

THE ROLE OF ATHERMAL FUSION IN FAST IGNITION DRIVEN BY ION BEAMS

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The fast ignition scheme is recognized as a potentially promising approach to achieve the high-energy-gain target performance needed for commercial inertial confinement fusion. The hot spot heating process by an assumed deuteron beam is evaluated in order to estimate the contribution of the energy produced by the deuteron beam-target fusion to the heating process. So, deuteron beam was considered with Maxwellian energy distribution at temperature of 3MeV. Then, the deuteron energy loss and range, Includes Coulomb and nuclear elastic interactions, in the uniformly pre-compressed fuel, with density 300gcm^{-3} , were calculated. By calculating the contribution of alpha particles produced by the athermal nuclear reactions and nuclear elastic scattering, power deposition of deuteron beams increased up 6% compared with the fast ignition by similar ion beams. This can lead to reduced energy delivered by the external beam.

Key words: fast ignition, ion beam, ion energy deposition, athermal fusion.

1. INTRODUCTION

Currently, fast ignition scheme has been considered as a robust approach to inertial confinement fusion (ICF) which basically takes different path to bring fuel to final ignition and burn state. It has a two stage process, first of it begins with the fuel pre-compressed state through illumination of a long-pulse (ns) driver (laser beams, x-rays) which then ignite by a short-pulse (ps) laser (particles) beam [1]. The technological advantage of the method goes back to less sensitivity of compression process to growth of the Rayleigh-Taylor instability. This is a milestone relative to the standard approach which suffers from hydro dynamical instabilities during implosion phase. Here, we also expect to achieve a higher energy gain in the case of same driver input energy [2].

In original idea of fast ignition, relativistic electrons produced in the course of laser target interaction are responsible to form an off-center hot spot by local energy deposition. A few years later, the idea gained more attraction and innovative

target designs like cone-guided target were proposed. Long range and focusing of hot electrons are issues that motivate researchers to assess the reliability of ion beams [3]. Protons offer better focusing by providing almost ballistic-like trajectories, but currently, the laser-to-ion converter foils used for proton ion generation give proton beam fluxes several orders of magnitude below the total fluxes required [4]. Heavier ions, such as carbon, have been proposed and also studied to further improve focusing and ion yields, but much higher laser intensities needed for this approach will not be available in the immediate future [5].

The transport of energetic ions in dense pre-compressed plasmas is an important problem in high gain target design [6]. If we restrict ourselves to the analysis of collisional phenomena, the dominant process in energetic ion interactions with plasmas is Coulomb scattering. Thus, most of the previous studies on energetic ion transport used the Fokker-Planck equation which describes well the long range nature of Coulomb interaction [7]. However, when the energy of the ions is in order of the nuclear force, discrete events can take place, such as large angle Coulomb scattering, nuclear elastic scattering (NES) and, eventually, athermal fusion. The first derived the Boltzmann-Fokker-Planck equation was capable of accounting for large angle Coulomb scattering [8]. Later calculations showed that large angle Coulomb scattering has a negligible effect on the ion energy deposition in plasmas [9]. In contrast, NES can be an important process; it enhances the heating of background ions, accelerates the slowing-down process and shortens the range of energetic ions.

Deuteron beams have the advantages over other competitors. Moreover, accelerated deuterons not only provide the required heating in the same condition but also coalesce with the target fuel (both deuteron and triton isotopes) as they are slowing down in the target. Therefore, the ignition energy carried by the deuteron beam can be reduced appreciably.

2. ENERGY LOSS MECHANISM IN HOT PLASMAS

To study ignition condition in fast ignition driven by ion beams, we have to incorporate the main interaction mechanism between driver and target particles. The most important energy loss process can be categorized as Coulomb scattering and nuclear elastic scattering.

2.1. COULOMB SCATTERING

In the case of low-density, high-temperature plasmas, Coulomb interactions can be approximated as small angle binary collisions [10]. However, large-angle scattering and collective effects needed to be included at high density cases [11]. These parameters enhance the stopping power of the incident (or produced)

charged particles transport in fusion plasma. Li and Petrasso derived an analytical expression for the charged particle stopping power by the assumption of including large angle scattering contribution in Fokker-Plank equation [12]. The energy loss per unit length of path (by coulomb interactions) of a fast charged particle with (or projectile particle with subscript p) mass m_p , velocity v_p and charge $Z_p e$, which moves through hot plasma (or background particle with subscript b) with ions of mass m_b , charge $Z_b e$ and number density n_b at a temperature T is presented by [12]:

