

MOMENTS OF INERTIA FOR ^{93}Mo , ^{194}Ir AND ^{196}Au I. AMI^{1,2}, M. FELLAH^{2,3}, N.H. ALLAL^{2,3}, M.R. OUDIH²¹Département de Physique, Faculté des Sciences, Université M'Hamed Bougara-Boumerdès, Campus Sud - 35000 Boumerds ALGERIE. *E-mail*: amiismahane@yahoo.fr²Laboratoire de Physique Théorique, Faculté de Physique, USTHB

BP 32, El Alia, 16111 Bab Ezzouar, Algiers, Algeria

³Centre de Recherche Nucléaire d'Alger,

2 Bd. Frantz Fanon, BP. 399 Alger-Gare, Algiers, Algeria

Received December 12, 2013

The moments of inertia for ^{93}Mo , ^{194}Ir and ^{196}Au are studied as a function of the nuclear deformation and the nuclear temperature. The calculations have been carried out in the framework of the Inglis cranking model by taking into account the pairing correlations using the path integral formalism. The single particle energies and eigen functions used are those of a deformed Woods-Saxon mean field. These nuclei were used in evaluations of induced neutron cross sections and revealed a strong dependence on momentum of inertia parameters.

Key words: Momentum of inertia, Cranking model, temperature.*PACS*: 21.60.Cs, 21.60.Ev.**1. INTRODUCTION**

The cross sections for nuclear reactions are generally considered to be reasonably well known in spite of many reactions for which the data are conflicting or incomplete to make possible validation of different model calculation. This is why is still a lot of interest in the model constraints that are responsible for statistical model (SM) calculation variance. Among the former SM parameters some of the most important concern the nuclear level density and its Gaussian spin distribution with the dispersion σ determined by the nuclear moment of inertia. Actually, the moment of inertia \mathfrak{J} plays a basic role within both nuclear reaction and spectroscopic studies, with different features pointed out within each of these fields. Thus, the spin cutoff parameter was related by Bloch [1] to the rotations of the whole nucleus as a rigid body, while Ericson [2] expressed $\sigma^2 = \mathfrak{J}T/\hbar^2$ in terms of \mathfrak{J} and the nuclear temperature T . At the same time, the rigid body value \mathfrak{J}_r was noted as a limit at high excitation, where the nucleus can be described as an ordinary Fermi gas (FG), while a constant temperature was found suitable at low excitation, similarly to a melting system. Nevertheless \mathfrak{J} values reduced by $\leq 30\%$, due to pairing interactions, were found with an uncertainty still of the same order as the expected effect [2].

On the other hand, Grover [3] gave a first comment on the difference between the moment of inertia that describes the spin dependence of the level density, and an

Rom. Journ. Phys., Vol. 59, Nos. 5-6, P. 504–513, Bucharest, 2014

”effective” moment of inertia which is defined by the energies of the lowest excited level at a given spin J of the nucleus (the yrast levels [4]) that is otherwise the rotational energy of the nucleus with the same spin. While the former is related to the FG of noninteracting particles, the latter is consistent with the opposite shell model calculations corresponding roughly to $\mathfrak{J}/\mathfrak{J}_r \sim 0.5$. The shell structure may also account for values $\mathfrak{J} < \mathfrak{J}_r$, for nuclei away from closed shells, and the opposed case for nuclei near closed shells [4]. A particular case was pointed out, *e.g.*, by Joly *et al.* by analysis of γ -ray strength functions deduced from neutron capture between 0.5 and 3 MeV [5], namely \mathfrak{J}_r values for near closed-shell nuclei ^{104}Rh and ^{198}Au , and half of \mathfrak{J}_r for the deformed nucleus ^{170}Tm .

A significant development of the theoretical approaches came out in the 1990s. Thus, a proper description of the energy–dependent spin cutoff factor deduced from both the spin distribution of the low-lying levels and resonance data proved the consistency of the phenomenological generalized superfluid model, taking into account the phase transition from the superfluid state to the normal state at a critical temperature lower than $\Delta_0=12/\sqrt{A}$ [6]. Moreover, a dramatic increase in the moment of inertia up to the \mathfrak{J}_r value was shown to be associated with the breaking of nucleon pairs for temperatures of 1.0–1.5 MeV, within Shell Model Monte Carlo (SMMC) calculations [7] as well as, later on, in a simplified spin cutoff model [8]. Further developments made possible conversion of the SMMC values of σ^2 to an energy–dependent \mathfrak{J} that is at low energies significantly smaller than the \mathfrak{J}_r value due to the pairing correlations [9].

