NEW COSMOLOGICAL SOLUTIONS IN NONLOCAL MODIFIED GRAVITY*

IVAN DIMITRIJEVIC[1], BRANKO DRAGOVICH[2], JELENA GRUJIC[3], ZORAN RAKIC [1]

[1]Faculty of Mathematics, University of Belgrade, Studentski trg 16, Belgrade, Serbia
E-mail: ivand@matf.bg.ac.rs
[2]Institute of Physics, University of Belgrade, Pregrevica 118, 11080 Zemun, Belgrade, Serbia
E-mail: dragovich@ipb.ac.rs
[3]Teachers Training Faculty, University of Belgrade, Kraljice Natalije 43, Belgrade, Serbia
E-mail: jelenagg@gmail.com

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We consider some cosmological aspects of nonlocal modified gravity with Λ term, where nonlocality is of the type $R\mathcal{F}(\Box)R$. Using ansatz of the form $\Box R = rR + s$, we find a few $a(t)$ nonsingular bounce cosmological solutions for all three values of spatial curvature parameter k. We also discuss this modified gravity model from $F(R)$ theory point of view.

Key words: Modified gravity, nonlocal gravity, $F(R)$ theory, cosmological solutions.

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1. INTRODUCTION

Modern theory of gravity is general theory of relativity, which was founded by Einstein at the end of 1915 and has been successfully confirmed for the Solar System. It is given by the Einstein equations of motion for gravitational field $g_{\mu\nu}$: $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$,which can be derived from the Einstein-Hilbert action $S = \frac{1}{16\pi G}\int\sqrt{-g}Rd^4x + \int\sqrt{-g}\mathcal{L}_{mat}d^4x$, where $g = det(g_{\mu\nu})$ and $c = 1$.

Attempts to modify Einstein's theory of gravity started already at its very beginning and it was mainly motivated by investigation of possible mathematical generalizations. During last decade there has been an intensive activity in gravity modification, motivated by discovery of accelerating expansion of the Universe, which has not yet generally accepted theoretical explanation. If Einstein's gravity is theory of gravity for the Universe as a whole then it has to be some new kind of matter with negative pressure, called *dark energy*, which is responsible for the accelerated Universe expansion. However, general relativity has not been verified at the cosmic

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scale (low curvature regime) and dark energy has not been confirmed in a laboratory. Hence, after discovery of the accelerated Universe there emerged a renewed interest in modification of Einstein's theory of gravity, which should be some kind of its generalization (for a recent review of various approaches, see [1]). However there is not a unique way how to modify the Einstein-Hilbert action. Among many approaches there are two of them, which have attracted more interest than the others: 1) $F(R)$ theories of gravity (for a review, see [2, 3]) and 2) nonlocal gravities (see, e.g. [3–9] and references therein).

In the case of $F(R)$ gravity, the Ricci scalar R in the action is replaced by a function $F(R)$. This has been extensively investigated for the various forms of function $F(R)$.

In the sequel we shall consider some cosmological aspects of a nonlocal gravity. Here, nonlocality means that Lagrangian contains an infinite number of space-time derivatives, i.e. derivatives up to an infinitive order in the form of d'Alembert operator $\Box$ which is argument of an analytic function. In string theory nonlocality emerges as a consequence of extendedness of strings. Since string theory contains gravity, as well as other kinds of interaction and matter, it is natural to expect nonlocality not only in the matter sector but also in geometrical sector of gravity. So the main motivation to consider an additional nonlocal term in the Einstein-Hilbert action comes from the string theory. On some developments in cosmology with nonlocality in the matter sector one can see, e.g., [10–14] and references therein. In the next sections we shall discuss a nonlocal modification of only geometry sector of gravity and its corresponding new cosmological solutions.

2. A NONLOCAL MODIFICATION OF GRAVITY

In our consideration, nonlocal modification of gravity is a replacement of the Ricci curvature R in the Einstein-Hilbert action by a suitable function $F(R,\Box)$, where $\Box = \frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu$ is d'Alembert-Beltrami operator.