$$\frac{dE^{p/b}}{dx} = -\frac{(Z_p e)^2}{v_p^2} \frac{4\pi n_b (Z_b e)^2}{m_b} \left(G(\beta^{p/b}) \ln \Lambda_b + \Theta(\beta^{p/b}) \ln(1.123\sqrt{\beta^{p/b}}) \right) \quad (1)$$

where $\beta^{p/b} = \frac{v_p^2}{v_b^2}$ ($v_b^2 = \frac{2kT_b}{m_b}$) and function G is defined as:

$$G(\beta^{p/b}) = \kappa(\beta^{p/b}) - \frac{m_b}{m_p} \left(\frac{d(\kappa(\beta^{p/b}))}{d\beta^{p/b}} - \frac{1}{\ln \Lambda_b} \left(\kappa(\beta^{p/b}) + \frac{d(\kappa(\beta^{p/b}))}{d\beta^{p/b}} \right) \right) \quad (2)$$

where

$$\kappa(\beta^{p/b}) = \frac{2}{\sqrt{\pi}} \int_0^{\beta^{p/b}} e^{-\xi} \sqrt{\xi} d\xi \quad (3)$$

is the Maxwell integral and $\Theta(\beta^{p/b})$ is a step function whose value receive 0 for $\beta^{p/b} \leq 1$ and 1 for $\beta^{p/b} > 1$. Note that in beam ion- plasma electron interaction $\beta^{p/b}$ is usually much less than 1, indicating that collective effects can be ignored. However, in beam ion- field ion interaction $\beta^{p/b}$ is much larger than 1, and therefore the collective effects are significant.

2.2. NUCLEAR ELASTIC SCATTERING

Loss of energy through Nuclear Elastic Scattering would be more crucial in the case high temperature plasma. The impact of particles velocity distribution functions due to nuclear elastic scattering may enhance the fusion cross section, and in turn fusion reaction rate [13]. So, we have to take into account their contributions in plasma heating. It accelerates the slowing down process and lowers the range of more energetic ions. The averaged energy loss of incident particle (subscript p) with mass m_p in a field of background charges (subscript b) with mass m_b due to NES can be estimated [14];

$$\left(\frac{dE_p}{dx}\right)^{NES} = -4\pi n_b E_p \frac{m_p m_f}{(m_p + m_b)^2} \int_{\theta=0}^{\pi} \left(\frac{d\sigma}{d\Omega}\right) (1 - \cos\theta) d\cos\theta \quad (4)$$

where $d\sigma/d\Omega$ represents the differential NES cross-section and is suggested as the exact polynomial expansion [15];

$$\begin{aligned} \frac{d\sigma}{d\Omega} = & -\frac{2\eta}{1 - \cos\theta} \operatorname{Re} \left[\exp \left(i\eta \ln \left(\frac{1 - \cos\theta}{2} \right) \right) \sum_0^{l_{\max}} \frac{2l+1}{2} a_l P_l(\cos\theta) \right] \\ & + \sum_0^{2l_{\max}} \frac{2l+1}{2} b_l P_l(\cos\theta) \end{aligned} \quad (5)$$

In which η is the Coulomb parameter and l_{\max} is the highest partial wave that participates in the nuclear scattering. The complex expansion coefficients a_l and real coefficients b_l are energy dependent and are interrelated in complicated ways that can only be imposed by a unitary parameterization of the collision matrix (such as the R-matrix or phase shifts). Finally, the total stopping power can be expressed as;

$$\left(\frac{dE}{dx}\right)_{total} = \left(\frac{dE}{dx}\right)_i^{(C)} + \left(\frac{dE}{dx}\right)_e^{(C)} + \left(\frac{dE}{dx}\right)_i^{(NES)} \quad (6)$$

The first two terms of the above equation indicate Coulomb stopping power of incident particles with field ions and electrons respectively, and the last one denotes the NES stopping power of incident ions with field ions. Range of incident deuterons in deuterium-tritium (DT) plasma can be calculated by the following formula:

$$R = \int_{E_D}^{E_{th}} \left[\left(\frac{dE}{dx}\right)_i^{(C)} + \left(\frac{dE}{dx}\right)_e^{(C)} + \left(\frac{dE}{dx}\right)_i^{(NES)} \right]^{-1} dE \quad (7)$$

3. RESULT AND DISCUSSION

It is assumed that as our initial configuration, we have a uniform pre-compressed DT mixture with state parameters of density $\rho = 300 \text{g.cm}^{-3}$, $\rho R = 0.6 \text{g.cm}^{-2}$ and equal initial electron and ion temperatures of T ($T_{i,0} = T_{e,0}$) = 1keV, which irradiated by a Maxwellian pencil beam of deuteron particles. These values are derived from detailed numerical simulation and lab experiments [16]. It should be noted that the deuterons in Maxwellian velocity distribution have different energies. So, In Fig.2, the range of incident beam against plasma temperature for different energy of deuteron has been shown.