On the other hand, recent Monte Carlo simulations of the fission fragment (FF) deexcitation process (*e.g.*, [10, 12–14] and Refs. therein), that are developed in order to analyze and predict nuclear data that are of crucial importance for basic and applied nuclear physics, involved deeply the FF moment of inertia. Thus, it has been considered that various models could be upgraded by using also realistic moment of inertia involved in the excitation energy limit for neutron evaporation [10], or to improve the approximations of the so-called centrifugal barriers [11]. This is the reason why we found useful within the present work the basic approach based on the cranking model for the study of the excitation energy dependence of the moment of inertia for the ^{93}Mo , ^{194}Ir and ^{196}Au nuclei. These nuclei were recently used in calculations that revealed a very high sensitivity to angular momentum parameters [15].

Several studies evaluate the nuclear moment of inertia by taking into account the pairing correlations between like-particles at zero and finite temperature [8, 16–20]. At finite temperature, this had been performed within FTBCS method [16, 19] and the Hartree-Fock-Bogoliubov framework (FTHFB) [21]. Let us note that Allal *et al.* [20] studied the effect of the particle-number fluctuations inherent to the BCS theory on the moment of inertia of some even-even actinide nuclei at finite temperature. In their study, they have established an explicit expression of the parallel and

perpendicular moments of inertia based on a discrete particle-number projection method which generalize the expression established at zero temperature. Their obtained results at finite temperature are similar to that obtained by Alhassid *et al.* [8] for the iron isotopes using a parity-number projection method. Moreover, it is well known that neutron-proton (np) pairing correlations play an important role in the calculation of the moment of inertia of nuclei with $N \simeq Z$. This, was carried out at zero temperature including isovector pairing correlations for proton-rich even-even rare earth nuclei [22] and also for several rare-earth and actinide nuclei [23, 24]. Recently, np pairing correlations effect on the moment of inertia have been performed at finite temperature for even-even proton rich nuclei [25–27].

In the present work, the temperature dependent moment of inertia [16] will be evaluated within a microscopic method in the framework of the cranking model. The partition function of the system including the pairing correlations between like-particles was derived within the Feynman path integral method. In the following section, the mean features of the model are discussed while in the last section the results concerning the three nuclei of interest are presented.

2. MOMENT OF INERTIA

Let us consider a system of nucleons which is cranked around Ox axis (Oz being the symmetry axis) of a rotating frame. The grand-partition function of a such a hot nuclear system is given by:

$$Z = Tr \exp -\beta[H - \lambda N - \hbar\omega J_x] \quad (1)$$

where β is the inverse of the temperature T and H is the Hamiltonian of the system given, in the second quantization formalism by:

$$H = \sum_{\nu>0} e_{\nu}(a_{\nu}^{\dagger}a_{\nu} + a_{\bar{\nu}}^{\dagger}a_{\bar{\nu}}) - G \sum_{\nu,\mu>0} a_{\nu}^{\dagger}a_{\bar{\nu}}^{\dagger}a_{\bar{\mu}}a_{\mu} \quad (2)$$

In the previous expression, a_{ν}^{\dagger} and a_{ν} represent respectively the creation and annihilation operators of a particle in the state $|\nu\rangle$ of energy e_{ν} ; $|\bar{\nu}\rangle$ is the time-reverse state of $|\nu\rangle$. The time-reversal invariance of H has been taken into account, which implies $e_{\nu} = e_{\bar{\nu}}$. G characterizes the pairing strength which is assumed to be constant. In all that follows, it is assumed that the single-particle energies are independent from the temperature [28]. λ is Lagrange parameter which represent the Fermi level energy and N is the particle-number operator given by:

$$N = \sum_{\nu>0} (a_{\nu}^{\dagger}a_{\nu} + a_{\bar{\nu}}^{\dagger}a_{\bar{\nu}}), \quad (3)$$

ω is the rotation frequency and J_x is the x projection of the angular momentum.

