A NOTE ON THE COVARIANT QUANTIZATION OF SELF-DUAL p-FORMS*

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Massive self-duals p-forms (p odd) are analysed from the point of view of the Hamiltonian path integral quantization in the framework of the gauge unfixing approach.

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In this paper the massive self-dual p-forms (p odd) [1] are analysed from the point of view of the Hamiltonian path integral quantization. The quantization procedure is based on the construction of a first-class system equivalent with the original second-class theory and then quantizing the resulting first-class system. The construction of the equivalent first-class system is achieved using the gauge unfixing method [2–6].

1. GAUGE UNFIXING METHOD

The starting point is a bosonic dynamic system with the phase-space locally parametrized by n canonical pairs $z^a = \left(q^i,\, p_i\right)$, endowed with the canonical Hamiltonian H_c and subject to the purely second-class constraints

$$\chi_{\alpha_0}\left(z^a\right) \approx 0, \qquad \alpha_0 = \overline{1, 2M_0}. \tag{1}$$

Assume that one can split the second-class constraint set (1) into two subsets

$$\chi_{\alpha_0}\left(z^a\right) \equiv \left(G_{\bar{\alpha}_0}\left(z^a\right), C^{\bar{\beta}_0}\left(z^a\right)\right) \approx 0, \qquad \bar{\alpha}_0, \bar{\beta}_0 = \overline{1, M_0}. \tag{2}$$

such that

$$\left[G_{\bar{\alpha}_0}, G_{\bar{\beta}_0}\right] = D^{\bar{\gamma}_0}_{\bar{\alpha}_0\bar{\beta}_0} G_{\bar{\gamma}_0}. \tag{3}$$

Relations (3) yield the subset

$$G_{\bar{\alpha}_0}\left(z^a\right) \approx 0 \tag{4}$$

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to be first-class. The second-class behaviour of the overall constraint set ensures that

$$C^{\bar{\alpha}_0}(z^a) \approx 0 \tag{5}$$

may be regarded as some gauge-fixing conditions for this first-class set.

We introduce an operator $\hat{X}$ [7] that associates an application $\hat{X}F$ with every smooth function F on the original phase-space

$$\hat{X}F = F - C^{\bar{\alpha}_0}[G_{\bar{\alpha}_0}, F] + \frac{1}{2}C^{\bar{\alpha}_0}C^{\bar{\beta}_0}\left[G_{\bar{\alpha}_0}, \left[G_{\bar{\beta}_0}, F\right]\right] - \cdots, \tag{6}$$

such that it is in strong involution with the functions of the first-class constraints subset

$$\left[\hat{X}F, G_{\bar{\alpha}_0}\right] = 0. \tag{7}$$

With the help of this operator we construct a first-class Hamiltonian

$$H_{GU} = \hat{X}H_c \tag{8}$$

with respect to the first-class constraints subset.

If we denote by $\mathcal{S}_O$ and $\mathcal{S}_{GU}$ the original and respectively the gauge-unfixed system, then they are classically equivalent since they possess the same number of physical degrees of freedom

$$\mathcal{N}_O = \frac{1}{2}(2n - 2M_0) = \mathcal{N}_{GU}, \tag{9}$$

and the corresponding algebras of classical observables are isomorphic

$$Phys(\mathcal{S}_O) = Phys(\mathcal{S}_{GU}). \tag{10}$$

Consequently, the two systems also become equivalent at the level of the path integral quantization and we can replace [8] the Hamiltonian path integral of the original second-class theory

$$Z_O = \int \mathcal{D}(z^a, \lambda^{\alpha_0}) \det\left(\left[G_{\bar{\alpha}_0}, C^{\bar{\beta}_0}\right]\right) \exp\left[\mathrm{i}\int dt\left(\dot{q}^i p_i - H_c - \lambda^{\alpha_0}\chi_{\alpha_0}\right)\right] \tag{11}$$

with that associated with the gauge unfixing first-class system

$$\begin{aligned} Z_{GU} = \int \mathcal{D}\left(z^a, \lambda^{\bar{\alpha}_0}\right)\left(\prod_{\bar{\alpha}_0}\delta\left(C^{\bar{\alpha}_0}\right)\right)\left(\det\left(\left[G_{\bar{\alpha}_0}, C^{\bar{\beta}_0}\right]\right)\right) \\ \times \exp\left[\mathrm{i}\int dt\left(\dot{q}^i p_i - H_{GU} - \lambda^{\bar{\alpha}_0}G_{\bar{\alpha}_0}\right)\right]. \end{aligned} \tag{12}$$

