THE QUANTUM UNIVERSE*

A. NICOLAIDIS

Theoretical Physics Department, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece
E-mail: nicolaid@auth.gr

Received February 20, 2013

We indicate that relational logic, established by C. S. Peirce in the years 1870–1880, may serve as the common foundation of quantum mechanics and string theory. A relation may be represented by a spinor and the Cartan-Penrose connection of spinor to geometry, allows to abstract geometry from a calculus of relations-spinors. With a single spinor related to the null cone of Minkowski space-time, we search for the geometry emerging when we entangle a left-handed spinor and a right-handed spinor. We find that the quantum entanglement generates an extra dimension with two branes coexisting in the extra dimension. One brane hosts left-handed particles (our brane), while the other brane hosts right-handed particles. A distinct phenomenology accompanies our proposal. The left-right symmetry is achieved with having two "mirror" branes and the neutrino appears as the ideal mediator between the branes. Our scheme brings closer logic – quantum theory – cosmology, while space-time, rather than an abstract and an a priori construction, appears as the outcome of a quantum logical act.

1. INTRODUCTION

Everything is under evolution. The entire universe appears as an ever-changing entity, with distinct stages in evolution. Ordinary matter and radiation, stars, galaxies, clusters of galaxies emerge as parts of an unfolding cosmic evolution. Next to the cosmological evolution, the biological evolution follows and finally the human language and culture appears. Do we dispose the necessary tools to understand the pivotal process of universal evolution? Can we detect and study the different modes of evolution? What kind of theory will bind very different processes together and will reproduce the observed time scales? Should we revisit the fundamental notions of space and time and seek for a dynamical emergence of geometry and time?

In another direction, there is the unification drive in physics, the effort of putting together within a unified framework distinct theoretical approaches. Along the unification of terrestrial and celestial Mechanics (Newton in the $17^{\rm th}$ century), of optics with the theories of electricity and magnetism (Maxwell in the $19^{\rm th}$ century),

*Paper presented at "The $8^{\rm th}$ Workshop on Quantum Field Theory and Hamiltonian Systems", September 19-22, 2012, Craiova, Romania.

Rom. Journ. Phys., Vol. 58, Nos. 5-6, P. 630–640, Bucharest, 2013

of spacetime geometry and gravity (Einstein in 1916), the next step in this process is the unification of quantum mechanics and general relativity. String theory, in this respect, appears as a promising example of unification. Not unrelated is the quest for the foundations and the internal architecture of a theory. While for Einstein's relativity theory we are guided by the equivalence principle, we are lacking a foundational principle for either quantum mechanics or string theory.

The logic of a physical theory reflects the structure of the propositions describing the physical system under study. The propositional logic of classical mechanics is Boolean logic, which is based on set theory. A set theory is deprived of any structure, being a plurality of structure-less individuals, qualified only by membership (or non-membership). Accordingly a set-theoretic enterprise is analytic, atomistic, arithmetic. It was noticed as early as 1936 by Neumann and Birkhoff that the quantum real needs a non-Boolean logical structure. On numerous cases the need for a novel system of logical syntax is evident. Quantum measurement bypasses the old disjunctions subject-object, observer-observed. The observer affects the system under observation and the borderline between ontological and epistemological is blurred. Correlations are not any more local and a quantum system embodies multiple entanglements. The particular-universal dichotomy is also under revision. While a single quantum event is particular, a plethora of quantum events leads to universal patterns. Viewing the quantum system as a system encoding information, we understand that the usual distinction between hardware and software is not relevant. Most importantly, if we consider the opposing terms being-becoming, we realize that the emphasis is sifted to the becoming, the movement, the process. The underlying dynamics is governed by relational principles and we have suggested [1] that the relational logic of C. S. Peirce may serve as the conceptual foundation of QM.

Peirce, the founder of American pragmatism, made important contributions in science, philosophy, semeiotics and notably in logic. Many scholars (Clifford, Schrder, Whitehead, Lukasiewicz) rank Peirce with Leibniz and Aristotle in the history of thought. Logic, in its most general sense, is the formal science of representation, coextensive with semeiotics. Algebraic logic attempts to express the laws of thought in the form of mathematical equations, and Peirce incorporated a theory of relations into algebraic logic [2, 3]. Relation is the primary irreducible datum and everything is expressed in terms of relations. A relational formulation is bound to be synthetic, holistic, geometric. Peirce invented also a notation for quantifiers and developed quantification theory, thus he is regarded as one of the principal founders of modern logic.

