CONSISTENT INTERACTIONS BETWEEN DUAL FORMULATIONS OF LINEARIZED GRAVITY

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A particular case of consistent interactions of a single massless tensor field with the mixed symmetry corresponding to a two-column Young diagram $(k,1)$, dual to linearized gravity in $D = k+3$, namely $k = 4$, is considered in the context of cross-interactions with another dual formulation of linearized gravity in terms of a massless tensor field with the mixed symmetry of the linearized Riemann tensor. The general approach relies on the deformation of the solution to the master equation from the antifield-BRST formalism by means of the local cohomology of the BRST differential.

Key words: BRST symmetry, consistent interactions, dual formulations of linearized gravity.

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1. INTRODUCTION

The purpose of this work is to investigate the consistent interactions between a single massless tensor field with the mixed symmetry $(4,1)$ and one massless tensor field with the mixed symmetry $(2,2)$. Our analysis relies on the deformation of the solution to the master equation [1] by means of cohomological techniques with the help of the local BRST cohomology [2–4], whose component in the $(4,1)$ can be solved along the same line like in [5] or [6] and in the $(2,2)$ sector has been investigated in [7, 8]. Apart from the duality of the massless tensor field with the mixed symmetry $(4,1)$ to the Pauli–Fierz theory (linearized limit of Einstein–Hilbert gravity) in $D = 7$ dimensions, it is interesting to mention the developments concerning the dual formulations of linearized gravity from the perspective of M-theory [9–11]. On the other hand, the massless tensor field with the mixed symmetry $(2,2)$ displays all the algebraic properties of the Riemann tensor, describes purely spin-two particles, and also provides a dual formulation of linearized gravity in $D = 5$. Actually, there is a revived interest in the construction of dual gravity theories, which led to

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several new results, viz. a dual formulation of linearized gravity in first order tetrad formalism in arbitrary dimensions within the path integral framework [12] or a reformulation of non-linear Einstein gravity in terms of the dual graviton together with the ordinary metric and a shift gauge field [13].

The method from [1] has been widely used in the literature at the construction of various interacting models, such as BF models [14], tensor fields of degree two [15], or $D = 11$ SUGRA [16].

Under the hypotheses of analyticity, locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the requirement that the interaction vertices contain at most two spatiotemporal derivatives of the fields, we prove that there exists a case where the deformation of the solution to the master equation provides non-trivial cross-couplings. This case corresponds to a seven-dimensional spacetime and is described by a deformed solution that stops at order two in the coupling constant. In this way we establish a new result, namely that dual linearized gravity in $D = 7$ gets coupled to a purely spin-two field with the mixed symmetry of the Riemann tensor. The interacting Lagrangian action contains only mixing-component terms of order one and two in the coupling constant. Both the gauge transformations and first-order reducibility functions of the tensor field $(4,1)$ are modified at order one in the coupling constant with terms characteristic to the $(2,2)$ sector. On the contrary, the tensor field with the mixed symmetry $(2,2)$ remains rigid at the level of both gauge transformations and reducibility functions. The gauge algebra and the reducibility structure of order two are not modified during the deformation procedure, being the same like in the case of the starting free action. It is interesting to note that if we require the PT invariance of the deformed theory, then no interactions occur. The results exposed here generalize the previous findings from [17] between a massless tensor field with the mixed symmetry $(3,1)$ and one with the mixed symmetry of the Riemann tensor.

2. FREE MODEL

We begin with a Lagrangian action describing a free massless tensor field with the mixed symmetry $(4,1)$ [18], $t_{\mu_1\mu_2\mu_3\mu_4|\alpha}$, and one with the mixed symmetry of the Riemann tensor, $r_{\mu_1\mu_2|\alpha_1\alpha_2}$

$$S^{\mathrm{L}}\left[t_{\mu_1\mu_2\mu_3\mu_4|\alpha}, r_{\mu_1\mu_2|\alpha_1\alpha_2}\right] = S^{\mathrm{L}}\left[t_{\mu_1\mu_2\mu_3\mu_4|\alpha}\right] + S^{\mathrm{L}}\left[r_{\mu_1\mu_2|\alpha_1\alpha_2}\right], \tag{1}$$

in $D \geq 6$ spatiotemporal dimensions, where

$$S^{\mathrm{L}}\left[t_{\mu_1\mu_2\mu_3\mu_4|\alpha}\right] = -\frac{1}{2\cdot 4!}\int d^D x\left[\left(\partial_\mu t_{\mu_1\mu_2\mu_3\mu_4|\alpha}\right)\partial^\mu t^{\mu_1\mu_2\mu_3\mu_4|\alpha}\right.$$
$$-\left(\partial^\alpha t_{\mu_1\mu_2\mu_3\mu_4|\alpha}\right)\partial_\beta t^{\mu_1\mu_2\mu_3\mu_4|\beta} - 4\left(\partial^\lambda t_{\lambda\mu_1\mu_2\mu_3|\alpha}\right)\partial_\rho t^{\rho\mu_1\mu_2\mu_3|\alpha}$$
$$-4\left(\partial_\mu t_{\mu_1\mu_2\mu_3}\right)\partial^\mu t^{\mu_1\mu_2\mu_3} - 8\left(\partial^\alpha t_{\mu_1\mu_2\mu_3\mu_4|\alpha}\right)\partial^{\mu_1} t^{\mu_2\mu_3\mu_4}$$
$$\left.+12\left(\partial^\lambda t_{\lambda\mu_1\mu_2}\right)\partial_\rho t^{\rho\mu_1\mu_2}\right], \tag{2}$$