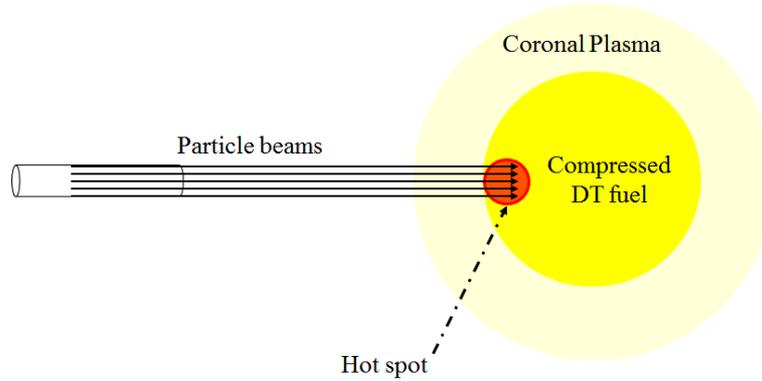


Fig. 1 – Fast ignition scheme of pre-compressed fuel bombard by incident deuteron beam.

The fuel configuration are assumed to be in state of mass density $\rho = 300\text{g.cm}^{-3}$. Recent results have shown that ion range will increase while electron velocities become comparable to the ion. This effect is important for the case of driver with Maxwellian distribution placed far from the fuel. Clearly, the decrease in ion kinetic energy with time is balanced by their range lengthening as the DT is heated up, keeping the ion range almost constant with time [17]. Dashed curve in Fig. 2 corresponds to the region $R < 1.2\text{g.cm}^{-2}$, where according to Atzeni's model study, the ignition parameters were found to depend very little on ions range [18].

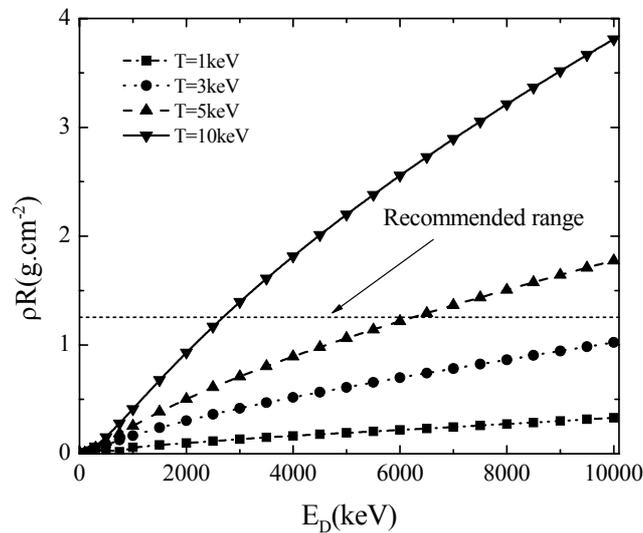


Fig. 2 – Deuteron stopping range in a DT target with density $\rho = 300\text{g.cm}^{-3}$ versus initial deuteron energy at different plasma temperatures (T). The stopping range below the dashed line is the desired stopping range.

We assumed that the deuteron beam with a radius of $15\mu\text{m}$ is characterized by the Maxwellian energy distribution;

$$\frac{dN_D}{dE_D} = \frac{2N_0}{\sqrt{\pi}T_D^{3/2}} \sqrt{E_D} \exp\left(-\frac{E_D}{T_D}\right) \quad (8)$$

$N_0 = 1.5 \times 10^{16}$ is the total number of deuterons. The temperature of the distribution is $T_D = 3\text{MeV}$ and the maximum of the deuteron's kinetic energy is 55MeV . Noting that the maximum kinetic energy of the deuteron is 55MeV , at these incident energies it is not required to use relativistic equations. The deuteron beam power with Maxwellian distribution is:

$$P_D(t) = \frac{8}{3\sqrt{\pi}} \frac{E_{tot}}{\tau} \left(\frac{\tau}{t}\right)^6 \exp\left[-\left(\frac{\tau}{t}\right)^2\right] \quad (9)$$

which E_{tot} is total energy of deuteron beam ($E_{tot} = (3/2)N_0kT_D$) and τ is characteristic time. During slowing down, the fast deuterons, athermal fusion reactions can be occurred with deuterons and tritons. The probability of athermal fusion reaction ($p(T)$) is depended on the fusion cross sections of the fast deuteron with the DT plasma fuel. It is also depended on the stopping power of incident deuterons in DT plasma fuel. Because of the deuteron- deuteron (DD) fusion cross section is much smaller than the DT reaction, for simplicity, we ignore the DD fusion contribution to this study. We can calculate $p(T)$ including by the fast deuterons as follows [19]:

$$p(T) = \int_0^R n_T \sigma_{DT} dx = \int_{E_D}^{E_{th}} n_T \sigma_{DT}(E) \frac{1}{\left(\frac{dE}{dx}\right)_{total}} dE \quad (10)$$