In the next section we present the structure of the relational logic and its affinity to category theory. In the third section we show that relational logic may serve as the foundation of quantum mechanics. In the fourth section we indicate how the algebra of relations leads to string theory. In the final section we indicate that a relation may be represented by a spinor and drawing from the Cartan-Penrose connection of spinors to geometry, we study the geometrical structures consistent with the entanglement of quantum spinors.

2. RELATIONAL LOGIC

The starting point is the binary relation $S_i R S_j$ between the two 'individual terms' (subjects) S_j and S_i. In a short hand notation we represent this relation by R_{ij}. Relations may be composed: whenever we have relations of the form R_{ij}, R_{jl}, a third transitive relation R_{il} emerges following the rule [2, 3]

$$R_{ij} R_{kl} = \delta_{jk} R_{il}. \tag{1}$$

In ordinary logic the individual subject is the starting point and it is defined as a member of a set. Peirce, in an original move, considered the individual as the aggregate of all its relations

$$S_i = \sum_j R_{ij}. \tag{2}$$

It is easy to verify that the individual S_i thus defined is an eigenstate of the R_{ii} relation

$$R_{ii} S_i = S_i. \tag{3}$$

The relations R_{ii} are idempotent

$$R_{ii}^2 = R_{ii} \tag{4}$$

and they span the identity

$$\sum_i R_{ii} = \mathbf{1}. \tag{5}$$

The Peircean logical structure bears great resemblance to category theory, a remarkably rich branch of mathematics developed by Eilenberg and Maclane in 1945 [4]. In categories the concept of transformation (transition, map, morphism or arrow) enjoys an autonomous, primary and irreducible role. A category [5] consists of objects A, B, C,... and arrows (morphisms) f, g, h,... . Each arrow f is assigned an object A as domain and an object B as codomain, indicated by writing $f : A \rightarrow B$. If g is an arrow $g : B \rightarrow C$ with domain B, the codomain of f, then f and g can be "composed" to give an arrow $gof : A \rightarrow C$. The composition obeys the associative law $ho(gof) = (hog)of$. For each object A there is an arrow $1_A : A \rightarrow A$ called the identity arrow of A. The analogy with the relational logic of Peirce is evident, R_{ij} stands as an arrow, the composition rule is manifested in Eq. (1) and the identity arrow for $A \equiv S_i$ is R_{ii}. There is an important literature on possible ways the category notions can be applied to physics; specifically to quantising space-time [6], attaching a formal language to a physical system [7], studying topological quantum field theories [8, 9], exploring quantum issues and quantum information theory [10].

A relation R_{ij} may receive multiple interpretations: as the proof of the logical proposition i starting from the logical premise j, as a transition from the j state to the i state, as a measurement process that rejects all impinging systems except those in the state j and permits only systems in the state i to emerge from the apparatus. We proceed to a representation of R_{ij}

$$R_{ij} = |r_i\rangle \langle r_j| \tag{6}$$

where state $\langle r_i|$ is the dual of the state$|r_i\rangle$ and they obey the orthonormal condition

$$\langle r_i| \, r_j\rangle = \delta_{ij}. \tag{7}$$

It is immediately seen that our representation satisfies the composition rule Eq. (1). The completeness, Eq.(5), takes the form

$$\sum_i |r_i\rangle \langle r_i| = \mathbf{1}. \tag{8}$$

All relations remain satisfied if we replace the state $|r_i\rangle$ by $|\varrho_i\rangle$, where

$$|\varrho_i\rangle = \frac{1}{\sqrt{N}} \sum_n |r_i\rangle \langle r_n| \tag{9}$$

with N the number of states. Thus we verify Peirce's suggestion, Eq. (2), and the state $|r_i\rangle$ is derived as the sum of all its interactions with the other states. R_{ij} acts as a projection, transferring from one r state to another r state