In this paper we consider nonlocal gravity model without matter, given by the action in the form

$$S = \int d^4x\sqrt{-g}\Big(\frac{R-2\Lambda}{16\pi G} + \frac{C}{2}R\mathcal{F}(\Box)R\Big), \tag{1}$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n\Box^n$ is an analytic function of the d'Alembert-Beltrami operator and C is a constant. Study of this model (1) was proposed in [4] and some further developments are presented in [5–8]. This model is attractive because it is ghost free and has some non-singular bounce solutions, which can solve the Big Bang cosmological singularity problem.

By variation of the action (1) with respect to metric $g_{\mu\nu}$ one obtains the corresponding equation of motion:

$$C\Big(2R_{\mu\nu}\mathcal{F}(\Box)R - 2(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)(\mathcal{F}(\Box)R) - \frac{1}{2}g_{\mu\nu}R\mathcal{F}(\Box)R + \sum_{n=1}^{\infty}\frac{f_n}{2}\sum_{l=0}^{n-1}\big(g_{\mu\nu}\Big(g^{\alpha\beta}\partial_\alpha\Box^l R\partial_\beta\Box^{n-1-l}R + \Box^l R\Box^{n-l}R\Big) - 2\partial_\mu\Box^l R\partial_\nu\Box^{n-1-l}R\big)\Big) = \frac{-1}{8\pi G}(G_{\mu\nu} + \Lambda g_{\mu\nu}). \tag{2}$$

The trace and 00 component of (2) are:

$$6\Box(\mathcal{F}(\Box)R) + \sum_{n=1}^{\infty}f_n\sum_{l=0}^{n-1}\Big(\partial_\mu\Box^l R\partial^\mu\Box^{n-1-l}R + 2\Box^l R\Box^{n-l}R\Big) = \frac{1}{8\pi GC}R - \frac{\Lambda}{2\pi GC}, \tag{3}$$

$$C\Big(2R_{00}\mathcal{F}(\Box)R - 2(\nabla_0\nabla_0 - g_{00}\Box)(\mathcal{F}(\Box)R) - \frac{1}{2}g_{00}R\mathcal{F}(\Box)R + \sum_{n=1}^{\infty}\frac{f_n}{2}\sum_{l=0}^{n-1}\big(g_{00}\Big(g^{\alpha\beta}\partial_\alpha\Box^l R\partial_\beta\Box^{n-1-l}R + \Box^l R\Box^{n-l}R\Big) - 2\partial_0\Box^l R\partial_0\Box^{n-1-l}R\big)\Big) = \frac{-1}{8\pi G}(G_{00} + \Lambda g_{00}). \tag{4}$$

We use Friedmann-Lemaître-Robertson-Walker (FLRW) metric $ds^2 = -dt^2 + a^2(t)\big(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\big)$ and investigate all three possibilities for curvature parameter k $(0, \pm 1)$.

3. ANSATZ AND SOLUTIONS

Investigation of equation (2) and finding its solutions is a very difficult task. In the case of the FLRW flat metric $(k = 0)$ two non-singular cosmological solutions for the scale factor are found: $a(t) = a_0\cosh\left(\sqrt{\frac{\Lambda}{3}}t\right)$, see [4, 5], and $a(t) = a_0 e^{\frac{1}{2}\sqrt{\frac{\Lambda}{3}}t^2}$, see [7].

To get some new solutions we also use ansatz of the form

$$\Box R = rR + s, \tag{5}$$

proposed in [5], where r and s are real parameters that will be fixed later. The first two consequences of this ansatz are

$$\Box^n R = r^n (R + \frac{s}{r}), n \geq 1, \qquad \mathcal{F}(\Box) R = \mathcal{F}(r) R + \frac{s}{r}(\mathcal{F}(r) - f_0). \tag{6}$$

Now we can search for a solution of the scale factor $a(t)$ in the form of a linear combination of $e^{\lambda t}$ and $e^{-\lambda t}$, i.e.

$$a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t}), \quad 0 < a_0, \lambda, \sigma, \tau \in \mathbb{R}. \tag{7}$$