The usual Inglis [29] expression of the energy may be easily generalized to include the temperature effects [20] that is:

$$E = \left(-\frac{\partial \ln Z}{\partial \beta} \right)_{\lambda\beta=cte} \quad (4)$$

Its expansion to the second order in ω is given by:

$$E \simeq E_0 - \omega^2 \hbar^2 \int_0^\beta \langle J_x(\beta) J_x(\chi) \rangle_0 d\chi \quad (5)$$

where

$$\begin{cases} J_x(\kappa) = e^{\kappa(H-\lambda N)} J_x e^{-\kappa(H-\lambda N)} \\ \langle J_x(\beta) J_x(\chi) \rangle = \frac{\text{Tr} e^{-\beta(H-\lambda N)} J_x(\beta) J_x(\chi)}{\text{Tr} e^{-\beta(H-\lambda N)}} \end{cases} ; \quad \kappa = \beta, \chi \quad (6)$$

$J_x(\kappa)$ is the Heisenberg transform of J_x and the thermal average in Eq. (5) is evaluated using the grand-canonical ensemble associated to the Hamiltonian without rotation. This thermal average value may easily be determined using the quasiparticle representation. In the latter, the Hamiltonian associated to $H - \lambda N$ has been approximately diagonalized by means of the Feynman path integral technique and using the Hubbard-Stratonovich transformation [30]. The moment of inertia \mathfrak{S}_\perp is defined by the expansion of the system energy to second order in ω :

$$\mathfrak{S}_\perp = 2\hbar^2 \int_0^\beta \langle J_x(\beta) J_x(\chi) \rangle_0 d\chi \quad (7)$$

One has, after some algebra [16, 19, 20]:

$$\begin{aligned} \mathfrak{S}_\perp = \hbar^2 \sum_{\nu\mu} |\langle \nu | J_x | \mu \rangle|^2 & \left\{ \frac{[u_\nu v_\mu - u_\mu v_\nu]^2}{E_\nu + E_\mu} \left(\tanh \frac{\beta E_\nu}{2} + \tanh \frac{\beta E_\mu}{2} \right) \right. \\ & \left. \frac{[u_\nu u_\mu + v_\nu v_\mu]^2}{E_\nu - E_\mu} \left(\tanh \frac{\beta E_\nu}{2} - \tanh \frac{\beta E_\mu}{2} \right) \right\} \end{aligned} \quad (8)$$

which is nothing but the usual FTBCS theory expression of the perpendicular moment of inertia for pairing between like-particles. Let us notice that in this expression, the chemical potential λ and the gap parameter Δ , as well as the u_ν and v_ν parameters, which are temperature dependent, are obtained by solving the usual FTBCS gap equations, that is [31]:

$$\frac{2}{G} = \sum_{\nu>0} \frac{1}{E_\nu} \tanh \left(\frac{1}{2} \beta E_\nu \right) \quad (9)$$

and

$$\langle N \rangle = \sum_{\nu>0} \left[1 - \left(\frac{e_\nu - \lambda - Gv_\nu^2}{E_\nu} \right) \tanh\left(\frac{\beta E_\nu}{2}\right) \right] \quad (10)$$

E_ν being the quasiparticle energies.

In particular, the moment of inertia of a rotating system calculated within the Inglis cranking method [29] without pairing is given by:

$$\mathfrak{S}_C = 2\hbar^2 \sum_{\nu \neq 0} \frac{|\langle \mu | J_x | 0 \rangle|^2}{\varepsilon_\nu - \varepsilon_0} \quad (11)$$

where ε_ν is the single particle energy of the excited state $|\nu\rangle$ and ε_0 that of the ground state $|0\rangle$. In the usual BCS theory, the moment of inertia is given by the like particle Belyaev formula [32]:

$$\mathfrak{S}_{BCS} = 2\hbar^2 \sum_{\nu\mu>0} |\langle \nu | J_x | \mu \rangle|^2 \frac{[u_\mu v_\nu - u_\nu v_\mu]^2}{E_\nu + E_\mu} \quad (12)$$

that correspond for the case $T = 0$ in our formalism.

