approach.

The *equation of state* (EOS) of hot nuclear matter has been extensively studied within various theoretical models such as the phenomenological nonrelativistic Skyrme force models [3-7] and the relativistic mean-field theory (RMT) [8]. However, the equation of cold nuclear matter has been explored by many physicists within the variational method [9-12], the nonrelativistic *Brueckner–Hartree–Fock* (BHF) approach [13, 14] and the Dirac–Brueckner theory [15, 16] based on realistic NN interactions. The selfconsistent or *Hartree–Fock* (HF) approach aims to deduce mathematically the nuclear potential from the NN interaction. In the case of the HF approaches, the trouble is not to find the mathematical function which describes best the nuclear potential, but that which describes best the NN interaction [1].

The density-dependent Skyrme-type effective NN interaction model has been one of the most popular microscopic tools to describe the ground-state properties of the finite nuclei as well as that of the *Symmetric Nuclear Matter* (SNM) and *Pure Neutron Matter* (PNM). The pioneering implementation of density dependent Skyrme type effective NN interaction in HF calculations is due to Vautherin and Brink [17]. Many different parameter sets for the Skyrme interaction have been proposed. In order to determine the parameter sets of all Skyrme interaction, known experimental quantities such as the nuclear binding energy and the saturation density can be used.

The aim of this study is to determine a new Skyrme parameter set which describe the properties of nuclear matter within a HF approach and consistent with the results obtained from the underlying realistic two-nucleon interaction potential using quantum Monte Carlo techniques [10,18,19]. By reason of using only the two-body interactions gives very great saturation densities for nuclear matter, in this study we use the realistic Urbana  $V_{14}$  potential for the NN interaction and we include the three-body interactions through a density dependent term.

A new set of Skyrme parameters is determined by fitting the energy results obtained from Variational Monte Carlo calculations to the Skyrme energy density functional. Also, we investigate the properties of SNM and ASNM by using the new Skyrme parameter set.

## 2. VMC CALCULATIONS OF NUCLEAR MATTER

### 2.1. INTERACTION POTENTIAL

The Hamiltonian operator of a free system of  $N$  particles can be written as a two-body interaction potential  $V_{ij}$

$$\mathbf{H} = -\frac{\hbar^2}{2m} \sum \nabla_i^2 + \sum_{i<j} V_{ij}. \quad (1)$$

The correct evaluation of the saturation point of nuclear matter is a requirement for any realistic two-body potential. Therefore, it is essential to have a reliable method to calculate the binding energy of the nuclear matter for a given two-body potential.

In this paper we use Urbana  $V_{14}$  potential, which was proposed by Lagaris and Pandharipande. The parameters of the potential were obtained by fitting the phase-shift data from low energy NN scattering experiments and the properties of the deuteron [20]. The phase-shift data varies greatly from channel to channel and it is necessary to have operator components, and Urbana  $V_{14}$  potential contains 14 operator components.

$$\begin{aligned} V_{ij} = & V^c + V^\sigma (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) + V^\tau (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) + V^{\sigma\tau} (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) + \\ & V^t S_{ij} + V^{t\tau} S_{ij} (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) + V^b (\mathbf{L} \cdot \mathbf{S})_{ij} + V^{b\tau} (\mathbf{L} \cdot \mathbf{S})_{ij} (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) + \\ & V^q L^2 + V^{q\sigma} L^2 (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) + V^{q\tau} L^2 (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) + \\ & V^{q\sigma\tau} L^2 (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) + V^{bb} (\mathbf{L} \cdot \mathbf{S})^2 + V^{bb\tau} (\mathbf{L} \cdot \mathbf{S})^2 (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j). \end{aligned} \quad (2)$$

Due to the translational invariance of the infinite nuclear matter the terms depending on the relative angular momentum operator  $\mathbf{L}$ , do not considerably effect the binding energy. Furthermore, as the contributions of latter terms are much smaller than those of the first four, their effect is smaller than the statistical fluctuations inherent to the Monte Carlo technique so the inclusion of these terms was pointless. Therefore only first four terms of the Urbana potential retained in the VMC calculations. Thus, we have

$$V_{ij} = V^c + V^\sigma (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) + V^\tau (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) + V^{\sigma\tau} (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j), \quad (3)$$

for the two-body interaction. Where  $V^c, V^\sigma, V^\tau$ , and  $V^{\sigma\tau}$  depend only on the distance between the nucleons  $i$  and  $j$ . In the Urbana potential each term in Eq. (2) has three parts