Then the corresponding expressions for the Hubble parameter $H(t) = \frac{\dot{a}}{a}$, scalar curvature $R(t) = \frac{6}{a^2}(a\ddot{a} + \dot{a}^2 + k)$ and $\Box R$ are:

$$\begin{aligned} H(t) &= \frac{\lambda(\sigma e^{\lambda t} - \tau e^{-\lambda t})}{\sigma e^{\lambda t} + \tau e^{-\lambda t}}, \\ R(t) &= \frac{6\left(2a_0^2\lambda^2\left(\sigma^2 e^{4t\lambda} + \tau^2\right) + k e^{2t\lambda}\right)}{a_0^2\left(\sigma e^{2t\lambda} + \tau\right)^2}, \\ \Box R &= -\frac{12\lambda^2 e^{2t\lambda}\left(4a_0^2\lambda^2\sigma\tau - k\right)}{a_0^2\left(\sigma e^{2t\lambda} + \tau\right)^2}. \end{aligned} \tag{8}$$

We can rewrite $\Box R$ as

$$\Box R = 2\lambda^2 R - 24\lambda^4, \qquad r = 2\lambda^2,\ s = -24\lambda^4. \tag{9}$$

Substituting parameters r and s from (9) into (6) we obtain

$$\begin{aligned} \Box^n R &= (2\lambda^2)^n (R - 12\lambda^2),\ n \geq 1, \\ \mathcal{F}(\Box) R &= \mathcal{F}(2\lambda^2) R - 12\lambda^2(\mathcal{F}(2\lambda^2) - f_0). \end{aligned} \tag{10}$$

Using this in (3) and (4) we obtain

$$\begin{aligned} &36\lambda^2 \mathcal{F}(2\lambda^2)(R - 12\lambda^2) + \mathcal{F}'(2\lambda^2)\left(4\lambda^2(R - 12\lambda^2)^2 - \dot{R}^2\right) \\ &- 24\lambda^2 f_0 (R - 12\lambda^2) = \frac{R - 4\Lambda}{8\pi G C}, \end{aligned} \tag{11}$$

$$\begin{aligned} &(2R_{00} + \frac{1}{2}R)\left(\mathcal{F}(2\lambda^2)R - 12\lambda^2(\mathcal{F}(2\lambda^2) - f_0)\right) - \frac{1}{2}\mathcal{F}'(2\lambda^2)\left(\dot{R}^2 + 2\lambda^2(R - 12\lambda^2)^2\right) \\ &- 6\lambda^2(\mathcal{F}(2\lambda^2) - f_0)(R - 12\lambda^2) + 6H\mathcal{F}(2\lambda^2)\dot{R} = -\frac{1}{8\pi G C}(G_{00} - \Lambda). \end{aligned} \tag{12}$$

Substituting $a(t)$ from (7) into equations (11) and (12) one obtains respectively the following two equations as polynomials in $e^{2\lambda t}$:

$$\begin{aligned} &\frac{a_0^4\tau^6}{4\pi G}\left(3\lambda^2 - \Lambda\right) + 3a_0^2\tau^4 Q_1 e^{2\lambda t} + 6a_0^2\sigma\tau^3 Q_2 e^{4\lambda t} - 2\sigma\tau Q_3 e^{6\lambda t} \\ &+ 6a_0^2\sigma^3\tau Q_2 e^{8\lambda t} + 3a_0^2\sigma^4 Q_1 e^{10\lambda t} + \frac{a_0^4\sigma^6}{4\pi G}\left(3\lambda^2 - \Lambda\right) e^{12\lambda t} = 0, \end{aligned} \tag{13}$$

$$\frac{\tau^6 a_0^4}{8\pi G}\left(3\lambda^2-\Lambda\right)+3\tau^4 a_0^2 R_1 e^{2\lambda t}+3\tau^2 R_2 e^{4\lambda t}+2\sigma\tau R_3 e^{6\lambda t}$$
$$+3\sigma^2 R_2 e^{8\lambda t}+3\sigma^4 a_0^2 R_1 e^{10\lambda t}+\frac{\sigma^6 a_0^4}{8\pi G}\left(3\lambda^2-\Lambda\right)e^{12\lambda t}=0, \quad (14)$$