and

$$S^{\mathrm{L}}\left[r_{\mu_1\mu_2|\alpha_1\alpha_2}\right] = \int d^D x\left[\frac{1}{8}\left(\partial^\lambda r^{\mu_1\mu_2|\alpha_1\alpha_2}\right)\left(\partial_\lambda r_{\mu_1\mu_2|\alpha_1\alpha_2}\right)\right.$$
$$-\frac{1}{2}\left(\partial_{\mu_1} r^{\mu_1\mu_2|\alpha_1\alpha_2}\right)\left(\partial^{\nu_1} r_{\nu_1\mu_2|\alpha_1\alpha_2}\right)$$
$$-\left(\partial_{\mu_1} r^{\mu_1\mu_2|\alpha_1\alpha_2}\right)\left(\partial_{\alpha_2} r_{\mu_2\alpha_1}\right) - \frac{1}{2}\left(\partial^{\mu_1} r^{\mu_2\alpha_1}\right)\left(\partial_{\mu_1} r_{\mu_2\alpha_1}\right)$$
$$\left.+\left(\partial_{\mu_1} r^{\mu_1\alpha_1}\right)\left(\partial^{\nu_1} r_{\nu_1\alpha_1}\right) - \frac{1}{2}\left(\partial_{\mu_1} r^{\mu_1\alpha_1}\right)\left(\partial_{\alpha_1} r\right) + \frac{1}{8}\left(\partial^{\mu_1} r\right)\left(\partial_{\mu_1} r\right)\right]. \tag{3}$$

Everywhere in this paper we employ the flat Minkowski metric of 'mostly plus' signature. The massless tensor field $t_{\mu_1\mu_2\mu_3\mu_4|\alpha}$ has the mixed symmetry $(4,1)$, and it is completely antisymmetric in its first 4 indices and satisfies the identity $t_{[\mu_1\mu_2\mu_3\mu_4|\alpha]} \equiv 0$. Here and in the sequel the notation $[\mu_1 \ldots \mu_n]$ signifies complete antisymmetry with respect to the (Lorentz) indices between brackets. The trace of $t_{\mu_1\mu_2\mu_3\mu_4|\alpha}$ is defined by $t_{\mu_1\mu_2\mu_3} = \sigma^{\mu_4\alpha} t_{\mu_1\mu_2\mu_3\mu_4|\alpha}$ and it is antisymmetric. The massless tensor field $r_{\mu_1\mu_2|\alpha_1\alpha_2}$ of degree 4 has the mixed symmetry of the linearized Riemann tensor, so it is separately antisymmetric in the pairs $\{\mu_1,\mu_2\}$ and $\{\alpha_1,\alpha_2\}$, symmetric under the interchange of these pairs, and satisfies the identity $r_{[\mu_1\mu_2|\alpha_1]\alpha_2} \equiv 0$ associated with the above diagram. The notation $r_{\mu_1\alpha_1}$ signifies the trace of the original tensor field, $r_{\mu_1\alpha_1} = \sigma^{\mu_2\alpha_2} r_{\mu_1\mu_2|\alpha_1\alpha_2}$, which is symmetric, while r denotes its double trace, which is a scalar.

A generating set of gauge transformations for action (1) can be taken of the form

$$\delta_{\chi,\epsilon} t_{\mu_1\mu_2\mu_3\mu_4|\alpha} = \partial_{[\mu_1}\chi_{\mu_2\mu_3\mu_4]|\alpha} + \partial_{[\mu_1}\epsilon_{\mu_2\mu_3\mu_4]\alpha} - 4\partial_\alpha \epsilon_{\mu_1\mu_2\mu_3\mu_4}, \tag{4}$$

$$\delta_\xi r_{\mu_1\mu_2|\alpha_1\alpha_2} = \partial_{\mu_1}\xi_{\alpha_1\alpha_2|\mu_2} - \partial_{\mu_2}\xi_{\alpha_1\alpha_2|\mu_1} + \partial_{\alpha_1}\xi_{\mu_1\mu_2|\alpha_2} - \partial_{\alpha_2}\xi_{\mu_1\mu_2|\alpha_1}. \tag{5}$$

The gauge parameters $\epsilon_{\mu_1\mu_2\mu_3\mu_4}$ are completely antisymmetric, $\chi_{\mu_1\mu_2\mu_3|\alpha}$ possess the mixed symmetry $(3,1)$, and $\xi_{\mu_1\mu_2|\alpha_1}$ display the mixed symmetry $(2,1)$, such that they are antisymmetric in their first 3 and respectively 2 indices and satisfy the identities $\chi_{[\mu_1\mu_2\mu_3|\alpha]} \equiv 0$ and $\xi_{[\mu_1\mu_2|\alpha_1]} \equiv 0$. The generating set of gauge transformations (4)–(5) is off-shell, third-order reducible, the accompanying gauge algebra being obviously Abelian.