$$R_{ij} |r_k\rangle = \delta_{jk} |r_i\rangle . \tag{10}$$

3. QUANTUM MECHANICS

Next to the above states characterized by an r-ness property, we may think also of another property characterizing our states, say q-ness, and define a corresponding operator

$$Q_{ij} = |q_i\rangle \langle q_j| \tag{11}$$

with

$$Q_{ij} |q_k\rangle = \delta_{jk} |q_i\rangle \tag{12}$$

and

$$\sum_i |q_i\rangle \langle q_i| = \mathbf{1}. \tag{13}$$

The individual states can be redefined

$$|r_i\rangle \quad \rightarrow \quad e^{i\varphi_i}\,|r_i\rangle \tag{14}$$

$$|q_i\rangle \quad \rightarrow \quad e^{i\theta_i}\,|q_i\rangle \tag{15}$$

without affecting the corresponding composition laws. However the overlap number $\langle r_i|\, q_j\rangle$ changes and therefore we need an invariant formulation for the transition $|r_i\rangle \rightarrow |q_j\rangle$. This is provided by the trace of the closed operation $R_{ii}Q_{jj}R_{ii}$

$$Tr\,(R_{ii}Q_{jj}R_{ii}) \equiv p\,(q_j,r_i) = |\langle r_i|\, q_j\rangle|^2\,. \tag{16}$$

The completeness relation, Eq. (13), guarantees that $p\,(q_j,r_i)$ may assume the role of a probability since

$$\sum_j p\,(q_j,r_i) = 1. \tag{17}$$

Thus we obtain the probability rule of quantum mechanics.

We know that quantum mechanics is characterized by the non-commutativity of conjugate operators. Consider a chain of N discrete states $|a_k\rangle$, with $k = 1,2,\ldots,N$. A relation R acts like a shift operator

$$R\,|a_k\rangle \quad = \quad |a_{k+1}\rangle \tag{18}$$

$$R\,|a_N\rangle \quad = \quad |a_1\rangle \tag{19}$$

N is the period of R

$$R^N = \mathbf{1}. \tag{20}$$

The numbers which satisfy $a^N = 1$ are given by

$$a_k = \exp\left(2\pi i\frac{k}{N}\right), \quad k = 1,2,\ldots,N \tag{21}$$

R has a set of eigenfunctions

$$R\,|b_i\rangle = b_i\,|b_i\rangle \tag{22}$$

with b_i the N-th root of unity $(b_i = a_i)$.

We introduce another relation Q acting like shift operator

$$\langle b_k|\,Q \quad = \quad \langle b_{k+1}| \tag{23}$$

$$\langle b_N|\,Q \quad = \quad \langle b_1|\,. \tag{24}$$

We may prove that the conjugate operators R and Q do not commute [11]

$$QR = \exp\left[2\pi i\frac{1}{N}\right]RQ. \tag{25}$$

In our discrete model the non-commutativity is determined by N. Let us define

$$L = Na, \qquad p = \frac{2\pi}{L}. \tag{26}$$

Then

$$\exp\left[2\pi i \frac{1}{N}\right] = \exp\left[ipa\right]. \tag{27}$$

What counts is the size of the available phase space and we may use Planck's constant h as a unit measuring the number of phase space cells. Using rather $\exp\left[\frac{i}{\hbar}pa\right]$, Eq.(25) becomes

$$QR = \exp\left[\frac{i}{\hbar}pa\right] RQ. \tag{28}$$

Approaching the continuum we may replace the discrete operators by exponential forms [11]

$$R = \exp\left[\frac{i}{\hbar}pX\right] \tag{29}$$

$$Q = \exp\left[\frac{i}{\hbar}aP\right]. \tag{30}$$

With R and Q unitary operators, X and P are hermitian operators. From Eqs. (28), (29), (30), we deduce

$$[X,P] = i\hbar. \tag{31}$$

The foundational non-commutative law of Quantum Mechanics testifies that there is a limit size $\hbar \sim pa$ in dividing the phase space. We discover that starting from the relational logic of Peirce we obtain all the essential laws of quantum mechanics.