$$V^i = V_\pi^i + V_I^i + V_S^i, \quad (4)$$

representing long-range ( $V_\pi^i$ ), intermediate-range ( $V_I^i$ ), and short-range ( $V_S^i$ ) interactions. The long range part of the interaction ( $V_\pi^i$ ) is nonzero only for  $i = \sigma\tau$  and is given by

$$V_\pi^{\sigma\tau} = 3.488 \frac{e^{-\mu r}}{\mu r} (1 - e^{-cr^2}), \quad (5)$$

where  $\mu = 0.7 \text{ fm}^{-1}$  is the inverse Compton wavelength for pions. The intermediate and short range parts are

$$V_I^i(r) = I^i \left[ \left( 1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2} \right) \frac{e^{-\mu r}}{\mu r} \left( 1 - e^{-cr^2} \right) \right]^2 \quad (6)$$

and

$$V_S^i(r) = \frac{S^i}{1 + e^{(r-R)/a}} \quad (7)$$

respectively. Values of the potential strengths  $I^i$  and  $S^i$  and the parameters  $c$ ,  $R$ ,  $a$  were given by Lagaris and Pandharipande [20].

It is well known that all two-nucleon interaction models estimate too large equilibrium densities for nuclear matter. Therefore, the three and more body interactions should be incorporated into any consistent nuclear matter calculation. In this study, we use the phenomenological approach assuming the density dependent term to be proportional to short ranged part of the Urbana potential and we assume that the total interaction, including the many body effects, is of the form

$$v_{14} + TNI = v_\pi + v_I + v_s + v_s(\alpha\rho)^\gamma, \quad (8)$$

where  $\rho$  is the number density of nucleons.  $\alpha$  and  $\gamma$  in the above equation are free parameters and adjusted so as to obtain the correct binding energy and saturation density of SNM.

## 2.2. THE VARIATIONAL MONTE CARLO METHOD

We use a Monte Carlo method which is same as our previous study [1]: In the VMC calculations to obtain the properties of bulk nuclear matter we consider a cubic box of side  $L$  containing  $N$  nucleons with periodic boundary conditions. The trial wave function used in the present study is a Jastrow type wave function in the form

$$\Psi_j(\mathbf{R}) = \prod_{i \langle j} f_j(r_{ij}) \Phi, \quad (9)$$

where  $\Phi$  is the many particle wave function for the system of non-interacting particles and  $\mathbf{R}$  is a  $3N$  dimensional vector representing the coordinates of particles, while  $f_j$  is the two particle correlation function. Jastrow suggests that this correlation function in general be an operator function [21]. However in most applications  $f_j$  is assumed to depend only on the interparticle distance,

$$r_{ij} = |r_i - r_j|.$$

One can use plane waves  $\phi(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}$  for the single particle wave functions of the nucleons in bulk matter. We consider nucleons to be restricted to a cubic box of side  $L$ , so that the wave number  $\mathbf{k} = 2\pi\mathbf{n}/L$  and  $\mathbf{n}$  is an integer vector. In order to conserve the rotational invariance of bulk nuclear matter we perform VMC calculations only for the numbers of neutrons (N) and protons (P) corresponding to completely filled energy shells. We assume that the space and spin parts of the wave function is separable. Under these conditions choosing a many particle trial wave function with

$$\Phi(\mathbf{R}) = D^{P\uparrow} D^{P\downarrow} D^{N\uparrow} D^{N\downarrow} \quad (10)$$

is quite reasonable because the spin-isospin dependent parts of the interaction potential is relatively weak. Also it is well known that the expectation value of the total energy is not very sensitive to small changes in the wave function. The determinants  $D^{P\uparrow}$ ,  $D^{P\downarrow}$ ,  $D^{N\uparrow}$  and  $D^{N\downarrow}$  in Eq. (10) are the slater determinants of single particle wave functions for corresponding spin, isospin state then

$$D^s = \det(d_{ij}^s), \quad (11)$$

where

$$d_{ij}^s = \phi_j((\mathbf{r}, s)_i). \quad (12)$$

The nuclear forces are short ranged and saturates very quickly, thus the radial distribution function is not expected to have very long range correlations therefore for the two particle correlation function  $f_j$  in eq.(9) we use a function in the form

$$f_j(r) = \left[ \frac{1}{1 + e^{(r_0 - r)/a}} \right]^t, \quad (13)$$

where  $t$ ,  $r_0$  and  $a$  are variational parameters. We define a pseudo potential  $u(r)$  for practical reasons such that  $f_j(r_{ij}) = \exp(-u(r_{ij}))$  then our variational wave function becomes

$$\Psi_j = \exp\left(-\sum_{i \in J} u(r_{ij})\right) D^{P\uparrow} D^{P\downarrow} D^{N\uparrow} D^{N\downarrow}. \quad (14)$$