Where n_T , is the number density of tritons in the fuel, $(dE/dx)_{total}$ is total stopping power (can be calculate from eq. 6), $\sigma_{DT}(E)$ is the cross section of DT fusion reaction which is a function of the deuterons kinetic energy, E_D and E_{th} are the initial kinetic and thermal energy of deuterons, respectively. In this study, we ignored the thermal motion of tritons in fuel, because they have much less than the velocity of deuteron projectiles. The probability of athermal fusion of incident deuteron with plasma field tritons is shown in Fig 3. As it can be seen, it increases with fuel temperature, as result of decrease in the stopping power of deuterons. The alpha particles from athermal nuclear reactions will deposit energy in the hot spot and supply additional energy and also power for further ignition. So, the total energy deposited in the hot spot includes the kinetic energy delivered by the deuteron beam and the energy deposited by alpha particles produced by the athermal nuclear reactions. These alpha particles are not monoenergetic because of the kinetic energy of deuterons just before the athermal nuclear fusions.

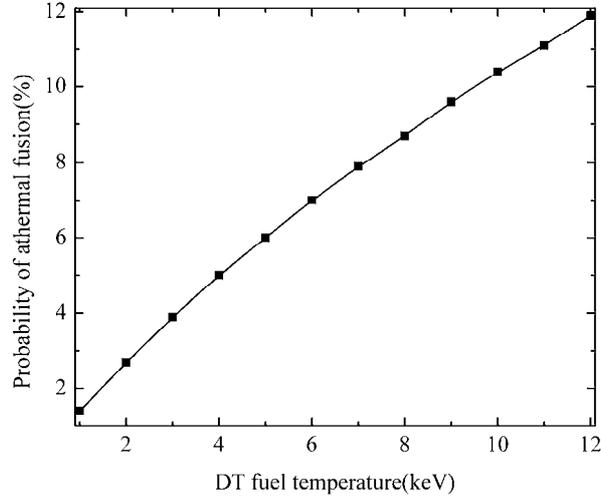


Fig. 3 – Probability of athermal fusion reactions of incident deuteron with plasma field tritons *versus* fuel temperature.

It should be noted that, at the center of the mass coordinate, the emission of the alpha particles produced by athermal nuclear reactions is isotropic. So in order to find the energy spectrum of alpha particle, athermal fusion reaction must be calculated in the center of mass (C.M) coordinate system, and then the results are expressed in the laboratory coordinate system. The distributed shape of energy spectrum is uniformly rectangular in the energy range ($E_{\alpha 1}(E)$, $E_{\alpha 2}(E)$) in which $E_{\alpha 1}(E)$ and $E_{\alpha 2}(E)$ are determined by the kinetic energy of deuteron (E) at the time of reacting (see the appendix). Produced alpha particles by athermal fusion of incident deuteron beams, do not deposit all their energies in the hot spot region. So, the fraction of the energy that the alpha particles deposited inside the considered hot spot can be estimated by the following formula [20];

$$f_{\alpha} = \begin{cases} \frac{3}{2}\tau_{\alpha} - \frac{4}{5}\tau_{\alpha}^2 & \left(\tau_{\alpha} \leq \frac{1}{2}\right) \\ 1 - \frac{1}{4\tau_{\alpha}} + \frac{1}{160\tau_{\alpha}^3} & \left(\tau_{\alpha} \geq \frac{1}{2}\right) \end{cases} \quad (11)$$

$$\tau_{\alpha} = 45 \times \frac{\ln \Lambda_{\alpha e}}{5} \frac{\rho_h R_h}{T_h^{3/2}} \sqrt{\frac{E_{0\alpha}}{E_{\alpha}}} \quad , \quad E_{0\alpha} = 3.5 \text{ MeV}$$

where $\ln \Lambda_{\alpha e}$ is the Coulomb logarithm, T_h and $\rho_h R_h$ are the temperature and areal density of the hot spot region, respectively. The deposited energy for the alpha particles in the hot spot region can be calculated by;

$$E_{dep} = \int_{E_{\alpha 1}}^{E_{\alpha 2}} E_{\alpha} f_{\alpha}(E_{\alpha}) \frac{dN}{dE_{\alpha}} dE_{\alpha} \quad (12)$$

Alpha energy deposition due to athermal fusion reactions causes the plasma temperature raises slightly faster, so it leads to faster formation of hot spots and therefore, helps to create ignition conditions. Energy spectrum of produced alpha particles in terms of energy deuterons, at the moment of fusion reactions for different hot spots temperatures are being shown in Fig.4. It is shown that, by increasing the kinetic energy of the incident deuteron, the produced energetic alpha particle and correspondingly deposited energy in the plasma field will be increased.