3. RESULTS AND DISCUSSION

The model described in the previous section corresponds to even systems. It has been extended to odd systems in an approximate way, by removing the contribution of the odd nucleon that corresponds to the blocked level. The moments of inertia were calculated for the three nuclei of interest: ^{93}Mo , ^{194}Ir and ^{196}Au . The results were calculated as function of the deformation parameter β_2 that characterizes the elongation of the nucleus. The values of β_4 were kept unchanged, and correspond to the ground state deformations reported in Ref. [33]. The range scanned by our elongations includes the ground states values of all investigated nuclei. The negative values correspond to oblate shapes while the positive ones to prolate configurations. We tested several values of the excitation energies by taking into account the temperature T . We estimated that at $T=0.5$ MeV, the excitation energies are about 10 MeV. The behavior of Mo is represented in Fig. 1, that of the Ir is given in Fig. 2 while the Au is represented in Fig. 3. For each nucleus, the temperatures taken into consideration, ranging from 0 to 5 MeV, were marked on the plot. As expected, for $\beta_2 \approx 0$, in the region of spherical nuclear shapes, the momentum of inertia has lower values.

The modification of the nuclear structure as function of the system deformation parameter β_2 is reflected by oscillations in the values of the inertia. Such examples can be found in all studied nuclei. In principle, the moments of inertia must increase monotonically with the deformation parameter. But, in the vicinity of $\beta_2 \approx 0.04-0.08$,

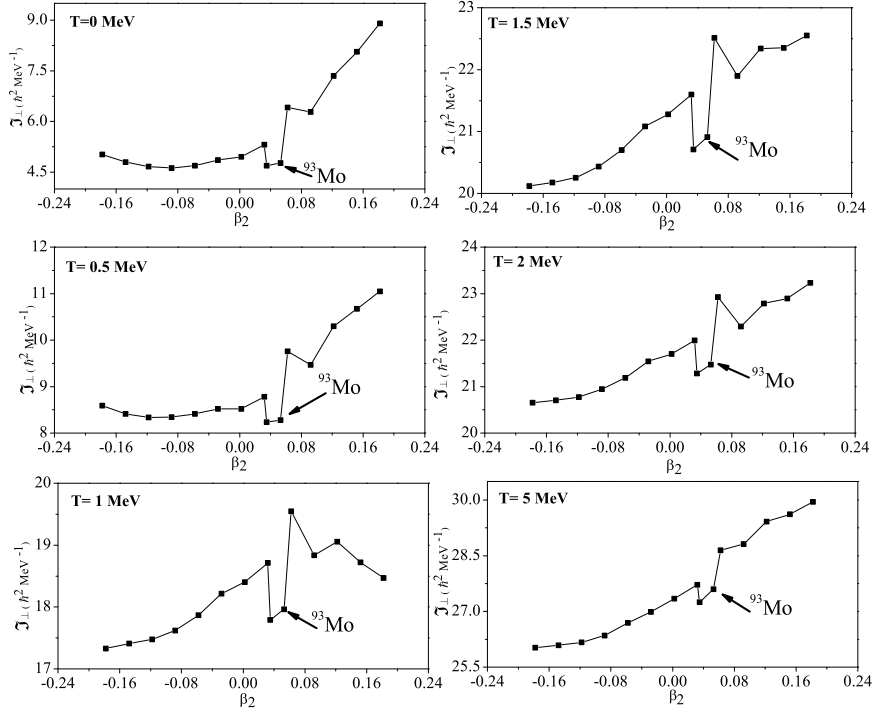


Fig. 1 – Dependence of the ^{93}Mo moment of inertia in $\hbar^2\text{MeV}^{-1}$ units as function of the deformation for different values of the temperature: 0, 0.5, 1, 1.5, 2, and 5 MeV. The values of the temperature are marked on the plot. The ground state position is marked by an arrow in each plot.

the moments of inertia become to decrease abruptly, especially for $T=0$, and for the low temperatures. We consider that this fluctuations are due to the rearrangement of the nuclear single particle levels taken into consideration in the pairing active level space. The ingredients of the model, as the pairing gap and the single particle occupation probabilities, depend strongly on the single particle level distribution. Therefore any change in these microscopic distributions must be reflected in the behavior of the moment of inertia. For larger temperatures, for example at $T=5$ MeV, these fluctuation due to the nuclear structure are attenuated. These fact show that the nucleus begin to behave as a rigid body system when the excitation energy increases. This phenomenon is a general one, being observed for the three investigated nuclei.