where

$$Q_1=36C\lambda^2 K\mathcal{F}(2\lambda^2)+a_0^2(-96Cf_0\lambda^4+\frac{\lambda^2}{\pi G}-\frac{\Lambda}{2\pi G})\sigma\tau$$
$$+24Cf_0k\lambda^2+\frac{k}{8\pi G}, \quad (15)$$
$$Q_2=72C\lambda^2 K\mathcal{F}(2\lambda^2)+a_0^2(-192Cf_0\lambda^4+\frac{7\lambda^2}{8\pi G}-\frac{5\Lambda}{8\pi G})\sigma\tau$$
$$+48Cf_0k\lambda^2+\frac{k}{4\pi G}, \quad (16)$$
$$Q_3=-324Ca_0^2\lambda^2\sigma\tau K\mathcal{F}(2\lambda^2)+144C\lambda^2K^2\mathcal{F}'(2\lambda^2)$$
$$-a_0^2k(216Cf_0\lambda^2+\frac{9}{8\pi G})\sigma\tau+a_0^4(864Cf_0\lambda^4-\frac{3\lambda^2}{\pi G}+\frac{5\Lambda}{2\pi G})\sigma^2\tau^2, \quad (17)$$

$$R_1=Q_1-\frac{3\lambda^2-\Lambda}{4\pi G}\sigma\tau a_0^2, \quad (18)$$
$$R_2=-6C\left(k-12a_0^2\lambda^2\sigma\tau\right)K\mathcal{F}(2\lambda^2)-36C\lambda^2K^2\mathcal{F}'(2\lambda^2)$$
$$+\frac{a_0^2k}{2\pi G}\left(192\pi GCf_0\lambda^2+1\right)\sigma\tau-\frac{a_0^4}{8\pi G}\left(3072\pi GCf_0\lambda^4+\lambda^2+5\Lambda\right)\sigma^2\tau^2, \quad (19)$$
$$R_3=-18C\left(k-6a_0^2\lambda^2\sigma\tau\right)K\mathcal{F}(2\lambda^2)+36C\lambda^2K^2\mathcal{F}'(2\lambda^2)$$
$$+\frac{9a_0^2k}{8\pi G}\left(192\pi GCf_0\lambda^2+1\right)\sigma\tau-\frac{a_0^4}{4\pi G}\left(3456\pi GCf_0\lambda^4+3\lambda^2+5\Lambda\right)\sigma^2\tau^2, \quad (20)$$

and $K=4a_0^2\lambda^2\sigma\tau-k$.

Equations (13) and (14) are satisfied when $\lambda=\pm\sqrt{\frac{\Lambda}{3}}$, as well as $Q_1=Q_2=Q_3=0$ and $R_1=R_2=R_3=0$. Note that this approach to find conditions under which solution exists differs with respect to approach used in [5] and [7].

The corresponding solutions can split into the following three cases.

Case 1.

$$\mathcal{F}\left(2\lambda^2\right)=0, \quad \mathcal{F}'\left(2\lambda^2\right)=0, \quad f_0=-\frac{1}{64\pi GC\Lambda}. \quad (21)$$

Case 2.

$$3k=4a_0^2\Lambda\sigma\tau. \quad (22)$$

Case 3.

$$\mathcal{F}\left(2\lambda^2\right) = \frac{1}{96\pi GC\Lambda} + \frac{2}{3}f_0, \quad \mathcal{F}'\left(2\lambda^2\right) = 0, \quad k = -4a_0^2\Lambda\sigma\tau. \tag{23}$$

In the first case we have family of solutions for arbitrary σ, τ and a_0

$$a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})$$

with function $\mathcal{F}$ satisfying conditions given in (21) and arbitrary $k = 0, \pm 1$. Thus it includes also solution $a(t) = a_0 \cosh\left(\sqrt{\frac{\Lambda}{3}}t\right)$, which was found in [4] with addition of some radiation [5] in the action.

The second case yields a family of solutions for arbitrary $\sigma \neq 0$ and a_0

$$a(t) = a_0\left(\sigma e^{\lambda t} + \frac{3k}{4a_0^2\Lambda\sigma}e^{-\lambda t}\right)$$

which are valid for arbitrary analytic function $\mathcal{F}$.

The third case yields another family of solutions

$$a(t) = a_0\left(\sigma e^{\lambda t} - \frac{k}{4a_0^2\Lambda\sigma}e^{-\lambda t}\right)$$

and function $\mathcal{F}$ has to satisfy conditions given in (23).