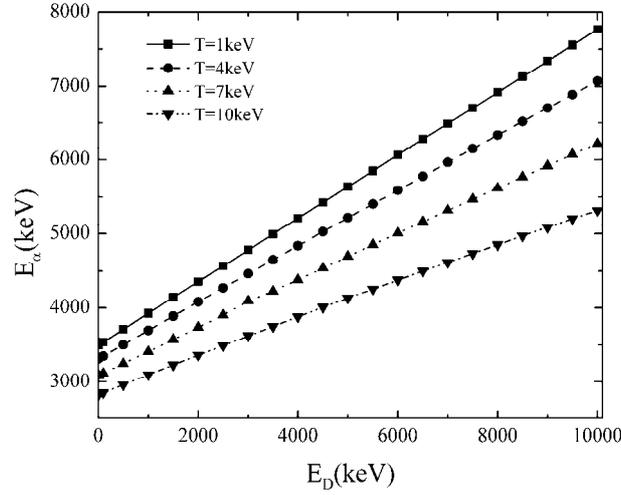


Fig. 4 – Energy deposition of alpha particles produced from athermal fusion versus incident deuteron energy (E_D) at different plasma temperatures (T).

The deposited power of the alpha particles in the hot spot region can be obtained by combining Eq. (10), (11) and (12):

$$P_{\alpha}(t) = \int_{E_0}^{E_{th}} \frac{dN}{dt} \sigma_{DT}(E) n_T \cdot \frac{1}{\left(\frac{dE}{dx}\right)_{total}} \left(\frac{1}{E_{\alpha 2}(E) - E_{\alpha 1}(E)} \int_{E_{\alpha 1}}^{E_{\alpha 2}} E_{\alpha} f_{\alpha}(E_{\alpha}) dE_{\alpha} \right) dE \quad (13)$$

The transport of charged ions in dense plasmas is highly complicated, so we have assumed that the number of deuterons per unit time, which are slowed down to the range (E_0, E_{th}) is the same with dN/dt in Eq (13). The variation of power of the

deuteron beam, both with and without considering the energy deposition of alpha particles, is shown in Fig.5. We can see that the increase in deposition power is about 24 TW at the peak power. This considerable power increase associated only with the deuteron show the advantages of deuteron beam as the fast ignition trigger in comparison to the other ions.

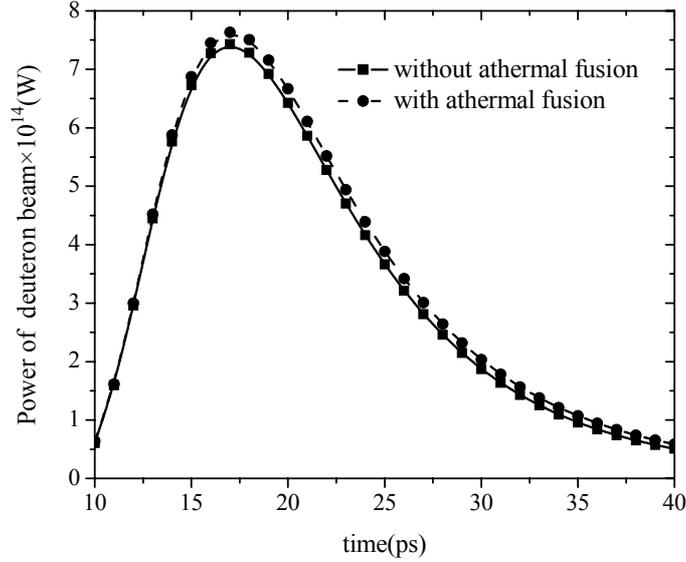


Fig. 5 – The variations of power of the deuteron beam (black-dashed) versus time, including the energy deposition for α particles (solid line).