In general, the moments of inertia increase with the mass. So, the Mo moment of inertia is smaller than those of the Ir and Au. In Fig. 4 we represent comparison

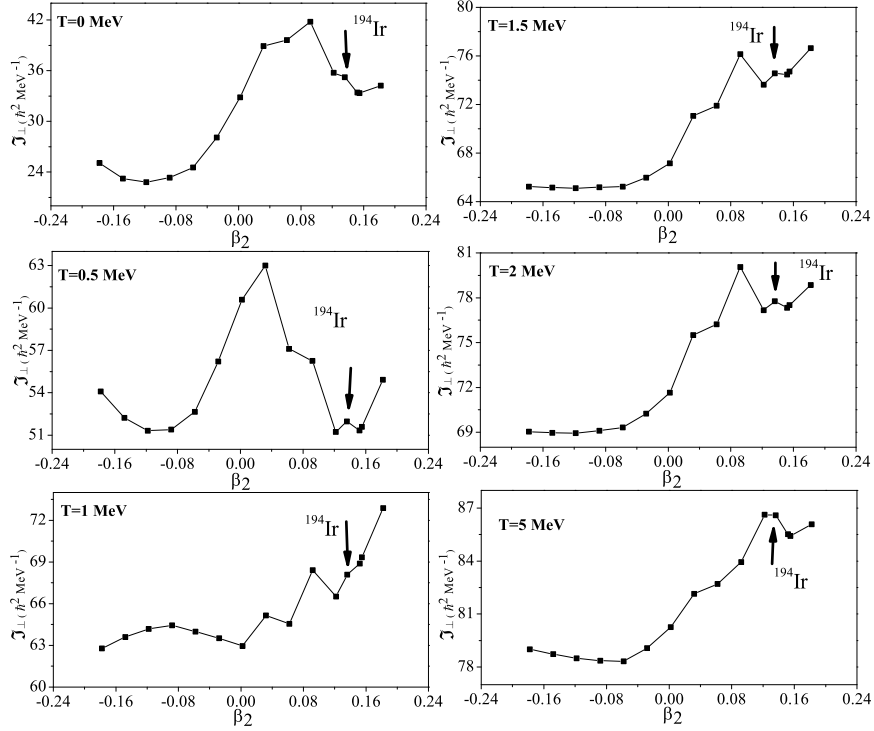


Fig. 2 – Same as 1 for ^{194}Ir .

between the moments of inertia of the Ir and Au at several temperatures. We will discuss the values of the moments of inertia in terms of the fundamental configurations. Despite the fact that the mass number of the Au is larger than that of the Ir, the moment of inertia in the ground state is lower by a ratio of approximately 0.7 at $T = 0$. This ratio will increase by increasing the temperature, reaching about 0.96 for $T = 5$ MeV. By increasing the temperatures, the nucleus behaves as a rigid body system, the internal structure plays a smaller role, so that the moments of inertia reach very close values. It is interesting to note the behavior displayed by the plot at $T=0.5$ MeV, where the moment of inertia of the Au surpasses that of the Ir in the fundamental configurations. This last value of the temperature approaches the critical value, where the gap vanishes. Having in mind that a 10 MeV excitation energy is supposed to exist in the calculations of Ref. [15], our results at $T=0.5$ MeV must be considered in the evaluation of his data. As in Ref. [15] we can report that the moment of inertia of the

Au must be larger than that of Ir at the same excitation energy.

In conclusion, within the cranking model we were able to show that the moment of inertia corresponding to Au is larger than that of the Ir, as found in Ref. [15] at a temperature of 0.5 MeV. This behavior is obtained with two different fundamental configurations. In general, the oblate configurations (negative values of β_2) have lower values of the moments of inertia that the prolate ones (positive values of β_2). In principle, we expect that the ground state configuration of the Au favor a lower value of the moment of inertia as that of Ir. It is not the case at $T=0.5$ MeV where the data of Ref. [15] are available.