3. FREE BRST SYMMETRY

The construction of the antifield-BRST symmetry for this free theory debuts with the identification of the algebra on which the BRST differential s acts. The generators of the BRST algebra are of two kinds: fields/ghosts and antifields

$$\left(t_{\mu_1\mu_2\mu_3\mu_4|\alpha}, r_{\mu_1\mu_2|\alpha_1\alpha_2}\right), \left(\overset{(1)}{C}_{\mu_1\mu_2\mu_3|\alpha}, \overset{(1)}{\eta}_{\mu_1\mu_2\mu_3\mu_4}, \mathcal{C}_{\mu_1\mu_2|\alpha}\right), \tag{6}$$

$$\left(\overset{(2)}{C}_{\mu_1\mu_2|\alpha}, \overset{(2)}{\eta}_{\mu_1\mu_2\mu_3}, \mathcal{C}_{\mu_1\mu_2}\right), \left(\overset{(3)}{C}_{\mu_1|\alpha}, \overset{(3)}{\eta}_{\mu_1\mu_2}\right), \left(\overset{(4)}{\eta}_{\mu_1}\right), \tag{7}$$

$$\left(t^{*\mu_1\mu_2\mu_3\mu_4|\alpha}, r^{*\mu_1\mu_2|\alpha_1\alpha_2}\right), \left(\overset{(1)}{C}{}^{*\mu_1\mu_2\mu_3|\alpha}, \overset{(1)}{\eta}{}^{*\mu_1\mu_2\mu_3\mu_4}, \mathcal{C}^{*\mu_1\mu_2|\alpha}\right), \tag{8}$$

$$\left(\overset{(2)}{C}{}^{*\mu_1\mu_2|\alpha}, \overset{(2)}{\eta}{}^{*\mu_1\mu_2\mu_3}, \mathcal{C}^{*\mu_1\mu_2}\right), \left(\overset{(3)}{C}{}^{*\mu|\alpha}, \overset{(3)}{\eta}{}^{*\mu_1\mu_2}\right), \left(\overset{(4)}{\eta}{}^{*\mu_1}\right). \tag{9}$$

The ghosts $\overset{(1)}{\eta}_{\mu_1\mu_2\mu_3\mu_4}$, $\overset{(1)}{C}_{\mu_1\mu_2\mu_3|\alpha}$, and $\mathcal{C}_{\mu_1\mu_2|\alpha}$ are all fermionic and respectively associated with the gauge parameters $\epsilon_{\mu_1\mu_2\mu_3\mu_4}$, $\chi_{\mu_1\mu_2\mu_3|\alpha}$, and $\xi_{\mu_1\mu_2|\alpha_1}$ and, meanwhile, display the same mixed symmetries like the corresponding parameters. The ghosts for ghosts $\overset{(2)}{\eta}_{\mu_1\mu_2\mu_3}$, $\overset{(2)}{C}_{\mu_1\mu_2|\alpha}$, and $\mathcal{C}_{\mu_1\mu_2}$ are all bosonic and due to the first-order reducibility relations. The first and the last are antisymmetric, while the second exhibits the mixed symmetry $(2,1)$. Regarding the ghosts for ghosts for ghosts $\overset{(3)}{C}_{\mu_1|\alpha}$ and $\overset{(3)}{\eta}_{\mu_1\mu_2}$, they are all fermionic, their presence being dictated by the second-order reducibility relations. The former is symmetric and the latter antisymmetric. Finally, the ghosts for ghosts for ghosts for ghosts $\overset{(4)}{\eta}_{\mu_1}$ are a consequence of the third-order reducibility specific to the $(4,1)$ sector. The antifields carry a star and there is one antifield for each field/ghost, of opposite statistics, but with the same mixed symmetry like the corresponding field/ghost.

Since both the gauge generators and reducibility functions for this model are field-independent, it follows that the BRST differential s simply reduces to

$$s = \delta + \gamma, \tag{10}$$

where δ represents the Koszul–Tate differential, graded by the antighost number agh ($\mathrm{agh}(\delta) = -1$), and γ stands for the exterior derivative along the gauge orbits, whose degree is named pure ghost number pgh ($\mathrm{pgh}(\gamma) = 1$). In this situation γ is a true differential. These two degrees do not interfere ($\mathrm{agh}\,(\gamma) = 0$, $\mathrm{pgh}\,(\delta) = 0$). The overall degree that grades the BRST complex is known as the ghost number (gh) and is defined like the difference between the pure ghost number and the antighost number, such that $\mathrm{gh}\,(s) = \mathrm{gh}\,(\delta) = \mathrm{gh}\,(\gamma) = 1$. According to the standard rules of the BRST method, the corresponding degrees of the generators from the BRST complex, (6)–(9), are valued like

$$\mathrm{pgh}\left(t_{\mu_1\mu_2\mu_3\mu_4|\alpha}\right) = 0 = \mathrm{pgh}\left(r_{\mu_1\mu_2|\alpha_1\alpha_2}\right), \tag{11}$$

$$\mathrm{pgh}\left(\overset{(1)}{C}_{\mu_1\mu_2\mu_3|\alpha}\right) = \mathrm{pgh}\left(\overset{(1)}{\eta}_{\mu_1\mu_2\mu_3\mu_4}\right) = \mathrm{pgh}\left(\mathcal{C}_{\mu_1\mu_2|\alpha}\right) = 1, \tag{12}$$