3.1. HOT SPOT DYNAMICS

Now, we calculate the time evolution of the hot spots, including athermal fusion of the incident deuterons. We take into account the gain and loss of power in the igniting sphere during the hot spot formation. We consider the laser-accelerated deuteron beam has a radius of $15\mu\text{m}$, and a hot spot with an equivalent radius of $15\mu\text{m}$ can be generated by the deuterons with the range not more than 1.1gcm^{-2} at the edge of the pre-compressed fuel. To study ignition condition, we must consider energy gain and loss mechanisms. Gain processes contribution came from external driver ions and plasma accelerated ions. On the other hand, for the loss processes, radiation, thermal conduction and mechanical work are the main mechanism of energy dissipation. When the plasma has a temperature of few keV's, charged particles may lose part of their energies through light emission while decelerating by plasma ions. Because of higher mobility of electrons than ions, this is more important radiation loss for electrons. For DT plasma the power radiated per unit volume is:

$$W_r = A_b \rho_h^2 T_h(t)^{1/2} \quad (14)$$

where ρ_h , T_h and $A_b = 3.05 \times 10^{16} \text{ Jcm}^3 (\text{g}^{-2} \text{s}^{-1} \text{ keV}^{-1/2})$ are hot spot density, temperature and radiation loss parameter, respectively. For the thermal conduction loss for DT plasma, the power per unit volume is:

$$W_e = (3A_e c_e / \ln \Lambda) r^{-2} T_h(t)^{7/2} \quad (15)$$

where r , $\ln \Lambda$, c_e and $A_e = 9.5 \times 10^{12} \text{ J}(\text{s}^{-1} \text{ cm}^{-1} \text{ keV}^{-7/2})$ are hot spot radius, electron Coulomb logarithm, a numerical coefficient close to unity and the parameter of the thermal conduction of electrons, respectively. The thermal conduction loss is quite small when the fuel temperature is low. Power density of the mechanical work is:

$$W_m = A_m \rho_h r_h^{-1} T_h(t)^{3/2} \quad (16)$$

where $A_m = 5.5 \times 10^{15} \text{ Jcm}(\text{g}^{-1} \text{s}^{-1} \text{ keV}^{-3/2})$ is the parameter mechanical work loss under isochoric condition. Because of the pressure in the isochoric fuel of fast ignition is much higher in the hot spot than in the surrounding fuel and a shock is driven into the cold fuel, so the loss of mechanical work must be considered. The equation of power balance can be given by;

$$\begin{aligned} P_D(t) + P_\alpha(t) - [W_r + W_m + W_e - \langle \sigma v \rangle_{DT} n_D n_T f_{\alpha 0} E_{\alpha 0}] \frac{4}{3} \pi r^3 = \\ = m_{DT} C_V \frac{dT_h(t)}{dt} \end{aligned} \quad (17)$$

In the above equations, $C_V = 1.15 \times 10^8 \text{ J}(\text{g}^{-1} \text{ keV}^{-1})$ is the specific heat at constant volume, $E_{\alpha 0} = 3.5 \text{ MeV}$ and $f_{\alpha 0}$ correspond to the fraction of deposited energy for alpha particles of 3.5 MeV. We solve the differential Eq. (17) and drive the temperature of the hot spot as a function of time. A simple conservative ignition criterion $\rho_h R_h T_h > 6 \text{ gcm}^{-2} \text{ keV}$ can be used to estimate that ignition occurs when the temperature of hot spot reaches 10 keV [20]. According to Fig.6, we can see that, by taking account the alpha particles energy deposition (from deuteron athermal fusion) and considering NES (from the incident deuteron beam with plasma ion field) leads to the temperature of hot spot reach 10 keV about 2ps earlier than that heated only by deuterons. The time evolution of hot spot temperature for different initial pre-compressed plasma density is shown in Fig.7. As we have seen, by increasing initial plasma density, the temperature of the hot spots, at the same time, further increases.

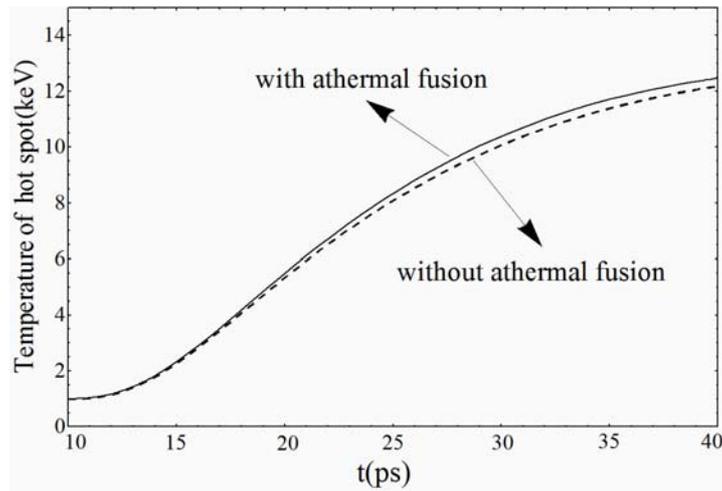


Fig. 6 – Time evolution of the hot spot temperature. The dashed line refers to the deuteron beam heating only, and the solid line refers to the deuteron beam heating and the energy deposition of α particle by considering NES.