We sample the  $3N$  dimensional space with the probability distribution

$$\frac{|\Psi(\mathbf{R})|^2}{\int d\mathbf{R} |\Psi(\mathbf{R})|^2} \quad (15)$$

using a random walk created by the usual Metropolis method. The method given above is a slightly modified version of the VMC method for fermions defined by Ceperley *et al.* [22]. They have also discussed in detail the use of a trial wave function of this form.

The expectation value of any operator  $F$  is then simply the average value of the operator evaluated for the coordinates of the random walk with  $M$  moves

$$\langle \hat{F} \rangle = \frac{\int d\mathbf{r} \Psi^*(\mathbf{r}) F(\mathbf{r}) \Psi(\mathbf{r})}{\int d\mathbf{r} |\Psi(\mathbf{r})|^2} \cong \frac{1}{M} \sum_{i=1}^M F(\mathbf{r}_i). \quad (16)$$

Thus the total energy of the system is calculated as an average over a sufficiently long random walk. The contribution of the NN interactions to total energy are calculated for interparticle separations up to a cut off distance of  $L/2$ . Because the NN interaction is very short ranged, the pair distribution function heals quickly and a reasonable approximation to include the contributions of the pairs farther apart is to assume that the density of particles is constant outside this interaction sphere.

For each density and asymmetry parameter the total energy corresponding to the Hamiltonian of the system is calculated for various values of the parameters in the trial wave function. Then the variational parameters  $r_0$ ,  $a$ , and  $t$  are determined from these calculations so that the total energy is a minimum. Then a final Monte Carlo calculation of the system with the optimized parameter set is performed.

As we have mentioned before, one must use fully occupied closed shells of plane waves for both neutrons and protons in order to preserve the isotropy of the system. Thus the number of neutrons or protons must be chosen from the set (2, 14, 38, 54, 66, 114, ... ). The isospin asymmetry parameter,  $\beta$ , is defined as

$$\beta = \frac{N_n - N_p}{N_n + N_p},$$

where  $N_n$  and  $N_p$  are the numbers of neutrons and protons in the

cubic box under consideration.

### 3. HARTREE-FOCK EQUATIONS

The effective interaction proposed by Skyrme [23] contains basically a two-body part of which is momentum dependent, and a zero-range three-body part. It was designed for Hartree-Fock (HF) calculations of nuclei. The role of three-body part is to simulate the effects of short-range correlations since it is equivalent in HF calculations to a two-body force with a linear density dependence.

Vautherin and Brink [17] has been used with great success a simplified form this interaction which containing six parameters. A characteristic feature of the Skyrme force is that it leads to a HF energy density which is local and has an analytic form simple enough to exhibit the role of the interaction parameters. This

structural simplicity of the force thus allows one to have some general and qualitative idea about the importance of such effects as those arising from the density dependence, the spin-orbit coupling and the neutron-proton asymmetry [23].

Here we will recall the main properties and formulae relevant for our aim which describe by Vautherin and Brink [17]. Consider a nucleus whose ground state is represented by a Slater determinant  $\phi$  of single particle states  $\phi_i$  :

$$\phi(x_1, x_2, \dots, x_A) = \frac{1}{\sqrt{A!}} \det |\phi_i(x_i)| \quad (17)$$

where  $x$  denotes the set  $\vec{r}$ ,  $\sigma$ ,  $q$  of space, spin, and isospin coordinates ( $q = +\frac{1}{2}$  for a proton,  $-\frac{1}{2}$  for a neutron). The expectation value of the total energy is

$$\begin{aligned} E = \langle \phi, (T + V)\phi \rangle &= \sum_i \left\langle i \left| \frac{p^2}{2m} \right| i \right\rangle + \frac{1}{2} \sum_{ij} \langle ij | \bar{v}_{12} | ij \rangle + \frac{1}{6} \sum_{ijk} \langle ijk | \bar{v}_{123} | jk \rangle, \quad (18) \\ &= \int H(\vec{r}) d^3 r \end{aligned}$$

where the notation  $\bar{v}$  denotes an antisymmetrized matrix element. For the Skyrme interaction the energy density  $H(\vec{r})$  is an algebraic function of the nucleon densities  $\rho_n(\rho_p)$ , the kinetic energy  $\tau_n(\tau_p)$  and spin densities  $\vec{J}_n(\vec{J}_p)$ . These quantities depend in turn on the single-particle states  $\phi_i$  defining the Slater-determinant wave function  $\phi$ ,

