BREMSSTRAHLUNG IN NONCOMMUTATIVE QUANTUM MECHANICS

SEVDA HEYDARNEZHAD

Payame Noor Univeristy (PNU), Orumieh, Iran
E-mail: sevda.heydarnezhad@live.com

Received September 30, 2013

The intensity cross section for a non-relativistic bremsstrahlung is calculated in noncommutative space. It is shown that for a soft bremsstrahlung the cross section is increased by a factor which is second order in noncommutativity parameter.

Key words: Soft bremsstrahlung, noncommutative quantum mechanics.

1. INTRODUCTION

In recent years, many aspects of the noncommutative quantum mechanics, as the low energy limit of the noncommutative quantum field theory, has been studied by the several authors [1-10]. The theory of noncommutative fields deals with the fields defined in space-time with noncommutating coordinates $\widehat{x}^i \quad (i = 1,2,3)$ satisfying

$$[\widehat{x}^i, \widehat{x}^j] = i\gamma^{ij}, \qquad \gamma^{ij} = -\gamma^{ji}. \tag{1}$$

where γ^{ij} is an antisymmetric 3×3 matrix. In a noncommutative space one must replace the product of the fields with Moyal product. For the fields $\phi_1(\mathbf{x})$ and $\phi_2(\mathbf{x})$ it is defined as

$$\phi_1(\widehat{\mathbf{x}}) \star \phi_2(\widehat{\mathbf{x}}) \equiv \phi_1(\mathbf{x}) \exp\left(\frac{i}{2}\gamma^{ij}\overleftarrow{\partial}_i \overrightarrow{\partial}_j\right)\phi_2(\mathbf{x}). \tag{2}$$

Using the above definition, one can easily show that [11]

$$e^{i\mathbf{k}_1\cdot\widehat{\mathbf{x}}} \star e^{i\mathbf{k}_2\cdot\widehat{\mathbf{x}}} = e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{x}} e^{-i\mathbf{k}_1\wedge\mathbf{k}_2}, \tag{3}$$

and

$$e^{i\mathbf{k}_1\cdot\widehat{\mathbf{x}}} \star e^{i\mathbf{k}_2\cdot\widehat{\mathbf{x}}} \star e^{i\mathbf{k}_3\cdot\widehat{\mathbf{x}}} = e^{i(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\cdot\mathbf{x}} e^{-i\mathbf{k}_1\wedge\mathbf{k}_2} e^{-i\mathbf{k}_1\wedge\mathbf{k}_3} e^{-i\mathbf{k}_2\wedge\mathbf{k}_3}, \tag{4}$$

where $\mathbf{k}\wedge\mathbf{k}' = \frac{1}{2}\gamma^{ij}k_i k'_j$.

In this note, we will derive the radiation cross section in a non-relativistic bremsstrahlung in noncommutative space. The correction on the radiation cross section, due to non-commutativity of space, is calculated to second order in γ. In noncommutative space, we find an increase in the intensity cross section. In this work, we will use the natural units, *i.e.* $\hbar = c = 1$.

2. NON-RELATIVISTIC BREMSSTRAHLUNG

Let us consider an incoming particle with momentum $\mathbf{p} = m\mathbf{v}$ and charge e, scattered by a static source with charge $-Ze$. The bremsstrahlung cross section is given by [12-14]

$$d\sigma_{\mathbf{p}'} = \frac{4e^2}{3\omega v} |\langle \mathbf{p}'|\ddot{\mathbf{x}}|\mathbf{p}\rangle|^2 \frac{d^3p'}{(2\pi)^3} = \frac{4mp'e^2}{3\omega v} |\langle \mathbf{p}'|\ddot{\mathbf{x}}|\mathbf{p}\rangle|^2 \frac{d\omega d\Omega_{\mathbf{p}'}}{(2\pi)^3}. \tag{5}$$