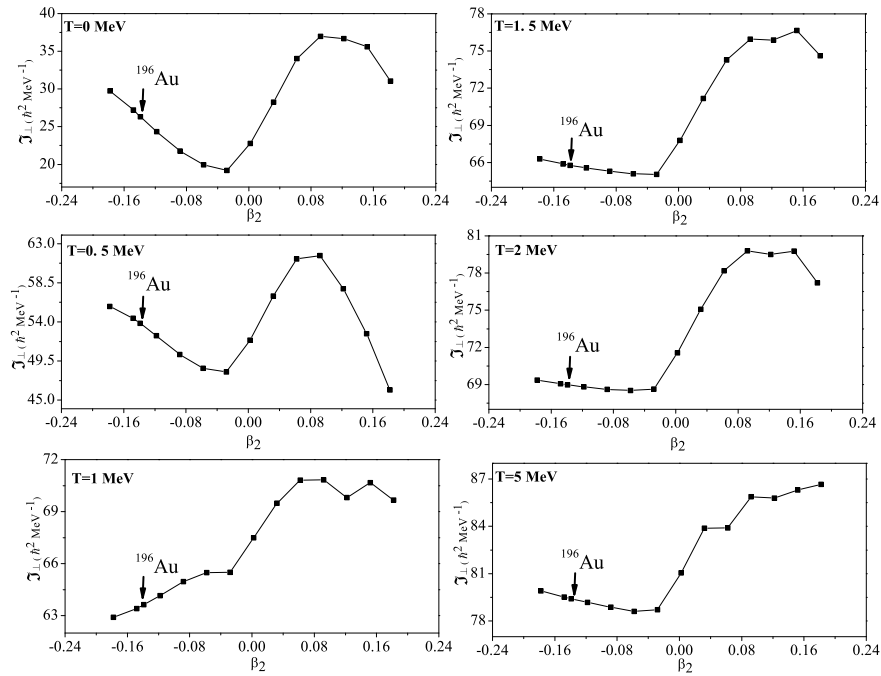


Fig. 3 – Same as 1 for ^{196}Au .

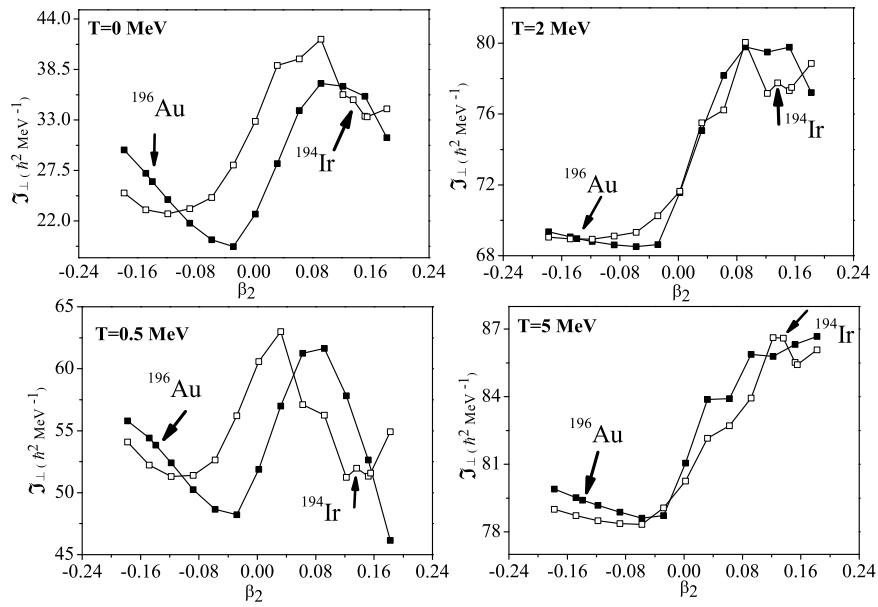


Fig. 4 – Comparison between the moment of inertia of Ir and Au. The open squares correspond to Ir and the filled ones to Au.

Acknowledgements. This work was performed under a Grant of the Faculte des Sciences, Universite M'hammed Bougara de Boumerdes UMBB. Discussions with V. Avrigeanu and M. Mirea were appreciated.