$$\begin{aligned} \rho_q(\vec{r}) &= \sum_{i,\sigma} |\phi_i(\vec{r}, \sigma, q)|^2, \\ \tau_q(\vec{r}) &= \sum_{i,\sigma} |\bar{\nabla} \phi_i(\vec{r}, \sigma, q)|^2, \\ \vec{J}_q(\vec{r}) &= (-i) \sum_{i,v,\sigma'} \phi_i^*(\vec{r}, \sigma, q) \left[ \bar{\nabla} \phi_i(\vec{r}, \sigma', q) \times \langle \sigma | \vec{\sigma} | \sigma' \rangle \right]. \quad (19) \end{aligned}$$

The sums in Eq. (19) are taken over all occupied single-particle states. The expression for  $H(\vec{r})$  is given:

$$H(\vec{r}) = \frac{\hbar^2}{2m} \tau(\vec{r}) + \frac{1}{2} t_0 \left[ \left( 1 + \frac{1}{2} x_0 \right) \rho^2 - \left( x_0 + \frac{1}{2} \right) (\rho_n^2 + \rho_p^2) \right] + \frac{1}{4} (t_1 + t_2) \rho^2 \tau + \frac{1}{8} (t_2 - t_1) (\rho_n \tau_n + \rho_p \tau_p)$$

$$\begin{aligned}
& + \frac{1}{16}(t_2 - 3t_1)\rho\nabla^2\rho + \frac{1}{32}(3t_1 + t_2)(\rho_n\nabla^2\rho_n + \rho_p\nabla^2\rho_p) + \frac{1}{16}(t_1 - t_2)(\bar{J}_n^2 + \bar{J}_p^2) \\
& + \frac{1}{4}t_3\rho_n\rho_p\rho + H_c(\bar{r}) - \frac{1}{2}W_0(\rho\bar{\nabla}\cdot\bar{J} + \rho_n\bar{\nabla}\cdot\bar{J}_n + \rho_p\bar{\nabla}\cdot\bar{J}_p)
\end{aligned} \quad (20)$$

where  $\rho = \rho_n + \rho_p$ ,  $\tau = \tau_n + \tau_p$ , and  $\bar{J} = \bar{J}_n + \bar{J}_p$ . The direct part of Coulomb interaction in  $H_c(\bar{r})$  is  $\frac{1}{2}V_c(\bar{r})\rho_p(\bar{r})$ , where

$$V_c(\bar{r}) = \int \rho_p(\bar{r}') \frac{e^2}{|\bar{r} - \bar{r}'|} d^3r' \quad (21)$$

### 3.1. SYMMETRIC AND ASYMMETRIC NUCLEAR MATTER

Nuclear matter is an idealized system of interacting nucleons (protons and neutrons). It is not the matter in a nucleus, but a hypothetical bulk system consisting of a huge number of protons and neutrons interacting by only nuclear force and no Coulomb force. Total volume and particle number are considered to be infinite, but their ratio is constant. Infinite volume implies no surface effects and translational invariance. A common idealization is SNM, which consists of equal numbers of protons and neutrons [1]. A detailed description of nuclear matter properties can be found in our previous publications [1, 11, 12, 24-26].

In symmetric nuclear matter we have

$$\rho_n = \rho_p = \frac{1}{2}\rho, \quad \tau_n = \tau_p = \frac{1}{2}\tau, \quad \bar{J}_n = \bar{J}_p = \frac{1}{2}\bar{J}, \quad (22)$$

and the expression for  $H(\bar{r})$  simplifies to

$$H(\bar{r}) = \frac{\hbar^2}{2m}\tau + \frac{3}{8}t_0\rho^2 + \frac{1}{16}t_3\rho^3 + \frac{1}{16}(3t_1 + 5t_2)\rho\tau + \frac{1}{64}(9t_1 - 5t_2)(\bar{\nabla}\rho)^2 - \frac{3}{4}W_0\rho\bar{\nabla}\cdot\bar{J}. \quad (23)$$

From this expression one can get immediately the binding energy per particle in nuclear matter. In nuclear matter  $\bar{\nabla}\rho = \bar{\nabla}\cdot\bar{J} = 0$ ,  $\rho = \left(\frac{2}{3}\pi^2\right)k_F^3$ ,  $\tau = \frac{3}{5}k_F^2$ , so that

$$\frac{E}{A}(\rho) = \frac{H}{\rho} = \frac{3}{5}T_F + \frac{3}{8}t_0\rho + \frac{1}{16}t_3\rho^2 + \frac{3}{80}(3t_1 + 5t_2)\rho k_F^2, \quad (24)$$