$$\mathrm{pgh}\left(\overset{(2)}{C}_{\mu_1\mu_2|\alpha}\right) = \mathrm{pgh}\left(\overset{(2)}{\eta}_{\mu_1\mu_2\mu_3}\right) = \mathrm{pgh}\left(\mathcal{C}_{\mu_1\mu_2}\right) = 2, \tag{13}$$

$$\mathrm{pgh}\left(\overset{(3)}{C}_{\mu_1|\alpha}\right) = 3 = \mathrm{pgh}\left(\overset{(3)}{\eta}_{\mu_1\mu_2}\right), \qquad \mathrm{pgh}\left(\overset{(4)}{\eta}_{\mu_1}\right) = 4, \tag{14}$$

$$\mathrm{agh}\left(t^{*\mu_1\mu_2\mu_3\mu_4|\alpha}\right) = 1 = \mathrm{agh}\left(r^{*\mu_1\mu_2|\alpha_1\alpha_2}\right), \tag{15}$$

$$\mathrm{agh}\left(\overset{(1)}{C}{}^{*\mu_1\mu_2\mu_3|\alpha}\right) = \mathrm{agh}\left(\overset{(1)}{\eta}{}^{*\mu_1\mu_2\mu_3\mu_4}\right) = \mathrm{agh}\left(\mathcal{C}^{*\mu_1\mu_2|\alpha}\right) = 2, \tag{16}$$

$$\mathrm{agh}\left(\overset{(2)}{C}{}^{*\mu_1\mu_2|\alpha}\right) = \mathrm{agh}\left(\overset{(2)}{\eta}{}^{*\mu_1\mu_2\mu_3}\right) = \mathrm{agh}\left(\mathcal{C}^{*\mu_1\mu_2}\right) = 3, \tag{17}$$

$$\mathrm{agh}\left(\overset{(3)}{C}{}^{*\mu|\alpha}\right) = 4 = \mathrm{agh}\left(\overset{(3)}{\eta}{}^{*\mu_1\mu_2}\right), \qquad \mathrm{agh}\left(\overset{(4)}{\eta}{}^{*\mu_1}\right) = 5. \tag{18}$$

It is understood that the missing degrees are all equal to zero.

Actually, (10) is a decomposition of the BRST differential according to the antighost number and it shows that s contains only components of antighost number equal to minus one and zero. The Koszul–Tate differential is imposed to realize a homological resolution of the algebra of smooth functions defined on the stationary surface of field equations, while the exterior longitudinal derivative is related to the gauge symmetries (see relations (4)–(5)) of action (1) through its cohomology at pure ghost number zero computed in the cohomology of δ, which is required to be the algebra of physical observables for the free model under consideration. The nontrivial actions of δ and γ on the generators from the BRST complex, which enforce all the above mentioned properties, are given by

$$\gamma t_{\mu_1\mu_2\mu_3\mu_4|\alpha} = \partial_{[\mu_1}\overset{(1)}{C}_{\mu_2\mu_3\mu_4]|\alpha} + \partial_{[\mu_1}\overset{(1)}{\eta}_{\mu_2\mu_3\mu_4\alpha]} - 5\partial_\alpha \overset{(1)}{\eta}_{\mu_1\mu_2\mu_3\mu_4}, \tag{19}$$

$$\gamma r_{\mu_1\mu_2|\alpha_1\alpha_2} = \partial_{\mu_1}\mathcal{C}_{\alpha_1\alpha_2|\mu_2} - \partial_{\mu_2}\mathcal{C}_{\alpha_1\alpha_2|\mu_1} + \partial_{\alpha_1}\mathcal{C}_{\mu_1\mu_2|\alpha_2} - \partial_{\alpha_2}\mathcal{C}_{\mu_1\mu_2|\alpha_1}, \tag{20}$$

$$\gamma \overset{(m)}{C}_{\mu_1\ldots\mu_{4-m}|\alpha} = \partial_{[\mu_1} \overset{(m+1)}{C}_{\mu_2\ldots\mu_{4-m}]|\alpha} + \partial_{[\mu_1} \overset{(m+1)}{\eta}_{\mu_2\ldots\mu_{4-m}\alpha]} \tag{21}$$

$$+(-)^{5-m}(5-m)\,\partial_\alpha \overset{(m+1)}{\eta}_{\mu_1\ldots\mu_{4-m}},\ m=1,2,\ \gamma \overset{(3)}{C}_{\mu_1|\alpha} = \partial_{(\mu_1} \overset{(4)}{\eta}_{\alpha)}, \tag{22}$$

$$\gamma \overset{(m)}{\eta}_{\mu_1\ldots\mu_{5-m}} = \frac{4-m}{6-m}\partial_{[\mu_1} \overset{(m+1)}{\eta}_{\mu_2\ldots\mu_{5-m}]},\ m=1,2,3,\ \gamma \overset{(4)}{\eta}_{\mu_1} = 0, \tag{23}$$

$$\gamma\mathcal{C}_{\mu_1\mu_2|\alpha} = 2\partial_\alpha\mathcal{C}_{\mu_1\mu_2} - \partial_{[\mu_1}\mathcal{C}_{\mu_2]\alpha},\ \gamma\mathcal{C}_{\mu_1\mu_2} = 0, \tag{24}$$