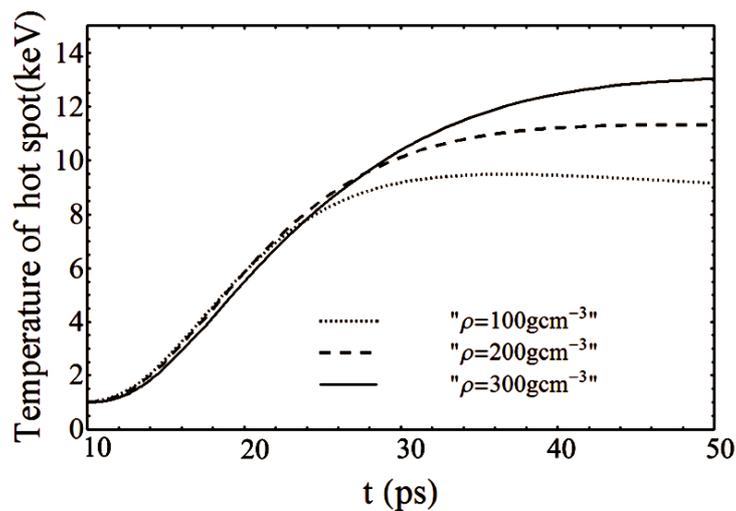


Fig. 7 – Time evolution of the hot spot temperature for different initial pre-compressed plasma density.

4. CONCLUSION

In this paper, we have investigated fast ignition scheme by a laser-accelerated deuteron beam with Maxwellian energy distribution at temperature of 3MeV. Also we suppose that the deuteron beam can be generated instantaneously by a source

located at a distance 500 μm from the pre-compressed fuel. Then, we calculate the stopping power and range of deuterons in the pre-compressed of uniform DT fuel with regard to Coulomb and nuclear elastic interactions of deuterons with plasma ions and electrons. We have found that by increasing the incident deuteron energy, the contribution of the nuclear elastic interaction increases. So, we consider this term, and the stopping power from the incident deuteron be more accurate and therefore, all related quantities improved. In comparison to with fast ignition scheme by other light ions, the suggested idea in here makes complete use of the deposited power of the alpha particles produced by the athermal nuclear reactions and can extra “bonus” up to 6% ion-beam deposition power.

APPENDIX

In order to find the energy spectrum of alpha particle produced by athermal nuclear reactions, we assume that $\vec{v}_D, \vec{v}_T, \vec{v}_\alpha, \vec{v}_n$ and \vec{v}_c are deuteron, triton, alpha particle neutron and center of mass velocity, respectively in laboratory frame. Also \vec{u}_α and \vec{u}_n are alpha particle and neutron velocity in center of mass frame. According to the conservation law of momentum in laboratory frame;

$$m_D \vec{v}_D + m_T \vec{v}_T = (m_D + m_T) \vec{v}_c \Rightarrow \vec{v}_c = \frac{m_D \vec{v}_D + m_T \vec{v}_T}{m_D + m_T} \quad (\text{A.1})$$

and

$$\begin{aligned} \frac{1}{2} m_\alpha v_c^2 &= \frac{1}{2} m_\alpha \frac{m_D^2 v_D^2 + m_T^2 v_T^2 + 2m_D m_T \vec{v}_D \cdot \vec{v}_T}{(m_D + m_T)^2} \\ &= \frac{m_\alpha}{(m_D + m_T)^2} [m_D E_D + m_T E_T + m_D m_T v_D v_T \cos \theta'] \end{aligned} \quad (\text{A.2})$$

In center of mass frame,

$$m_D \vec{u}_D + m_T \vec{u}_T = m_\alpha \vec{u}_\alpha + m_n \vec{u}_n = 0 \quad (\text{A.3})$$

$$\frac{1}{2} m_D u_D^2 + \frac{1}{2} m_T u_T^2 + Q = \frac{1}{2} m_n u_n^2 + \frac{1}{2} m_\alpha u_\alpha^2 \quad (\text{A.4})$$

in which $Q=17.6$ MeV.