Here $|\mathbf{p}\rangle$ and $|\mathbf{p}'\rangle$ denote the exact wave functions of the incoming and outgoing particles with momentum $\mathbf{p}$ and $\mathbf{p}'$, respectively. The outgoing particle has the momentum $\mathbf{p}' = m\mathbf{v}'$ and the energy of emitted photon is $\omega = \frac{p^2}{2m} - \frac{p'^2}{2m}$. By engaging the classical equation of motion

$$m\ddot{\mathbf{x}} = Ze^2 \nabla \frac{1}{|\mathbf{x}|}, \tag{6}$$

(5) takes the form

$$d\sigma_{\mathbf{p}'} = \frac{4mp'e^2}{3\omega v} \Big(\frac{Ze^2}{m}\Big)^2 |\langle \mathbf{p}'|\nabla \frac{1}{|\mathbf{x}|}|\mathbf{p}\rangle|^2 \frac{d\omega d\Omega_{\mathbf{p}'}}{(2\pi)^3}. \tag{7}$$

In Born approximation, one replaces the exact wave function with the plane wave *i.e.* $\phi_{\mathbf{p}} \approx e^{i\mathbf{p}\cdot\mathbf{x}}$. Thus, by taking into account

$$\langle \mathbf{p}'|\nabla \frac{1}{|\mathbf{x}|}|\mathbf{p}\rangle = \int d^3x e^{-i\mathbf{p}'\cdot\mathbf{x}} \nabla \frac{1}{|\mathbf{x}|} e^{i\mathbf{p}\cdot\mathbf{x}} = 4\pi i \frac{\mathbf{p}' - \mathbf{p}}{|\mathbf{p}' - \mathbf{p}|^2}, \tag{8}$$

and integrating over the solid angle $d\Omega_{\mathbf{p}'} = d\phi d\theta \sin\theta$ one arrives at

$$d\sigma_\omega = \frac{16}{3} \frac{Z^2e^6}{mv^2} \log\Big(\frac{v+v'}{v-v'}\Big) \frac{d\omega}{\omega}, \tag{9}$$

where θ denotes the angle between the incoming and outgoing particles. The intensity of emitted photon is $dI = \omega d\sigma_\omega$. So, for a soft bremsstrahlung, *i.e.* $\omega \ll 1$ and $p' \approx p$, the intensity per frequency is found to be [14]

$$\frac{dI}{d\omega} = \frac{16}{3} \frac{Z^2e^6}{mv^2} \log\Big(\frac{2mv^2}{\omega}\Big). \tag{10}$$

3. BREMSSTRAHLUNG IN NONCOMMUTATIVE SPACE

Now, we consider the bremsstrahlung cross section in noncommutative space:

$$d\sigma_{\mathbf{p}'} = \frac{4mp'e^2}{3\omega v} \Big(\frac{Ze^2}{2m}\Big)^2 |\langle \mathbf{p}'|\nabla \frac{1}{|\mathbf{x}|}|\mathbf{p}\rangle_\star|^2 \frac{d\omega d\Omega_{\mathbf{p}'}}{(2\pi)^3}, \tag{11}$$

where the usual product is replaced by the Moyal product, namely

$$\begin{aligned}\langle \mathbf{p}'|\nabla \frac{1}{|\mathbf{x}|}|\mathbf{p}\rangle_\star &= \int d^3x e^{-i\mathbf{p}'\cdot\mathbf{x}} \star \nabla\Big(\frac{1}{|\mathbf{x}|}\Big) \star e^{i\mathbf{p}\cdot\mathbf{x}} \\ &= 4\pi i \frac{\mathbf{p}-\mathbf{p}'}{|\mathbf{p}-\mathbf{p}'|^2} e^{i\mathbf{p}\wedge\mathbf{p}'}.\end{aligned} \tag{12}$$

Also, we have invoked

$$\nabla \frac{1}{|\mathbf{x}|} = i \int \frac{d^3k}{(2\pi)^3} \frac{\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{x}}}{|\mathbf{k}|^2}. \tag{13}$$