4. STRING THEORY

For the general case of N available states the R_{ij} satisfy the W_∞ algebra

$$[R_{ij}, R_{kl}] = \delta_{jk}R_{il} - \delta_{li}R_{kj}. \tag{32}$$

The W_∞ algebras are bosonic extensions of the Virasoro algebra, containing generating currents of higher conformal-spin, in addition to the spin-2 stress tensor of Virasoro (for a review see [12]). They are linked to the area-preserving diffeomorphisms of two dimensional surfaces [13, 14]. W_∞ symmetries are exhibited by a number of systems, among them, QCD_2 [15, 16], gravity in two-dimensions [17], bosonic string in four-dimensional Minkowski space [18]. We may proceed to a pictorial representation of the

operation R_{ij}. Each distinct state i is represented by a specific line (solid, dashed,...), with a downward (upward) arrow attached to the annihilated (created) state. In this sense we picture R_{12} by a double line, Fig. 1, while the composition rule, for example

$$R_{12}R_{21} = R_{11}, \tag{33}$$

is represented by the diagram of Fig. 2. The similarity with string theory, string joining and string splitting, is obvious. The "cubic-string" interaction may be repeated

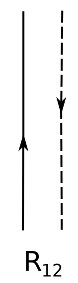

Fig. 1 – The relation R_{12}. Solid (dashed) line stands for the state 1 (2). A downward (upward) arrow is attached to an impinging (emerging) state.

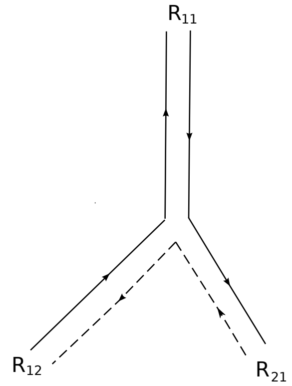

Fig. 2 – Pictorial representation of the composition rule $R_{12}R_{21} = R_{11}$

an indefinite number of times, with vertices connected together and giving rise to different forms of polygons (see Fig. 3). These types of structures can be generated by a random matrix model [19]

$$Z = \int [dM] exp\{-Ntr(\frac{1}{2}M^2 + gM^3)\} \tag{34}$$

where M are $N \times N$ random matrices. A perturbative expansion of this integral leads to 't Hooft-type Feynman diagrams with cubic vertices. Each such diagram specifies a unique surface topology, with faces arbitrary n-gons. The corresponding dual lattice has n lines meeting at a point but the faces are triangles. The result is a triangulated Riemann surface (Fig. 3). We observe that the algebra of relations gives rise to a discrete string theory. At the same time we notice the affinity between quantum mechanics and string theory, since both share the same inner syntax.

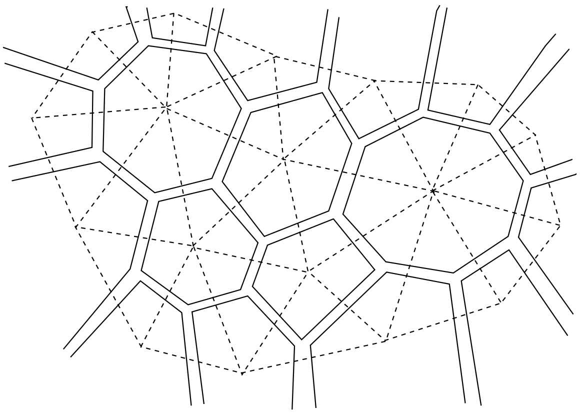

Fig. 3 – Random partition of a surface. Each triangle (dashed lines) is dual to a cubic vertex.

5. THE EMERGENCE OF A QUANTUM UNIVERSE

Let us consider the simplified case of only two states (i=1,2). We define

$$R_z = \frac{1}{2}(R_{11} - R_{22}) \tag{35}$$

and

$$R_+ = R_{12} \quad R_- = R_{21}. \tag{36}$$

These operators satisfy the SU(2) commutation relations

$$[R_z, R_\pm] = \pm R_\pm \quad [R_+, R_-] = 2R_z \tag{37}$$

and the quadratic Casimir operator

$$R^2 = R_z^2 + \frac{1}{2}(R_+R_- + R_-R_+) \tag{38}$$

can be written as

$$R^2 = \frac{1}{2}(\frac{1}{2}+1)\mathbf{1}. \tag{39}$$

The underlying dynamics is analogous to an “angular momentum $1/2$ particle” [20, 21]. Considering that the answer to a logical proposition is a “yes” or a “no” statement, analogous to a “spin up” or a “spin down” measurement, we conclude that we may represent a proposition-relation by a spinor.