REFERENCES

1. C. Bloch, Phys. Rev. **93**, 1094–1106 (1954).
2. T. Ericson, Nucl. Phys. **11**, 481–491 (1959).
3. J.R. Grover, Phys. Rev. **127**, 2142–2150 (1962).
4. J.R. Grover, Phys. Rev. **157**, 832–847 (1967).
5. S. Joly, D.M. Drake, and L. Nilsson, Phys. Rev. **C 20**, 2072–2083 (1979).
6. A.V. Ignatyuk, J.L. Weil, S. Raman, and S. Kahane, Phys. Rev. **C 47**, 1504–1513 (1993).
7. D.J. Dean, S.E. Koonin, K. Langanke, P.B. Radha and Y. Alhassid, Phys. Rev. Lett. **74**, 2909–2912 (1995);
K. Langanke, D.J. Dean, S.E. Koonin, and P.B. Radha, Nucl. Phys. **A 613**, 253–266 (1997).
8. Y. Alhassid, G.F. Bertsch, L. Fang, and S. Liu, Phys. Rev. **C 72**, 064326–11 (2005).
9. Y. Alhassid, S. Liu, and H. Nakada, Phys. Rev. Lett. **99**, 162504–4 (2007).
10. O. Litaize and O. Serot, Phys. Rev. **C 82**, 054616–11 (2010).
11. A. Diaf, M. Lassaut, and R.J. Lombard, Rom. J. Phys. **57**, 159–164 (2012).
12. C. Maniulescu, A. Tudora, F.-J. Hamsch, C. Morariu, and S. Oberstedt, Nucl. Phys. **A 867**, 12–40 (2011).
13. B. Becker, P. Talou, T. Kawano, Y. Danon, and I. Stetcu, Phys. Rev. **C 87**, 014617–16 (2013).
14. I. Stetcu, P. Talou, T. Kawano, and M. Jandel, Phys. Rev. **C 88**, 044603–8 (2013).
15. M. Avrigeanu, V. Avrigeanu, M. Diakaki, and R. Vlastou, Phys. Rev. **C 85**, 044618–10 (2012).
16. M. Brack, J. Damgaard, A.S. Jensen, H.C. Pauli, V.M. Strutinsky and C.Y. Wong, Rev. Mod. Phys. **44**, 320–405 (1972).
17. N.H. Allal and M. Fellah, Phys. Rev. **C 43**, 2648–2657 (1991).
18. M. Hasegawa and S. Tazaki, Phys. Rev. **C 47**, 188–193 (1993).
19. M. Brack, T. Ledergerber, H.C. Pauli, A.S. Jensen, Nucl. Phys. **A 234**, 185–215 (1974).
20. N.H. Allal, M. Fellah, N. Benhamouda, and M.R. Oudih, Phys. Rev. **C 77**, 054310–8 (2008).
21. A.L. Goodman, Phys. Rev. **C 48**, 2679–2684 (1993).
22. D. Mokhtari, I. Ami, M. Fellah and N.H. Allal, Int. J. Mod. Phys. **E 17**, 655–667 (2008).
23. E. Meftunoğlu *et al.*, J. Phy. G : Nucl. Part. Phys. **24**, 107–115 (1998).
24. M. Gerçeklioğlu and A.E. Calik, Acta Phys. Slo. **55**, 197–209 (2005).
25. I. Ami, M. Fellah, N.H. Allal, N. Benhamouda, M. Belabbas and M.R. Oudih, Int. J. Mod. Phys. **E 20**, 1947–1959 (2011).
26. I. Ami, M. Fellah, N.H. Allal, N. Benhamouda and M. Belabbas, J. Phys.: Conf. Ser. 338, 012019 (2012).
27. I. Ami, M. Fellah, N. Benhamouda, N.H. Allal, J. Phys.: Conf. Ser. 413, 012032 (2013).
28. M. Brack and P. Quentin, Nucl. Phys. **A 361**, 35–82 (1981).
29. D.R. Inglis, Phys. Rev. **96**, 1059–1065 (1954).
30. J. Hubbard, Phys. Rev. Lett. **3**, 77–78 (1959).
31. L.G. Moretto, Nucl. Phys. **A 182**, 641–668 (1972).
32. P. Ring and P. Schuk, “*The Nuclear Many Body Problem*”, (Springer, Berlin, 2000).
33. P. Moller, J.R. Nix, At. Data. Nucl. Data Tabl. **59**, 185–381 (1995).