Now, we expand the exponent in the second line of (12) to first order in noncommutativity parameter

$$e^{i\mathbf{p}\wedge\mathbf{p}'} \approx 1 + \frac{i}{4}\gamma p p' \sin\theta. \tag{14}$$

Thus, one arrives at

$$|\langle \mathbf{p}'|\nabla \frac{1}{|\mathbf{x}|}|\mathbf{p}\rangle_\star|^2 \approx |\langle \mathbf{p}'|\nabla \frac{1}{|\mathbf{x}|}|\mathbf{p}\rangle|^2 + \frac{\pi^2 p^2 p'^2 \sin^2\theta}{|\mathbf{p}-\mathbf{p}'|^2}\gamma^2. \tag{15}$$

On substituting the above expression in (11) and using

$$\int d\Omega_{\mathbf{p}'} \frac{p^2 p'^2 \sin^2\theta}{|\mathbf{p}-\mathbf{p}'|^2} = \pi(p^2+p'^2) - \frac{2\pi m^2\omega^2}{pp'} \ln \frac{(p+p')^2}{2m\omega}, \tag{16}$$

and noting that for a soft bremsstrahlung $p \simeq p'$, we obtain the emission intensity per frequency in noncommutative space to second order in γ as

$$\begin{aligned}\frac{dI_\theta}{d\omega} &= \frac{dI}{d\omega} + \gamma^2 \frac{dI'}{d\omega} \\ &= \frac{16}{3}\frac{Z^2e^6}{mv^2}\log\Big(\frac{2mv^2}{\omega}\Big) + \frac{Z^2e^6}{3}\Big[\frac{m^2v^2}{\omega} - \frac{\omega}{v^2}\log\Big(\frac{2mv^2}{\omega}\Big)\Big]\gamma^2.\end{aligned} \tag{17}$$

Therefore, we observe that the intensity in usual space, *i.e.* equation (10), is corrected by the term arising from the noncommutativity of space implying an increase in the intensity cross section. In $\gamma \to 0$ limit, *i.e.* when the noncommutativity of space disappears, equation (17) reduces to (10) and one recovers the standard textbook result.

REFERENCES

1. J. Gamboa, M. Loewe, F. Mendez and J. C. Rojas, Mod. Phys. Lett. A **16**, 2075 (2001).
2. M. Demetrian, D. Kochan, Acta Physica Slovaca **52**, (2002).
3. J. Gamboa, M. Loewe, F. Mendez and J.C. Rojas, Phys. Rev. D **64**, 067901 (2001).
4. L. Mezincescu, *Star Operation in Quantum Mechanics*, arXiv:hep-th/0007046.
5. J. Gamboa, M. Loewe, F. Mendez and J.C. Rojas, Int. J. Mod. Phys. A **17**, 2555 (2002).

6. A.E.F. Djemai, H. Smail, Commun.Theor. Phys. **41**, 837 (2004).
7. C. Acatrinei, JHEP **0109**, 007 (2001).
8. I. Jabbari, A. Jahan, Z. Riazi, Turk. J. Phys **33**, 149 (2009).
9. A. Jahan, Braz. J. Phys. **38**, 144 (2008).
10. A. Alavi, Mod. Phys. Lett. **A20**, 1013 (2005).
11. A. Jahan, Europhys. Lett **78**, 50001 (2007).
12. V.B. Berestetskii, E.M. Lifshitz, L.P. Pitaevskii, *Quantum Electrodynamics* (2nd edn., Pergamon Press, Oxford, 1982).
13. A.I. Akhiezer, V.B. Berestetskii, *Quantum Electrodynamics* (2nd edn., John Wiley & Sons, New York, 1965).
14. T. Padmanabhan, *Theoretical Astrophysics* (vol. 1, Cambridge University Press, Cambridge, 2000).