Also, the density functional Skyrme energy can be written for ASNM from Eqs. (20) [17]



$$\begin{aligned} \frac{E}{A}(\rho) = & \frac{3}{5}T_F + \frac{1}{2}t_0 \left[ \left(1 + \frac{1}{2}x_0\right)\rho^2 - \left(x_0 + \frac{1}{2}\right)(\rho_n^2 + \rho_p^2) \right] + \frac{1}{4}(t_1 + t_2)\rho^2\tau + \\ & + \frac{1}{8}(t_2 - t_1)(\rho_n\tau_n + \rho_p\tau_p) + \frac{1}{4}t_3\rho_n\rho_p\rho \end{aligned} \quad (25)$$

where  $T_F = \hbar^2 k_F^2 / 2m$  is the kinetic energy of a particle at the Fermi surface. Differentiating the previous expression twice with respect to the Fermi momentum  $k_F$  yields the following value of the nuclear matter incompressibility  $K$ :

$$K = k_F^2 \frac{\partial^2 (E/A)}{\partial k_F^2} = \frac{6}{5}T_F + \frac{9}{4}t_0\rho + \frac{15}{8}t_3\rho^2 + \frac{3}{4}(3t_1 + 5t_2)\rho k_F^2 \quad (26)$$

If one now adds to these two equations the saturation condition  $\partial(E/A)/\partial k_F = 0$ , one gets a system of three linear equations for the quantities  $t_0$ ,  $t_3$  and  $3t_1 + 5t_2$ . Next, solving this system allows one to express the parameters of the interaction in terms of the nuclear matter constants  $E/A$ ,  $k_F$ ,  $K$  [17]:

$$\begin{aligned} t_0\rho &= \frac{40}{9}(E/A) + \frac{4}{9}K - \frac{16}{5}T_F, \\ \frac{3}{16}t_3\rho^2 &= 15(E/A) + K - \frac{9}{5}T_F, \\ \frac{1}{16}(3t_1 + 5t_2)\rho k_F^2 &= 2T_F - 15(E/A) - \frac{5}{6}K. \end{aligned} \quad (27)$$

The another important property of asymmetric nuclear matter is symmetry energy coefficient  $a_s$ , defined as follows:

$$a_s = \frac{1}{3} \frac{\hbar^2}{2m} \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{2/3} - \frac{1}{4}t_0 \left( x_0 + \frac{1}{2} \right) \rho - \frac{1}{16}t_3\rho^2 + \frac{1}{6}t_2 \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{5/3}. \quad (28)$$

## 4. RESULTS

### 4.1. VMC CALCULATION RESULTS

Here, we present the results obtained from Monte Carlo calculations for ASNM with various isospin symmetry parameters. The realistic Urbana NN interaction potential of Lagaris and Pandharipande was used for VMC calculations of ASNM. Also many-body interactions are included as a density-dependent term in the potential. The total energy per nucleon are obtained for densities between

$\rho = 0.01 \text{ fm}^{-3}$  and  $\rho = 0.20 \text{ fm}^{-3}$  steps of  $0.01 \text{ fm}^{-3}$  for each isospin asymmetry parameter. The total energy per nucleon obtained from the Monte Carlo simulations is presented in Table 1. In Table 1,  $\beta = 0.0$  corresponds to SNM.

The new Skyrme parameter set is generated by fitting the energy results per nucleon, which contains 80 energy values obtained from VMC calculations, to the Skyrme energy density functional. The new set obtained is  $t_0 = -488.716 \text{ MeV fm}^3$ ,  $t_1 = -5819.885 \text{ MeV fm}^5$ ,  $t_2 = 1994,994 \text{ MeV fm}^5$ ,  $t_3 = 63800 \text{ MeV fm}^6$ ,  $x_0 = 0.22$ . The results for the new Skyrme parameter set will be discussed below.