1.1. THE CONSTRUCTION OF THE FIRST-CLASS SYSTEM

Self-dual p-forms are described by the Lagrangian action [1]

$$S = \int d^{2p+1}x\left(-\alpha\varepsilon_{\mu_1\ldots\mu_{2p+1}}F^{\mu_1\ldots\mu_{p+1}}A^{\mu_{p+2}\ldots\mu_{2p+1}} - \frac{m^2}{2p!}A_{\mu_1\ldots\mu_p}A^{\mu_1\ldots\mu_p}\right), \tag{13}$$

The canonical analysis [9] displays the constraints [10]

$$\chi^{(1)i_1\ldots i_{p-1}} \equiv \pi^{0i_1\ldots i_{p-1}} \approx 0, \tag{14}$$

$$\chi^{(2)}_{i_1\ldots i_{p-1}} \equiv 2p\partial^i\pi_{ii_1\ldots i_{p-1}} - \tfrac{m^2}{(p-1)!}A_{0i_1\ldots i_{p-1}} \approx 0, \tag{15}$$

$$\chi^{i_1\ldots i_p} \equiv \pi^{i_1\ldots i_p} + \alpha(p+1)\varepsilon^{0i_1\ldots i_p j_1\ldots j_p}A_{j_1\ldots j_p} \approx 0, \tag{16}$$

and the canonical Hamiltonian

$$H_c = \int d^{2p}x \left(-2pA_{0i_1\ldots i_{p-1}}\partial_i\pi^{ii_1\ldots i_{p-1}} + \tfrac{m^2}{2p!}A_{\mu_1\ldots\mu_p}A^{\mu_1\ldots\mu_p}\right), \tag{17}$$

Eliminating the second-class constraints (16) (the coordinates of the reduced phase-space are $A_{i_1\ldots i_p}$, $A_{0i_1\ldots i_{p-1}}$ and $\pi_{0i_1\ldots i_{p-1}}$) we are left with a system subject to the second-class constraints

$$C^{i_1\ldots i_{p-1}} \equiv \pi^{0i_1\ldots i_{p-1}} \approx 0, \tag{18}$$

$$G_{i_1\ldots i_{p-1}} \equiv \tfrac{1}{m^2}\left(2\alpha p!\varepsilon_{0i_1\ldots i_{2p}}\partial^{[i_p}A^{i_{p+1}\ldots i_{2p}]} + m^2A_{0i_1\ldots i_{p-1}}\right) \approx 0, \tag{19}$$

while the canonical Hamiltonian (17) takes the form

$$\bar{H}_c = \int d^{2p}x \left(2\alpha pA_{0i_1\ldots i_{p-1}}\varepsilon^{0i_1\ldots i_{2p}}\partial_{[i_p}A_{i_{p+1}\ldots i_{2p}]} + \tfrac{m^2}{2p!}A_{\mu_1\ldots\mu_p}A^{\mu_1\ldots\mu_p}\right). \tag{20}$$

According to the gauge unfixing method we consider (19) as the first-class constraint set and the remaining constraints (18) as the corresponding canonical gauge conditions. The first-class Hamiltonian with respect to (19) follows from relation (6)

$$\begin{aligned} H_{GU} = \int d^{2p}x \,\Big[& 2\alpha pA_{0i_1\ldots i_{p-1}}\varepsilon^{0i_1\ldots i_{p-1}j_1\ldots j_{p+1}}\partial_{[j_1}A_{j_2\ldots j_{p+1}]} \\ &+ \tfrac{m^2}{2p!}A_{\mu_1\ldots\mu_p}A^{\mu_1\ldots\mu_p} + \tfrac{1}{p}\left(A_{i_1\ldots i_p} + \tfrac{1}{m^2}\tfrac{(p-1)!}{2}\partial_{[i_1}\pi_{i_2\ldots i_p]0}\right)\partial^{[i_1}\pi^{i_2\ldots i_p]0}\Big]. \end{aligned} \tag{21}$$

2. COVARIANT PATH INTEGRAL FOR THE FIRST-CLASS SYSTEM

In order to obtain a manifestly Lorentz-covariant path integral we pass to another first-class system equivalent with the original second-class one at both classical and path integral levels.