$$\delta t^{*\mu_1\mu_2\mu_3\mu_4|\alpha} = -\frac{\delta S^{\mathrm{L}}\left[t_{\mu_1\mu_2\mu_3\mu_4|\alpha}\right]}{\delta t_{\mu_1\mu_2\mu_3\mu_4|\alpha}},\ \delta r^{*\mu_1\mu_2|\alpha_1\alpha_2} = -\frac{\delta S^{\mathrm{L}}\left[r_{\mu_1\mu_2|\alpha_1\alpha_2}\right]}{\delta r_{\mu_1\mu_2|\alpha_1\alpha_2}}, \tag{25}$$

$$\delta \overset{(1)}{C}{}^{*\mu_1\mu_2\mu_3|\alpha} = -\partial_\lambda\left(4t^{*\lambda\mu_1\mu_2\mu_3|\alpha} + t^{*\mu_1\mu_2\mu_3\alpha|\lambda}\right), \tag{26}$$

$$\delta \overset{(2)}{C}{}^{*\mu_1\mu_2|\alpha} = \partial_\lambda\left(3\overset{(1)}{C}{}^{*\lambda\mu_1\mu_2|\alpha} - \overset{(1)}{C}{}^{*\mu_1\mu_2\alpha|\lambda}\right), \tag{27}$$

$$\delta \overset{(3)}{C}{}^{*\mu_1|\alpha} = -\partial_\lambda \overset{(2)}{C}{}^{*\lambda(\mu_1|\alpha)},\ \delta \overset{(1)}{\eta}{}^{*\mu_1\mu_2\mu_3\mu_4} = 5\partial_\alpha t^{*\mu_1\mu_2\mu_3\mu_4|\alpha}, \tag{28}$$

$$\begin{aligned}\delta \overset{(m)}{\eta}{}^{*\mu_1\ldots\mu_{5-m}} &= (6-m)\left(\partial_\alpha \overset{(m-1)}{C}{}^{*\mu_1\ldots\mu_{5-m}|\alpha}\right.\\ &\left.+(-)^m\frac{5-m}{7-m}\partial_\lambda \overset{(m-1)}{\eta}{}^{*\lambda\mu_1\ldots\mu_{5-m}}\right),\ m=2,3,4,\end{aligned} \tag{29}$$

$$\delta\mathcal{C}^{*\mu_1\mu_2|\alpha} = -4\partial_\beta r^{*\beta\alpha|\mu_1\mu_2},\ \delta\mathcal{C}^{*\mu_1\mu_2} = 3\partial_\alpha\mathcal{C}^{*\mu_1\mu_2|\alpha}. \tag{30}$$

By convention, we take δ and γ to act like right derivations and omit the automatically vanishing actions on the BRST generators.

The definitions of δ and γ acting on the BRST generators can be written in a more compact form if we perform some appropriate transformations on the ghosts and the corresponding antifields

$$\overset{(m)}{C'}_{\mu_1\ldots\mu_{4-m}||\alpha} \equiv \overset{(m)}{C}_{\mu_1\ldots\mu_{4-m}|\alpha} + (6-m)\overset{(m)}{\eta}_{\mu_1\ldots\mu_{4-m}\alpha}, \tag{31}$$

$$\overset{(m)}{C'}{}^{*\mu_1\ldots\mu_{4-m}||\alpha} \equiv \overset{(m)}{C}{}^{*\mu_1\ldots\mu_{4-m}|\alpha} + \frac{1}{6-m}\overset{(m)}{\eta}{}^{*\mu_1\ldots\mu_{4-m}\alpha}, \tag{32}$$

with $m=\overline{1,3}$. The double bar "$||$" signifies that the corresponding variable satisfies no additional identity except of being antisymmetric (where appropriate) with respect to its indices located before the bars. Under these considerations, some of formulas (19)–(30) take the simpler form

$$\gamma t_{\mu_1\mu_2\mu_3\mu_4|\alpha} = \partial_{[\mu_1} \overset{(1)}{C'}_{\mu_2\mu_3\mu_4]||\alpha} - \frac{1}{5}\partial_{[\mu_1} \overset{(1)}{C'}_{\mu_2\mu_3\mu_4||\alpha]}, \tag{33}$$

$$\gamma \overset{(m)}{C'}_{\mu_1\ldots\mu_{4-m}||\alpha} = \partial_{[\mu_1} \overset{(m+1)}{C'}_{\mu_2\ldots\mu_{4-m}]||\alpha},\ m = 1,2, \tag{34}$$

$$\gamma \overset{(3)}{C'}_{\mu_1||\alpha} = 2\partial_{\mu_1} \overset{(4)}{\eta}_{\alpha},\ \delta \overset{(1)}{C'}^{*\mu_1\mu_2\mu_3||\alpha} = -4\partial_\lambda t^{*\lambda\mu_1\mu_2\mu_3|\alpha}, \tag{35}$$

$$\delta \overset{(m)}{C'}^{*\mu_1\ldots\mu_{4-m}||\alpha} = (-)^m (5-m)\,\partial_\lambda \overset{(m-1)}{C'}^{*\lambda\mu_1\ldots\mu_{4-m}||\alpha},\ m = 2,3, \tag{36}$$

$$\delta \overset{(4)}{\eta}^{*\alpha} = 2\partial_{\mu_1} \overset{(3)}{C'}^{*\mu_1||\alpha}. \tag{37}$$