Table 1

Expectation values of total energies per nucleon for ASNM obtained from VMC for various isospin asymmetry parameters

$\rho \text{ (fm}^{-3}\text{)}$	$\beta=0.5 \text{ (MeV)}$	$\beta=0.35714 \text{ (MeV)}$	$\beta=0.17392 \text{ (MeV)}$	$\beta=0.0 \text{ (MeV)}$
0,01	1,32069	0,87883	0,41851	0,53757
0,02	0,91851	0,16132	-0,61732	-0,42186
0,03	0,12414	-0,92273	-1,98471	-1,72072
0,04	-0,88323	-2,16527	-3,47151	-3,16458
0,05	-1,93515	-3,49360	-5,03666	-4,71151
0,06	-3,06944	-4,84494	-6,60980	-6,24752
0,07	-4,18120	-6,16864	-8,16101	-7,77962
0,08	-5,24924	-7,42884	-9,64250	-9,25410
0,09	-6,22001	-8,63061	-11,02191	-10,63470
0,10	-7,12596	-9,69399	-12,25119	-11,91990
0,11	-7,88808	-10,60981	-13,35397	-13,08257
0,12	-8,47292	-11,33257	-14,27374	-13,98779
0,13	-8,86204	-11,89319	-15,00446	-14,78032
0,14	-9,06424	-12,19190	-15,47167	-15,30654
0,15	-9,05303	-12,26387	-15,65175	-15,61422
0,16	-8,74170	-12,10253	-15,60797	-15,67273
0,17	-8,17787	-11,58609	-15,22975	-15,43771
0,18	-7,33894	-10,79302	-14,52957	-14,88654
0,19	-6,21559	-9,63044	-13,43081	-14,00599
0,20	-4,72726	-8,13561	-12,02648	-12,80795

#### 4.2. RESULTS FOR THE NEW SKYRME PARAMETER SET

In this section we present the results obtained by the new Skyrme parameter set (which we will call SKaan5) for nuclear matter. Using these new parameters we obtain saturation energy  $E_0 = -15.69 \text{ MeV}$  at  $\rho_0 = 0.151 \text{ fm}^{-3}$  ( $k_F = 1.307 \text{ fm}^{-1}$ ) in the SNM case. These values are in good agreement with our expectations from semi-empirical mass formulas of known nuclei. Various semi-empirical mass formulas estimate the saturation point of the SNM to have an energy per particle between  $-15$  and  $-17 \text{ MeV}$  and Fermi momentum  $k_F$  in the range  $1.29-1.44 \text{ fm}^{-1}$  [27]. For the new Skyrme parameter set we have obtained the symmetry energy coefficient as of nuclear matter as  $19.44 \text{ MeV}$ , this value is different from experimental value. However, the coefficient  $t_3$  of the density dependent term

decreases linearly with the symmetry energy coefficient in nuclear matter. Due to the obtained coefficient  $t_3$  is great in this study, the symmetry energy coefficient value is different than the experimentally quoted value of  $30 \pm 4$  MeV [28]. Also, this symmetry energy coefficient value is close to the results obtained from Skyrme parameterizations SkSC2 and SkSC10 in the literature [2].

Another property of the ASNM, the incompressibility factor, appears in some sophisticated mass formulas, however it cannot be precisely determined from these formulas and the quoted values in the literature have a wide range from 240 to 300 MeV with error estimates of  $\pm 50$  MeV [12]. We have obtained the incompressibility factor of SNM as 244.2 MeV, this value is agreement with the quoted values in the literature. Also, this incompressibility value is very close to the results obtained from Skyrme parameterizations MSk5\*, SLy10 and SkI5 in the literature [2]. These values are given comparatively along with the values obtained from other Skyrme parameterizations [2] in Table 2. Also, the binding energies of nuclear matter are close to each other and to the empirical value for all parameterizations. SIII, SIV and SLy9 parameterizations and this study given in Table 2 produce almost similar binding energies per particle at saturation point.

We have obtained the total energies per nucleon by the parameter set SKaan5 for ASNM are presented in Table 3. The results of saturation density and energy for SNM ( $\beta = 0.0$ ) by the new Skyrme parameter set are in good agreement with empirical values and VMC results. Comparison of other results in Table 4 with the VMC calculations indicate that the new Skyrme parameter set SKaan5 can be acceptable.