The profound connection between spinors and geometry was established a hundred years ago by Cartan, who introduced spinors as representation of the rotation group [22]. Penrose used spinor as the building block of discrete space-time and as a powerful tool to study physics issues [23]. We further extended the Cartan-Penrose argument. If a single spinor, represented by a point on a Riemann sphere, is connected to the Minkowski null cone, we may search for the geometry emerging when two propositions-spinors are entangled.

We consider a Dirac-type entanglement involving a left-handed Weyl spinor and a right-handed Weyl spinor. Writing

$$|\chi_L\rangle = \begin{pmatrix} a \\ b \end{pmatrix} \qquad |\psi_R\rangle = \begin{pmatrix} c \\ d \end{pmatrix} \tag{40}$$

and

$$|\Psi_D\rangle = \begin{pmatrix} |\chi_L\rangle \\ |\psi_R\rangle \end{pmatrix} \tag{41}$$

we obtain

$$X_1 = \langle \Psi_D |\gamma_1| \Psi_D\rangle = a^*b + b^*a - c^*d - d^*c \tag{42}$$

$$X_2 = \langle \Psi_D |\gamma_2| \Psi_D\rangle = i\,(-a^*b + b^*a + c^*d - d^*c) \tag{43}$$

$$X_3 = \langle \Psi_D |\gamma_3| \Psi_D\rangle = |a|^2 - |b|^2 - |c|^2 + |d|^2 \tag{44}$$

$$X_0 = \langle \Psi_D |\gamma_0| \Psi_D\rangle = -\left(|a|^2 + |b|^2\right) - \left(|c|^2 + |d|^2\right). \tag{45}$$

The quantity $X_1^2 + X_2^2 + X_3^2 - X_0^2$ is not anymore zero. We obtain [27]

$$X_1^2 + X_2^2 + X_3^2 - X_0^2 \equiv -M_D^2 = -4\,|(a^*c + b^*d)|^2 . \tag{46}$$

With

$$X_4 = i\,\langle \Psi_D | \Psi_D\rangle = -2\,Im\,(a^*c + b^*d)$$

$$X_5 = \langle \Psi_D |\gamma_5| \Psi_D\rangle = -2\,Re\,(a^*c + b^*d)$$

we find

$$M_D^2 = \left(X_4^2 + X_5^2\right). \tag{47}$$

The absence of entanglement and the re-establishement of the Riemann-Bloch sphere is obtained, when the condition

$$a^*c + b^*d = 0 \tag{48}$$

is fulfilled.

Let us define $T = X_0$, $t = M_D$. The Dirac entanglement, Eq.(46), takes then the form of a space-like hyperboloid

$$T^2 - \sum_{i=1}^{3} X_i^2 = t^2. \tag{49}$$

A comparison with the null cone geometry, indicates that quantum entanglement, specified and quantified by t, generates an extra dimension. The distance along this extra dimension indicates how far we are from the null cone. Furthermore our space-time acquires a double-sheet structure, reminding the ekpyrotic model where two branes coexist [24–26]. There is though a distinct difference. In our model, by construction, one brane hosts left-handed particles (our brane), while the other brane hosts right-handed particles.

For a specific quantum spinor the amount of entanglement t is fixed. Still we may imagine that we "tune" t and consider t as a continuous variable. A varying t will offer us a continuous foliation of the internal space of the null cone. We can adopt a reparametrization, satisfying automatically Eq.(49) and leading us to the four independent dynamical degrees of freedom

$$T = t\cosh\rho, \quad X_i = n_i t\sinh\rho, \quad \sum_{i=1}^{3} n_i^2 = 1. \tag{50}$$

The induced metric becomes then

$$\begin{aligned} ds^2 &= dT^2 - \sum_{i=1}^{3} dX_i^2 \\ &= dt^2 - t^2(d\rho)^2 - t^2\sinh^2\rho\, d\Omega_2^2 \end{aligned} \tag{51}$$

representing a Milne universe [27].