The Lagrangian BRST differential admits a canonical action in a structure named antibracket and defined by decreeing the fields/ghosts conjugated with the corresponding antifields, $s\cdot = (\cdot, S)$, where $(,)$ signifies the antibracket and S denotes the canonical generator of the BRST symmetry. It is a bosonic functional of ghost number zero, involving both field/ghost and antifield spectra, that obeys the master equation

$$(S,S) = 0. \tag{38}$$

The master equation is equivalent with the second-order nilpotency of s, where its solution S encodes the entire gauge structure of the associated theory. Taking into account formulae (19)–(30) as well as the standard actions of δ and γ in canonical form, we find that the complete solution to the master equation for the free model under study is given by

$$\begin{aligned} S = {} & S^{\mathrm{L}}\left[t_{\mu_1\mu_2\mu_3\mu_4|\alpha}, r_{\mu_1\mu_2|\alpha_1\alpha_2}\right] \\ & + \int d^D x \left[t^{*\mu_1\mu_2\mu_3\mu_4|\alpha} \left(\partial_{[\mu_1} \overset{(1)}{C}_{\mu_2\mu_3\mu_4]|\alpha} + \partial_{[\mu_1} \overset{(1)}{\eta}_{\mu_2\mu_3\mu_4\alpha]} - 5\partial_\alpha \overset{(1)}{\eta}_{\mu_1\mu_2\mu_3\mu_4} \right) \right. \\ & + \sum_{m=1}^{2} \overset{(m)}{C}^{*\mu_1\ldots\mu_{4-m}|\alpha} \left(\partial_{[\mu_1} \overset{(m+1)}{C}_{\mu_2\ldots\mu_{4-m}]|\alpha} + \partial_{[\mu_1} \overset{(m+1)}{\eta}_{\mu_2\ldots\mu_{4-m}\alpha]} \right. \\ & \left. + (-)^{5-m}(5-m)\,\partial_\alpha \overset{(m+1)}{\eta}_{\mu_1\ldots\mu_{4-m}} \right) + \overset{(3)}{C}^{*\mu_1|\alpha} \partial_{(\mu_1} \overset{(4)}{\eta}_{\alpha)} \\ & + \sum_{m=1}^{3} \frac{4-m}{6-m} \overset{(m)}{\eta}^{*\mu_1\ldots\mu_{5-m}} \partial_{[\mu_1} \overset{(m+1)}{\eta}_{\mu_2\ldots\mu_{5-m}]} \end{aligned}$$

$$+r^{*\mu_1\mu_2|\alpha_1\alpha_2}\left(\partial_{\mu_1}\mathcal{C}_{\alpha_1\alpha_2|\mu_2}-\partial_{\mu_2}\mathcal{C}_{\alpha_1\alpha_2|\mu_1}+\partial_{\alpha_1}\mathcal{C}_{\mu_1\mu_2|\alpha_2}-\partial_{\alpha_2}\mathcal{C}_{\mu_1\mu_2|\alpha_1}\right)$$
$$+\mathcal{C}^{*\mu_1\mu_2|\alpha_1}\left(2\partial_{\alpha}\mathcal{C}_{\mu_1\mu_2}-\partial_{[\mu_1}\mathcal{C}_{\mu_2]\alpha}\right)\Big], \tag{39}$$

such that it contains pieces with the antighost number ranging from zero to four.

4. BRIEF REVIEW OF BRST DEFORMATION THEORY

There are three main types of consistent interactions that can be added to a given gauge theory: *(i)* the first type deforms only the Lagrangian action, but not its gauge transformations, *(ii)* the second kind modifies both the action and its transformations, but not the gauge algebra, and *(iii)* the third, and certainly most interesting category, changes everything, namely, the action, its gauge symmetries and the accompanying algebra.

The reformulation of the problem of consistent deformations of a given action and of its gauge symmetries in the antifield-BRST setting is based on the observation that if a deformation of the classical theory can be consistently constructed, then the solution to the master equation for the initial theory can be deformed into the solution of the master equation for the interacting theory

$$\bar{S}=S+\lambda S_1+\lambda^2 S_2+O\left(\lambda^3\right), \qquad \varepsilon\left(\bar{S}\right)=0, \qquad \mathrm{gh}\left(\bar{S}\right)=0, \tag{40}$$

such that

$$\left(\bar{S},\bar{S}\right)=0. \tag{41}$$

Here and in the sequel $\varepsilon(F)$ denotes the Grassmann parity of F. The projection of (41) on the various powers of the coupling constant induces the following tower of equations:

$$\lambda^0:(S,S)=0, \tag{42}$$
$$\lambda^1:(S_1,S)=0, \tag{43}$$
$$\lambda^2:\frac{1}{2}(S_1,S_1)+(S_2,S)=0, \tag{44}$$
$$\lambda^3:(S_1,S_2)+(S_3,S)=0, \tag{45}$$
$$\vdots$$

The first equation is satisfied by hypothesis. The second one governs the first-order deformation of the solution to the master equation, S_1, and it expresses the fact that S_1 is a BRST co-cycle, $sS_1=0$, and hence it exists and is local. The remaining equations are responsible for the higher-order deformations of the solution to the master equation. No obstructions arise in finding solutions to them as long as no further restrictions, such as spatiotemporal locality, are imposed. Obviously, only non-trivial first-order deformations should be considered, since trivial ones ($S_1 = sB$) lead to trivial deformations of the initial theory, and can be eliminated by convenient redefinitions of the fields. Ignoring the trivial deformations, it follows that S_1 is a non-trivial BRST-observable, $S_1 \in H^0(s)$ (where $H^0(s)$ denotes the cohomology space of the BRST differential at ghost number zero). Once the deformation equations ((43)–(45), etc.) have been solved by means of specific cohomological techniques, from the consistent non-trivial deformed solution to the master equation one can extract all the information on the gauge structure of the resulting interacting theory.