Note that if $k = 0$ then equation (22) and third equation in (23) coincide, σ or τ has to be zero, and we have 2 solutions

$$a_1(t) = a_0 e^{\lambda t}, \qquad a_2(t) = a_0 e^{-\lambda t},$$

where σ is absorbed into a_0. These are the de Sitter solutions, see also [8].

4. $F(R)$ THEORY

Action (1) with ansatz (5) gives an $F(R)$ theory and we get another way to analyze the above solutions . The corresponding action for $F(R)$ is

$$S' = \int \frac{\sqrt{-g}}{16\pi G}F(R)d^4x, \qquad F(R) = \alpha R^2 + \beta R - 2\Lambda, \tag{24}$$

$$\alpha = 8\pi GC\mathcal{F}(2\lambda^2), \quad \beta = 1 - 96\pi GC\lambda^2(\mathcal{F}(2\lambda^2) - f_0). \tag{25}$$

By variation of the action (24) we get the following equation [2]:

$$F'(R)R_{\mu\nu} - \frac{1}{2}F(R)g_{\mu\nu} - [\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box]F'(R) = 8\pi GT_{\mu\nu}. \tag{26}$$

The generalized Friedmann equations are

$$\left(\frac{\dot{a}}{a}\right)^2+\frac{k}{a^2}=\frac{8\pi G}{3F'(R)}(\rho+\overline{\rho}),$$

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3F'(R)}(\rho+3p+\overline{\rho}+3\overline{p}).$$

From the equations of motion we derive expressions for effective density $\overline{\rho}$ and pressure $\overline{p}$

$$\begin{aligned}\overline{\rho}&=\frac{-1}{8\pi G}\Big(\frac{1}{2}F(R)-\frac{R}{2}F'(R)+3\frac{\dot{a}}{a}\dot{R}F''(R)\Big),\\ \overline{p}&=\frac{1}{8\pi G}\Big(\frac{1}{2}F(R)-\frac{R}{2}F'(R)-(3\frac{\dot{a}}{a}\dot{R}+\ddot{R})F''(R)-\dot{R}^2F'''(R)\Big).\end{aligned} \tag{27}$$

In the case of action S' we have

$$\begin{aligned}\overline{\rho}&=\frac{1}{8\pi G}\Big(\frac{\alpha}{2}R^2+\Lambda-6\alpha\frac{\dot{a}}{a}\dot{R}\Big),\\ \overline{p}&=\frac{-1}{8\pi G}\Big(\frac{\alpha}{2}R^2+\Lambda+2\alpha(3\frac{\dot{a}}{a}\dot{R}+\ddot{R})\Big),\\ \overline{w}&=\frac{\overline{p}}{\overline{\rho}}=-1-\frac{2\alpha(6\frac{\dot{a}}{a}\dot{R}+\ddot{R})}{\frac{\alpha}{2}R^2+\Lambda-6\alpha\frac{\dot{a}}{a}\dot{R}}.\end{aligned} \tag{28}$$

It seems to be natural that any $F(R)$ theory should satisfy [2]

$$F'(R)>0,\qquad -1.1\le\bar{w}\le-0.98. \tag{29}$$

In the *Case* 1, we have $\mathcal{F}\left(2\lambda^2\right)=0$ and consequently $\alpha=0$ and therefore $F'(R)=\beta=\frac{1}{2}$, $\bar{w}=-1$.

In the *Case* 2, we have a solution of the form $a(t)=a_0(\sigma e^{\lambda t}+\tau e^{-\lambda t})$, where the constants a_0, σ, τ, λ are subject to the constraints

$$\lambda^2=\frac{1}{3}\Lambda,\quad 3k=4a_0^2\Lambda\sigma\tau. \tag{30}$$

From this set of parameters we see that

$$R(t)=4\Lambda,\qquad \overline{w}=-1. \tag{31}$$

In the *Case* 3, we obtain

$$\alpha=\frac{1}{12\Lambda}+\frac{16\pi GC}{3}f_0,\quad \beta=\frac{2}{3}(1+16\pi GC\Lambda f_0), \tag{32}$$

and

$$\overline{w}=-1+h(k,u,v), \tag{33}$$

where

$$h(k,u,v)=\frac{512ku\left(k^2+12ku+144u^2\right)(v+1)}{(k^4+20736u^4)V_1+(48k^3u+6912ku^3)V_2+288k^2u^2V_3}, \qquad (34)$$

$$V_1=2v+5, \quad V_2=14v+11, \quad V_3=70v+79, \qquad (35)$$

$$u=\frac{1}{3}a_0^2\Lambda\sigma^2e^{2\sqrt{\frac{\Lambda}{3}}t}, \quad v=64\pi GCf_0\Lambda. \qquad (36)$$