2.1. STÜCKELBERG-LIKE COUPLING

We supplement (19) with the constraints

$$G_{i_1\ldots i_{p-2}} \equiv \tfrac{m}{(p-2)!}\partial^jG_{ji_1\ldots i_{p-2}} \approx 0, \tag{22}$$

such that the new constraints set

$$G_{i_1\ldots i_{p-1}} \equiv \tfrac{1}{m^2}\left(2\alpha p!\varepsilon_{0i_1\ldots i_{2p}}\partial^{[i_p}A^{i_{p+1}\ldots i_{2p}]}+m^2A_{0i_1\ldots i_{p-1}}\right)\approx 0, \tag{23}$$

$$G_{i_1\ldots i_{p-2}} \equiv \tfrac{m}{(p-2)!}\partial^j A_{0ji_1\ldots i_{p-2}}\approx 0, \tag{24}$$

remains first-class and becomes off-shell reducible of order $(p-1)$. Performing the canonical transformation

$$A_{0i_1\ldots i_{p-1}}\to \tfrac{(p-1)!}{m}\Pi_{i_1\ldots i_{p-1}},\quad \pi^{0i_1\ldots i_{p-1}}\to -\tfrac{m}{(p-1)!}B^{i_1\ldots i_{p-1}}, \tag{25}$$

the constraints (23) and (24) become

$$G_{i_1\ldots i_{p-1}} \equiv \tfrac{(p-1)!}{m^2}\left(2\alpha p\varepsilon_{0i_1\ldots i_{2p}}\partial^{[i_p}A^{i_{p+1}\ldots i_{2p}]}+mA_{0i_1\ldots i_{p-1}}\right)\approx 0, \tag{26}$$

$$G_{i_1\ldots i_{p-2}} \equiv (p-1)\,\partial^j A_{0ji_1\ldots i_{p-2}}\approx 0, \tag{27}$$

while the first-class Hamiltonian (21) takes the form

$$\begin{aligned} H_{GU} = \int d^{2p}x\,\Big[&\tfrac{2\alpha p!}{m}\Pi_{i_1\ldots i_{p-1}}\varepsilon^{0i_1\ldots i_{p-1}j_1\ldots j_{p+1}}\partial_{[j_1}A_{j_2\ldots j_{p+1}]}\\ &+\tfrac{(p-1)!}{2}\Pi_{i_1\ldots i_{p-1}}\Pi^{i_1\ldots i_{p-1}}\\ &+\tfrac{1}{2p!}\left(\partial_{[i_1}B_{i_2\ldots i_p]}-mA_{i_1\ldots i_p}\right)\left(\partial^{[i_1}B^{i_2\ldots i_p]}-mA^{i_1\ldots i_p}\right)\Big]. \end{aligned} \tag{28}$$

The argument of the exponential from the Hamiltonian path integral of the reducible first-class system reads as