5. MAIN INGREDIENTS OF THE LOCAL BRST COHOMOLOGY

At this point we solve the deformation equations, (43)–(45), etc., in order to construct all consistent interactions that can be added to the free model (2), (4)–(5). We consider only analytical, local, and manifestly covariant deformations and, meanwhile, restrict to Poincaré-invariant quantities, *i.e.* we do not allow explicit dependence on the spatiotemporal coordinates. The analyticity of deformations refers to the fact that the deformed solution to the master equation, (40), is analytical in the coupling constant λ and reduces to the original solution (39) in the free limit ($\lambda = 0$). Moreover, we ask that the deformed gauge theory preserves the Cauchy order of the uncoupled model, which enforces the requirement that the interacting Lagrangian is of maximum order equal to two in the spatiotemporal derivatives of the fields at each order in the coupling constant.

If we make the notation $S_1 = \int d^D x\, a$, with a a local function, then the local form of Eq. (43), which we have seen that controls the first-order deformation of the solution to the master equation, becomes

$$sa = \partial_\mu m^\mu, \qquad \mathrm{gh}(a) = 0, \qquad \varepsilon(a) = 0, \tag{46}$$

for some local m^μ, and it shows that the non-integrated density of the first-order deformation pertains to the local cohomology of s at ghost number zero, $a \in H^0(s|d)$, where d denotes the exterior spatiotemporal differential. In order to analyze the above equation, we develop a according to the antighost number

$$a = \sum_{k=0}^{I} a_k, \qquad \mathrm{agh}(a_k) = k, \qquad \mathrm{gh}(a_k) = 0, \qquad \varepsilon(a_k) = 0, \tag{47}$$

and assume, without loss of generality, that the above decomposition stops at some finite value of the antighost number, I. By taking into account the splitting (10) of the BRST differential, Eq. (46) becomes equivalent to a tower of local equations, corresponding to the different decreasing values of the antighost number

$$\gamma a_I = \partial_\mu \overset{(I)}{m}{}^\mu, \tag{48}$$

$$\delta a_I + \gamma a_{I-1} = \partial_\mu \overset{(I-1)}{m}{}^\mu, \tag{49}$$

$$\delta a_k + \gamma a_{k-1} = \partial_\mu \overset{(k-1)}{m}{}^\mu, \qquad I-1 \geq k \geq 1, \tag{50}$$

where $\left(\overset{(k)}{m}{}^\mu\right)_{k=\overline{0,I}}$ are some local currents with $\mathrm{agh}\left(\overset{(k)}{m}{}^\mu\right) = k$. It can be proved that we can replace Eq. (48) at strictly positive antighost numbers with

$$\gamma a_I = 0, \qquad \mathrm{agh}\,(a_I) = I > 0. \tag{51}$$

In conclusion, under the assumption that $I > 0$, the representative of highest antighost number from the non-integrated density of the first-order deformation can always be taken to be γ-closed, such that Eq. (46) associated with the local form of the first-order deformation is completely equivalent to the tower of equations given by (49)–(50) and (51).

Now, we pass to the investigation of the solutions to Eqs. (51) and (49)–(50). We have seen that the solution to Eq. (51) belongs to the cohomology of the exterior longitudinal derivative, such that we need to compute $H(\gamma)$ in order to construct the component of highest antighost number from the first-order deformation. This matter is solved with the help of definitions (19)–(24). In order to determine the cohomology $H(\gamma)$, we split the differential γ into two pieces

$$\gamma = \gamma_{\mathrm{t}} + \gamma_{\mathrm{r}}, \tag{52}$$

where γ_{t} acts non-trivially only on the fields/ghosts from the $(4,1)$ sector, while γ_{r} does the same thing, but with respect to the $(2,2)$ sector. From the above splitting it follows that the nilpotency of γ is equivalent to the nilpotency and anticommutation of its components

$$(\gamma_{\mathrm{t}})^2 = 0 = (\gamma_{\mathrm{r}})^2, \qquad \gamma_{\mathrm{t}}\gamma_{\mathrm{r}} + \gamma_{\mathrm{r}}\gamma_{\mathrm{t}} = 0, \tag{53}$$

so by Künneth's formula we finally find the isomorphism

$$H(\gamma) = H(\gamma_{\mathrm{t}}) \otimes H(\gamma_{\mathrm{r}}). \tag{54}$$

It can be shown that $H(\gamma)$ is generated on the one hand by $\Theta^{*\Delta}$, $K_{\mu_1\mu_2\mu_3\mu_4\mu_5|\alpha_1\alpha_2}$ and $R_{\mu_1\mu_2\mu_3|\alpha_1\alpha_2\alpha_3}$ as well as by their spatiotemporal derivatives and, on the other hand, by the ghosts $\mathcal{F}_{\mu_1\mu_2\mu_3\mu_4\mu_5} \equiv \partial_{[\mu_1} \overset{(1)}{\eta}{}_{\mu_2\mu_3\mu_4\mu_5]}$, $\overset{(4)}{\eta}{}_{\mu_1}$, $\mathcal{C}_{\mu\nu}$ and $\partial_{[\mu}\mathcal{C}_{\nu\alpha]}$, where