Obviously, in the case $k=0$ we have $\overline{w}=-1$. Note that $h(-1,u,v)=h(1,-u,v)$. Hence it is sufficient to analyze only $h(1,u,v)$ and it will be presented elsewhere with some other properties of this $F(R)$ theory.

5. DISCUSSION AND CONCLUDING REMARKS

In this paper we have considered a nonlocal gravity model with cosmological constant Λ and without matter. We found three types of non-singular bouncing solutions for cosmological scale factor in the form $a(t)=a_0(\sigma e^{\lambda t}+\tau e^{-\lambda t})$. Solutions exist for all three values of spatial curvature constant $k=0,\pm1$. All these solutions depend on cosmological constant Λ, which is here an arbitrary positive parameter.

Note that trace equation (11) can be rewritten in the form

$$A_1R+A_2(4\lambda^2R^2-\dot{R}^2)+A_3=0, \qquad (37)$$

where

$$A_1=-\frac{1}{8\pi GC}+12\lambda^2(3\mathcal{F}(2\lambda^2)-2f_0)-96\lambda^4\mathcal{F}'(2\lambda^2), \qquad (38)$$

$$A_2=\mathcal{F}'(2\lambda^2), \qquad (39)$$

$$A_3=\frac{\Lambda}{2\pi GC}-144\lambda^4(3\mathcal{F}(2\lambda^2)-2f_0)+576\lambda^6\mathcal{F}'(2\lambda^2). \qquad (40)$$

From $A_1=A_2=A_3=0$ one obtains the following system of equations:

$$12\lambda^2(3\mathcal{F}(2\lambda^2)-2f_0)=\frac{1}{8\pi GC}, \qquad \mathcal{F}'(2\lambda^2)=0, \qquad (41)$$

$$144\lambda^4(3\mathcal{F}(2\lambda^2)-2f_0)=\frac{\Lambda}{2\pi GC}. \qquad (42)$$

Finally, we get

$$\lambda^2=\frac{\Lambda}{3}, \quad \mathcal{F}(2\lambda^2)=\frac{1}{96\Lambda\pi GC}+\frac{2}{3}f_0, \quad \mathcal{F}'(2\lambda^2)=0, \qquad (43)$$

and this is related to the corresponding conditions in [5] and to the first two equations of our *Case* 3. However in *Case* 3 there is additional condition $k=-4a_0^2\Lambda\sigma\tau$, which does not permit solution of hyperbolic cosine type when $k=0$. In [4,5] this problem was solved adding some radiation in 00 equation of motion.

Note that in the above solutions for the scale factor one can write more general expression by replacement $t \to t - t_0$, *i.e.*

$$a(t) = a_0(\sigma e^{\lambda(t-t_0)} + \tau e^{-\lambda(t-t_0)}). \tag{44}$$

When $\sigma > 0$ and $\tau > 0$, and $t_0 = \frac{1}{2\lambda}\ln(\frac{\sigma}{\tau})$ then (44) can be rewritten as

$$a(t) = 2a_0\sqrt{\sigma\tau}\cosh(\lambda t). \tag{45}$$

If $\tau < 0 < \sigma$ and $t_0 = \frac{1}{2\lambda}\ln(-\frac{\sigma}{\tau})$ then the scale factor (44) becomes

$$a(t) = 2a_0\sqrt{-\sigma\tau}\sinh(\lambda t). \tag{46}$$

Note that one can construct a solution $a(t)$ for a negative value of Λ. In this article we used linear ansatz (5), but some other ansätze [15] can be also useful in search of new cosmological solutions. Here we considered a nonlocal gravity model without matter, however modern cosmology will probably need both – modified gravity and an exotic matter (see, e.g. zeta strings [16] and p-adic matter [17]).

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