$$\begin{aligned} S_{GU} = \int d^{2p+1}x\,\Big[&-\alpha\,(p+1)\left(\partial_0 A_{i_1\ldots i_p}\right)\varepsilon^{0i_1\ldots i_p j_1\ldots j_p}A_{j_1\ldots j_p}\\ &+\left(\partial_0 B_{i_1\ldots i_{p-1}}\right)\Pi^{i_1\ldots i_{p-1}}+\tfrac{(p-1)!}{2}\Pi_{i_1\ldots i_{p-1}}\Pi^{i_1\ldots i_{p-1}}\\ &-\tfrac{1}{2\cdot p!}\left(\partial_{[i_1}B_{i_2\ldots i_p]}-mA_{i_1\ldots i_p}\right)\left(\partial^{[i_1}B^{i_2\ldots i_p]}-mA^{i_1\ldots i_p}\right)\\ &-\tfrac{1}{(p-1)!}\bar{\lambda}^{i_1\ldots i_{p-1}}\left(2\alpha p!\varepsilon_{0i_1\ldots i_{p-1}j_1\ldots j_{p+1}}\partial^{[j_1}A^{j_2\ldots j_{p+1}]}+m\,(p-1)!\Pi_{i_1\ldots i_{p-1}}\right)\\ &-(p-1)\,\lambda^{i_1\ldots i_{p-2}}\left(\partial^j\Pi_{ji_1\ldots i_{p-2}}\right)\Big], \end{aligned} \tag{29}$$

where

$$\bar{\lambda}^{i_1\ldots i_{p-1}} = \tfrac{(p-1)!}{m^2}\left(\lambda^{i_1\ldots i_{p-1}}+m\Pi^{i_1\ldots i_{p-1}}\right). \tag{30}$$

Performing some partial integrations over all the momenta and making the notations

$$\bar{\lambda}^{i_1\ldots i_{p-1}}\equiv \bar{A}^{i_1\ldots i_{p-1}0},\quad \lambda_{i_1\ldots i_{p-2}}\equiv B_{i_1\ldots i_{p-2}0}, \tag{31}$$

we get to the argument of the exponential in the form

$$S_{GU} = \int d^{2p+1}x \left[-\alpha (p+1) \alpha \varepsilon^{0 i_1 \dots i_p j_1 \dots j_p} \bar{F}_{0 i_1 \dots i_p} A_{j_1 \dots j_p} \right.$$
$$- \alpha p \varepsilon^{i_1 \dots i_{p+1} 0 j_1 \dots j_{p-1}} F_{i_1 \dots i_{p+1}} \bar{A}_{0 j_1 \dots j_{p-1}}$$
$$- \tfrac{1}{2(p-1)!} \left(F_{0 i_1 \dots i_{p-1}} - m \bar{A}_{0 i_1 \dots i_{p-1}} \right) \left(F^{0 i_1 \dots i_{p-1}} - m \bar{A}^{0 i_1 \dots i_{p-1}} \right)$$
$$\left. - \tfrac{1}{2p!} \left(F_{i_1 \dots i_p} - m A_{i_1 \dots i_p} \right) \left(F^{i_1 \dots i_p} - m A^{i_1 \dots i_p} \right) \right], \quad (32)$$

where

$$\bar{F}_{0 i_1 \dots i_p} = \partial_0 A_{i_1 i_2 i_p} - \partial_{[i_1} \bar{A}_{i_2 \dots i_p] 0}, \quad (33)$$

$$F_{i_1 \dots i_p} = \partial_{[i_1} B_{i_2 \dots i_p]}, \quad F_{0 i_1 \dots i_{p-1}} = \partial_0 B_{i_1 \dots i_{p-1}} + \partial_{[i_1} B_{i_2 \dots i_{p-1}] 0}, \quad (34)$$

The functional (32) associated with the reducible first-class system takes a manifestly Lorentz-covariant form

$$S_{GU} = \int d^{2p+1}x \left[-\alpha \varepsilon^{\mu_1 \dots \mu_{2p+1}} \bar{F}_{\mu_1 \dots \mu_{p+1}} \bar{A}_{\mu_{p+2} \dots \mu_{2p+1}} \right.$$
$$\left. - \tfrac{1}{2p!} \left(\partial_{[\mu_1} B_{\mu_2 \dots \mu_p]} - m \bar{A}_{\mu_1 \dots \mu_p} \right) \left(\partial^{[\mu_1} B^{\mu_2 \dots \mu_p]} - m \bar{A}^{\mu_1 \dots \mu_p} \right) \right], \quad (35)$$

and describes a (Lagrangian) Stückelberg-like coupling between the $(p-1)$-form $B_{\mu_1 \dots \mu_{p-1}}$ and the p-form $\bar{A}_{\mu_1 \dots \mu_p}$ [11, 12].