Table 2

Comparison of the calculations of values of saturation density  $\rho_0$ , Fermi momentum  $k_F$ , total binding energy per nucleon  $E_0$  at saturation density, asymmetry coefficient  $a_s$  and incompressibility  $K$  of symmetric nuclear matter for different Skyrme parametrization

Skyrme	$\rho_0(\text{fm}^{-3})$	$k_F(\text{fm}^{-1})$	$E_0(\text{MeV})$	$a_s(\text{MeV})$	$K(\text{MeV})$
SI	0.156	1.320	-15.99	29.25	372.1
SkI5	0.156	1.322	-15.85	36.69	256.7
SLy10	0.156	1.321	-15.90	32.02	230.4
Skyrme1'	0.156	1.320	-15.99	29.37	372.1
SkX	0.156	1.321	-16.05	31.11	272.0
MSk5*	0.156	1.322	-15.78	28.01	244.6
SGI	0.155	1.318	-15.89	28.35	262.6
SV	0.155	1.320	-16.05	32.86	306.8
SkXce	0.155	1.320	-15.86	30.16	269.1
SIV	0.151	1.308	-15.96	31.24	325.4
SIII*	0.151	1.307	-16.57	32.67	372.9
SLy9	0.151	1.308	-15.79	32.02	230.3
SII	0.148	1.300	-15.96	34.15	341.6
SkT	0.148	1.298	-15.40	24.90	334.0
SKaan5	0.151	1.307	-15.69	19.44	244.2

Table 3

Expectation values of total energies per nucleon for ASNM obtained by the new Skyrme parameter set SKaan5 for various isospin asymmetry parameters

$\rho$ (fm <sup>-3</sup> )	$\beta=0.5$ (MeV)	$\beta=0.35714$ (MeV)	$\beta=0.17392$ (MeV)	$\beta=0.0$ (MeV)
0,01	1,39555	1,19718	1,03908	0,98984
0,02	0,99710	0,53416	0,16479	0,04970
0,03	0,14916	-0,60492	-1,20709	-1,39481
0,04	-0,90759	-1,96307	-2,80655	-3,06961
0,05	-2,05439	-3,41147	-4,49676	-4,83537
0,06	-3,21770	-4,86949	-6,19144	-6,60405
0,07	-4,34637	-6,28059	-7,82971	-8,31342
0,08	-5,40212	-7,60219	-9,36560	-9,91646
0,09	-6,35496	-8,80076	-10,76272	-11,37589
0,10	-7,18050	-9,84891	-11,99131	-12,66120
0,11	-7,85839	-10,72371	-13,02639	-13,74676
0,12	-8,37130	-11,40560	-13,84657	-14,61064
0,13	-8,70424	-11,87756	-14,43326	-15,23375
0,14	-8,84405	-12,12466	-14,77011	-15,59928
0,15	-8,77904	-12,13360	-14,84252	-15,69225
0,16	-8,49875	-11,89246	-14,63740	-15,49920
0,17	-7,99374	-11,39044	-14,14290	-15,00794
0,18	-7,25540	-10,61773	-13,34821	-14,20735
0,19	-6,27588	-9,56531	-12,24342	-13,08725
0,20	-5,04793	-8,22490	-10,81938	-11,63822

Table 4

Comparison of saturation densities and energies of ASNM obtained from VMC and by the new Skyrme parameter set SKaan5 for various isospin asymmetry parameters

Isospin asymmetry parameters	Saturation density $\rho_0$ (fm <sup>-3</sup> )		Saturation energy $E_0$ (MeV)	
	VMC	SKaan5	VMC	SKaan5
$\beta$				
0.0	0.158	0.151	-15.67	-15.69
0.17392	0.153	0.150	-15.65	-14.84
0.35714	0.148	0.148	-12.26	-12.13
0.5	0.144	0.144	-9.06	-8.84
0.58824	0.142	0.141	-8.15	-6.31

In Fig. 1, the binding energies ( $E/A$ ) of ASNM obtained from the VMC calculations along with the Skyrme SKaan5 calculations for various isospin asymmetry parameters are given. Triangles indicate the VMC calculations and squares indicate the results for the new Skyrme parameter set SKaan5. A comparison of VMC results used in the fits and energy values calculated by the new Skyrme parameter set for ASNM are shown in Fig. 1. It can be seen from Fig. 1 that values of binding energies obtained by the new Skyrme parameter set are agreement with VMC calculations.

Saturation of asymmetric nuclear matter is a fundamental nuclear property which dominates the properties of unstable nuclei under astrophysical conditions. While the saturation density and energy of symmetric nuclear matter are determined empirically from masses and radii of stable nuclei, the properties of asymmetric nuclear matter are still uncertain [29]. For this reason, the agreement between VMC and SKaan5 calculations for near symmetric nuclear matter is notable.