Our approach explores alternative ways to study space-time geometries, next to the renown Einstein's general relativity. Geometry and the number of dimensions are not abstract, mathematical constructions, but the outcome of a quantum event. We may gain fresh insight by studying these geometries, consistent with quantum entanglement, and analyze the phenomenological implications. One of the most important issues is the communication between the two branes, and the neutrino appears as the best mediator. Work along these lines is in progress [28].

Acknowledgments. The paper was presented during the workshop "Quantum Field Theory and Hamiltonian Systems", held in Craiova (September 2012) and I would like to thank Prof. Radu Constantinescu for a warm hospitality. The work is supported by the Templeton Foundation.

REFERENCES

1. A. Nicolaidis, Int. J. Mod. Phys. A **24**, 1175–1183 (2009).
2. C.S. Peirce, Memoirs Amer. Acad. Sci. **9**, 317–378 (1870).
3. C.S. Peirce, Amer. J. Math. **3**, 15–57 (1880).
4. S. Eilenberg, S. Maclane, Trans. Am. Math. Soc. **58**, 239–294 (1945).
5. F. Lawvere, S. Schanuel, "*Conceptual Mathematics: A first indroduction to categories*" (Cambridge University Press, Cambridge, 1997).
6. C. Isham, Adv. Theor. Math. Phys. **7**, 331–367 (2003).
7. A. Doring, C. Isham, J. Math. Phys. **49**, 053515 (2008)
8. J. Baez, J. Dolan, J. Math. Phys. **36**, 6073–6105 (1995).

9. J. Baez, "*Quantum quandaries: A category-theoretic perspective*", arXiv:quant-ph/0404040, to appear in "*Structural Foundations of Quantum Gravity*" (Oxford University Press, Oxford, 2013).
10. S. Abramsky, B. Coecke, "*A categorical sematics of quantum protocols*", in "*Proceedings of the 19th IEEE conference on Logic in Computer Science (LiCS'04)*" (IEEE Computer Science Press, 2004).
11. A. Nicolaidis, "*Relational Quantum Mechanics*", arXiv:1211.2706 (2012).
12. X. Shen, Int. J. Mod. Phys. A **7**, 6953-6994 (1992).
13. I. Bakas, Phys. Lett. B **228**, 57–63 (1989).
14. E. Witten, Nucl. Phys. B **373**, 187–213 (1992).
15. S. Wadia, "*W_∞ algebra and geometric formulation of QCD_2*", arXiv:hep-th/9411213 (1994).
16. A. Dhar, G. Mandal, S. Wadia, Phys. Lett. B **329**, 15–26 (1994).
17. A. Polyakov, Mod. Phys. Lett. A **2**, 893–898 (1987).
18. I. Bakas, Phys. Lett. B **319**, 457–465 (1993).
19. P. Ginsparg, "*Matrix models of 2d gravity*", arXiv:hep-th/9112013 (1991).
20. J. Schwinger, "*On Angular Momentum*", in "*Quantum Theory of Angular Momentum*", eds. L.C. Biedenharn and H. Van Dam, p. 229 (Academic Press, New York, 1965).
21. J.J. Sakurai, "*Modern Quantum Mechanics*", p. 217 (Addison-Wesley, New York, 1990).
22. E. Cartan, Bull. Soc. Math. France **41**, 53–96 (1913); E. Cartan, "*Leçons sur la théorie des spineurs vols. 1 and 2*" (Hermann, Paris, 1938).
23. R. Penrose, Ann. Phys, **10**, 171–201 (1960); R. Penrose, W. Rindler, "*Spinors and space - time, vol. 1*" (Cambridge University Press, Cambridge, 1984).
24. Justin Khoury, Burt A. Ovrut, Nathan Seiberg, Paul J. Steinhardt, Neil Turok, Phys. Rev. D **65**, 086007 (2002).
25. Paul J. Steinhardt, Neil Turok, Phys. Rev. D **65**, 126003 (2002).
26. Justin Khoury, Paul J. Steinhardt, Neil Turok, Phys. Rev. Lett. **92**, 031302 (2004).
27. A. Nicolaidis, V. Kiosses, "*Quantum Entanglement on Cosmological Scale*", arXiv:1211.0356 (2012).
28. A. Nicolaidis, "*The neutrinos from the neighboring brane*", to appear.