2.2. CHERN-SIMONS-LIKE COUPLING

Starting from the first-class system (the gauge unfixing system) constructed in the above we arrive to another first-class theory whose field spectrum comprise two types of p-form gauge fields. We consider the following field combinations

$$\mathcal{F}_{i_1 \dots i_p} = A_{i_1 \dots i_p} + \tfrac{(p-1)!}{m^2} \partial_{[i_1} \pi_{i_2 \dots i_p] 0}, \quad \mathcal{F}_{0 i_1 \dots i_{p-1}} = A_{0 i_1 \dots i_{p-1}}, \quad (36)$$

which are in (strong) involution with first-class constraints (19)

$$\left[\mathcal{F}_{i_1 \dots i_p}, G_{j_1 \dots j_{p-1}} \right] = \left[\mathcal{F}_{0 i_1 \dots i_{p-1}}, G_{j_1 \dots j_{p-1}} \right] = 0. \quad (37)$$

By direct computation we obtain that $\mathcal{F}_{\mu_1 \dots \mu_p}$ satisfy the equation

$$\partial^\nu \partial_{[\nu} \mathcal{F}_{\mu_1 \dots \mu_p]} = - \tfrac{m^4}{4\alpha^2 (p!)^2 [(p+1)!]^2} \mathcal{F}_{\mu_1 \dots \mu_p} + \mathcal{O}\left(G_{i_1 \dots i_{p-1}} \right), \quad (38)$$

and it is divergence-less

$$\partial^\nu \mathcal{F}_{\nu \mu_1 \dots \mu_{p-1}} = 0, \quad (39)$$

on the first-class surface $G_{i_1 \dots i_{p-1}} \approx 0$.

Based on the gauge invariance and divergence-less of the $\mathcal{F}_{\mu_1 \dots \mu_p}$ we introduce a p-form $V_{\mu_1 \dots \mu_p}$ through the relation

$$\mathcal{F}_{\mu_1 \dots \mu_p} = \tfrac{1}{(p+1)!} \varepsilon_{\mu_1 \dots \mu_p \nu_1 \dots \nu_{p+1}} \partial^{[\nu_1} V^{\nu_2 \dots \nu_{p+1}]}. \quad (40)$$

Consequently, we enlarge the phase-space by adding the bosonic fields/momenta $\{V^{\nu_1\dots\nu_p}, P_{\nu_1\dots\nu_p}\}$. When we replace (40) in (19) the constraints set takes the form

$$\tfrac{1}{m^2}\varepsilon_{0i_1\dots i_{p-1}j_1\dots j_{p+1}}\left(2\alpha p!\partial^{[j_1}A^{i_2\dots i_{p+1}]}+\tfrac{m^2}{(p+1)!}\partial^{[j_1}V^{j_2\dots j_{p+1}]}\right)\approx 0, \tag{41}$$

remains first-class and becomes reducible of order $(p-1)$.

$$\begin{matrix} the\,old\,phase-space & the\,new\,phase-space \\ \left\{\begin{matrix} A_{i_1\dots i_p} \\ A_{0i_1\dots i_{p-1}}, \quad \pi^{0i_1\dots i_{p-1}}\end{matrix}\right\} & \left\{\begin{matrix} A_{i_1\dots ip} \\ V_{\mu_1\dots\mu_p}, \quad P^{\mu_1\dots\mu_p}\end{matrix}\right\}\end{matrix} \tag{42}$$

The number of physical degrees of freedom will be conserved if we impose the constraints

$$P_{0i_1\dots i_{p-1}}\approx 0, \qquad -p\partial^j P_{ji_1\dots i_{p-1}}\approx 0. \tag{43}$$

Constraints (41) and (43) are first-class and reducible of order $(p-1)$.