We assume that $K = \frac{1}{2} m_D u_D^2 + \frac{1}{2} m_T u_T^2$. The kinetic energy and velocity of alpha particle in center of mass frame can be obtained:

$$\frac{1}{2}m_\alpha u_\alpha^2 = \frac{m_n}{m_n + m_\alpha}(Q + K) \Rightarrow u_\alpha = \sqrt{\frac{2m_n(K + Q)}{m_\alpha(m_\alpha + m_n)}} \quad (\text{A.5})$$

By assuming $\vec{v}_\alpha = \vec{v}_c + \vec{u}_\alpha$ and the angle between \vec{v}_c and \vec{u}_α is θ , the kinetic energy of alpha particle in laboratory frame:

$$\begin{aligned} E_\alpha &= \frac{1}{2}m_\alpha v_\alpha^2 = \frac{1}{2}m_\alpha (\vec{v}_c + \vec{u}_\alpha)^2 = \frac{1}{2}m_\alpha v_c^2 + \frac{1}{2}m_\alpha u_\alpha^2 + m_\alpha v_c u_\alpha \cos \theta \\ &= \frac{1}{2}m_\alpha v_c^2 + \frac{m_n}{m_n + m_\alpha}(Q + K) + v_c \cos \theta \sqrt{\frac{2m_n(K + Q)}{m_\alpha(m_\alpha + m_n)}} \end{aligned} \quad (\text{A.6})$$

Also we could calculate;

$$\vec{u}_D = \vec{v}_D - \vec{v}_c = \vec{v}_D - \frac{m_D \vec{v}_D + m_T \vec{v}_T}{m_D + m_T} = \frac{m_T(\vec{v}_D - \vec{v}_T)}{m_D + m_T} \quad (\text{A.7})$$

$$\vec{u}_T = -\frac{m_D}{m_T} \vec{u}_D = \frac{m_D(\vec{v}_T - \vec{v}_D)}{m_D + m_T} \quad (\text{A.8})$$

$$\begin{aligned} K &= \frac{1}{2}m_D u_D^2 + \frac{1}{2}m_T u_T^2 = \\ &= \frac{m_D m_T}{(m_D + m_T)^2} \left[\left(\frac{m_T}{m_D} + 1 \right) E_D + \left(\frac{m_D}{m_T} + 1 \right) E_T - (m_D + m_T) v_D v_T \cos \theta' \right] \end{aligned} \quad (\text{A.9})$$

Alpha particle kinetic energy in laboratory frame;

$$\frac{1}{2}m_\alpha v_c^2 = \frac{m_\alpha}{(m_D + m_T)^2} [m_D E_D + m_T E_T + m_D m_T v_D v_T \cos \theta'] \quad (\text{A.10})$$

where θ' is the angle between \vec{v}_D and \vec{v}_T in laboratory frame.

In center of mass frame the triton is isotropic. We can get the average value of $\frac{1}{2}m_\alpha v_c^2$ for simplification:

$$\left\langle \frac{1}{2}m_\alpha v_c^2 \right\rangle_{\theta'} = \frac{m_\alpha}{(m_D + m_T)^2} [m_D E_D + m_T E_T] \quad (\text{A.11})$$

and then

$$E_{\alpha} = \left\langle \frac{1}{2} m_{\alpha} v_c^2 \right\rangle_{\theta'} + \frac{m_n}{m_{\alpha} + m_n} (K + Q) + \cos \theta \sqrt{\frac{2m_{\alpha} v_c^2 m_n (K + Q)}{m_{\alpha} + m_n}} \quad (\text{A.12})$$

Because the alpha particle is isotropic in center of mass frame, the distribution of alpha particle is equiprobable in the range $(E_{\alpha 1}(E_D), E_{\alpha 2}(E_D))$. In which,

$$\begin{aligned} E_{\alpha 1}(E_D) &= \left\langle \frac{1}{2} m_{\alpha} v_c^2 \right\rangle_{\theta'} + \frac{m_n}{m_{\alpha} + m_n} (K + Q) - \sqrt{\frac{2m_{\alpha} v_c^2 m_n (K + Q)}{m_{\alpha} + m_n}} \\ E_{\alpha 2}(E_D) &= \left\langle \frac{1}{2} m_{\alpha} v_c^2 \right\rangle_{\theta'} + \frac{m_n}{m_{\alpha} + m_n} (K + Q) + \sqrt{\frac{2m_{\alpha} v_c^2 m_n (K + Q)}{m_{\alpha} + m_n}} \end{aligned} \quad (\text{A.13})$$

So the distributed shape of energy spectrum of alpha particle is uniformly rectangular in the range $(E_{\alpha 1}(E_D), E_{\alpha 2}(E_D))$.

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