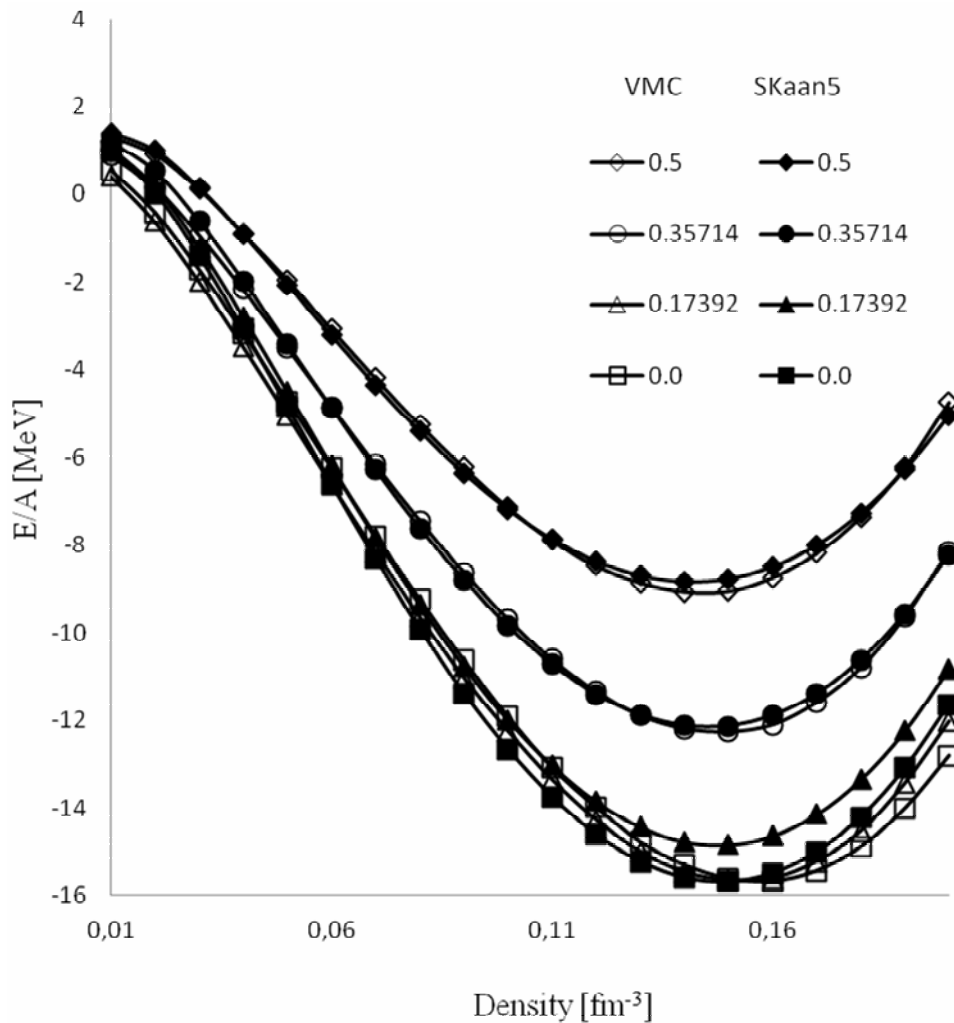


Fig. 1 – Comparison of VMC and SKaan5 calculations for the binding energies of ASNM for  $\beta = 0.5$ , 0.35714, 0.17392, 0.0. The  $\beta$  values increase from bottom to top with the lowermost curve corresponding to SNM ( $\beta = 0.0$ ) and the uppermost one corresponding to  $\beta = 0.5$ . The corresponding asymmetry parameters of the binding energy are shown by various symbols.

## 5. CONCLUSIONS

In order to calculate bulk properties of nuclear matter, the Skyrme potential and other phenomenological potential models are very convenient and useful. However, the reliability of the potential model should be established before using such potentials. Monte Carlo simulation calculations based on the first principles and NN interaction potential may serve as a means to evaluate relative merits of various phenomenological models [1]. We have observed that the results of VMC simulations of the SNM [11, 25] and ASNM [1, 12, 24–26] obtained in our previous studies reasonably agree with the experimental and theoretical studies.

In this study, the results obtained from Monte Carlo simulation are used to determine a new parameter set for Skyrme energy density functional. We have discussed the determination of parameters of the Skyrme interaction leading to a correct properties of nuclear matter. Also, we have attempt to obtain properties of nuclear matter by the new Skyrme parameter set SKaan5 which contains five parameters. The focus has been on the total binding energies and nuclear saturation densities. The results obtained with the new Skyrme parameter set SKaan5 for nuclear matter are compared with other selected Skyrme parameter sets and in the literature, and it was observed that the results obtained in this study agree reasonably well the results found in the literature.

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