The first-class Hamiltonian takes the form

$$\begin{aligned} H'_{\rm GU} = \int d^{2p}x \Big[& \tfrac{1}{(p-1)!(p+1)!}\varepsilon^{0i_1\dots i_{p-1}j_1\dots j_{p+1}}\partial_{[j_1}V_{j_2\dots j_{p+1}]} \\ & \times\varepsilon_{0i_1\dots i_{p-1}k_1\dots k_{p+1}}\left(2p!\partial^{[k_1}A^{k_2\dots k_{p+1}]}+\tfrac{m^2}{(p+1)!}\partial^{[k_1}V^{k_2\dots k_{p+1}]}\right) \\ & -\tfrac{m^2}{2(p+1)!}\partial_{[i_1}V_{i_2\dots i_{p+1}]}\partial^{[i_1}V^{i_2\dots i_{p+1}]}+\tfrac{m^2}{2p!}A_{i_1\dots i_p}A^{i_1\dots i_p} \\ & -\tfrac{1}{p!}\varepsilon_{0i_1\dots i_pj_1\dots j_p}A^{i_1\dots i_p}P^{j_1\dots j_p}+\tfrac{p!}{2m^2}P_{i_1\dots i_p}P^{i_1\dots i_p}\Big]. \end{aligned}$$

The first-class Hamiltonian (44) outputs the argument of the exponential from the Hamiltonian path integral of the reducible first-class system as

$$S'_{GU}=\int d^{2p+1}x\left[-\alpha(p+1)\left(\partial_0 A_{i_1\dots i_p}\right)\varepsilon^{0i_1\dots i_pj_1\dots j_p}A_{j_1\dots j_p}\right. \tag{44}$$

$$+\left(\partial_0 V_{\mu_1\dots\mu_p}\right)P^{\mu_1\dots\mu_p}-\mathcal{H}'^{(odd)}_{GU} \tag{45}$$

$$-\lambda^{(1)i_1\dots i_{p-1}}P_{0i_1\dots i_{p-1}}+p\lambda^{(2)i_1\dots i_{p-1}}\partial^j P_{ji_1\dots i_{p-1}} \tag{46}$$

$$-\tfrac{1}{m^2}\lambda^{i_1\dots i_{p-1}}\varepsilon_{0i_1\dots i_{p-1}j_1\dots j_{p+1}} \tag{47}$$

$$\left.\times\left(2\alpha p!\partial^{[j_1}A^{j_2\dots j_{p+1}]}+\tfrac{m^2}{(p+1)!}\partial^{[j_1}V^{j_2\dots j_{p+1}]}\right)\right], \tag{48}$$

After integrating out the auxiliary fields and performing some field redefinitions, we obtain that the argument of the exponential takes a manifestly Lorentz-covariant form

$$\begin{aligned} S'_{GU}=\int d^{2p+1}x\Big(& -\alpha\varepsilon^{\mu_1\dots\mu_{2p+1}}\bar{F}_{\mu_1\dots\mu_{p+1}}\bar{A}_{\mu_{p+2}\dots\mu_{2p+1}} \\ & +\tfrac{1}{2(p+1)!}\partial_{[\mu_1}V_{\mu_2\dots\mu_{p+1}]}\partial^{[\mu_1}V^{\mu_2\dots\mu_{p+1}]} \\ & -\tfrac{m}{p!(p+1)!}\varepsilon^{\mu_1\dots\mu_{2p+1}}\partial_{[\mu_1}V_{\mu_2\dots\mu_{p+1}]}\bar{A}_{\mu_{p+2}\dots\mu_{2p+1}}\Big) \end{aligned} \tag{49}$$

and describes a Chern-Simons-like coupling between the p-forms $\bar{A}_{\mu_1\ldots\mu_p}$ and $\bar{V}_{\mu_1\ldots\mu_p}$ [11, 12].

3. CONCLUSIONS

Using gauge unfixing method starting from the original second-class theory, self-dual p-forms, we constructed an equivalent first-class theory. The Hamiltonian path integral of the first-class system takes a manifestly Lorentz-covariant form after integrating out the auxiliary fields and performing some field redefinitions. In order to obtain a manifestly covariant path integral we enlarge the original phase-space. For different kinds of phase-space extensions we identify the Lagrangian path integral for $(p-1)$ and p-forms with Stückelberg-like coupling or the Lagrangian path integral for two kinds of p-forms with Chern-Simons-like coupling.

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