$\Theta^{*\Delta}$ denote all the antifields (from both sectors) and

$$K_{\mu_1\mu_2\mu_3\mu_4\mu_5|\alpha_1\alpha_2} = \partial_{[\mu_1} t_{\mu_2\mu_3\mu_4\mu_5]|[\alpha_2,\alpha_1]}, \tag{55}$$

$$R_{\mu_1\mu_2\mu_3|\alpha_1\alpha_2\alpha_3} = \partial_{[\mu_1} r_{\mu_2\mu_3]|[\alpha_1\alpha_2,\alpha_3]} \tag{56}$$

represent the curvature tensors of $t_{\mu_1\mu_2\mu_3\mu_4|\alpha_1}$ and respectively $r_{\mu_1\mu_2|\alpha_1\alpha_2}$. We used the standard notation $f_{,\mu} = \partial_\mu f$. So, the most general, non-trivial representative from $H(\gamma)$ for the overall theory (1) reads as

$$\begin{aligned} a_I &= \alpha_I\left(\left[K_{\mu_1\mu_2\mu_3\mu_4\mu_5|\alpha_1\alpha_2}\right],\left[R_{\mu_1\mu_2\mu_3|\alpha_1\alpha_2\alpha_3}\right],\left[\Theta^{*\Delta}\right]\right)\times \\ &\times\omega^I\left(\mathcal{F}_{\mu_1\mu_2\mu_3\mu_4\mu_5},\mathcal{C}_{\mu\nu},\partial_{[\mu}\mathcal{C}_{\nu\alpha]},\overset{(4)}{\eta}_{\mu_1}\right), \end{aligned} \tag{57}$$

where the notation $f([q])$ means that f depends on q and its derivatives up to a finite order, while ω^I denotes the elements of pure ghost number I (and antighost number zero) of a basis in the space of polynomials in the corresponding ghosts and some of their first-order derivatives. The objects α_I (obviously non-trivial in $H^0(\gamma)$) were taken to have a bounded number of derivatives, and therefore they are polynomials in the antifields $\Theta^{*\Delta}$, in the curvature tensors, as well as in their derivatives. Due to their γ-closeness, they are called invariant polynomials. At zero antighost number, the invariant polynomials are polynomials in the curvature tensors and in their derivatives.

Replacing the solution (57) into Eq. (49) and taking into account definitions (19)–(24), we remark that a necessary (but not sufficient) condition for the existence of (non-trivial) solutions a_{I-1} is that the invariant polynomials α_I are (non-trivial) objects from the local cohomology of the Koszul–Tate differential $H(\delta|d)$ at antighost number $I>0$ and pure ghost number equal to zero, *i.e.*,

$$\delta\alpha_I = \partial_\mu \overset{(I-1)^\mu}{j}, \qquad \mathrm{agh}\left(\overset{(I-1)^\mu}{j}\right) = I-1 \geq 0, \qquad \mathrm{pgh}\left(\overset{(I-1)^\mu}{j}\right) = 0. \tag{58}$$

The above notation is generic, in the sense that α_I and $\overset{(I-1)^\mu}{j}$ may actually carry supplementary Lorentz indices. Consequently, we need to investigate some of the main properties of the local cohomology of the Koszul–Tate differential $H(\delta|d)$ at pure ghost number zero and strictly positive antighost numbers in order to fully determine the component a_I of highest antighost number from the first-order deformation. As the free model under study is a linear gauge theory of Cauchy order equal to five, the general results from [2, 3] ensure that $H(\delta|d)$ (at pure ghost number zero) is trivial at antighost numbers strictly greater than its Cauchy order

$$H_I(\delta|d) = 0, \qquad I > 5. \tag{59}$$

Moreover, if the invariant polynomial α_I, with $\mathrm{agh}(\alpha_I) = I \geq 5$, is trivial in $H_I(\delta|d)$, then it can be taken to be trivial also in $H_I^{\rm inv}(\delta|d)$

$$\left(\alpha_I = \delta b_{I+1} + \partial_\mu \overset{(I)}{c}{}^{\mu}, \qquad \mathrm{agh}(\alpha_I) = I \geq 5\right) \Rightarrow \alpha_I = \delta\beta_{I+1} + \partial_\mu \overset{(I)}{\gamma}{}^{\mu}, \tag{60}$$

with β_{I+1} and $\overset{(I)}{\gamma}{}^{\mu}$ invariant polynomials. (An element of $H_I^{\rm inv}(\delta|d)$ is defined via an equation similar to (58), but with the corresponding current also an invariant polynomial.) The result (60) can be proved like in the Appendix B, Theorem 3, from [5]. This is important since together with (59) ensures that the entire local cohomology of the Koszul–Tate differential in the space of invariant polynomials (characteristic cohomology) is trivial in antighost number strictly greater than five

$$H_I^{\rm inv}(\delta|d) = 0, \qquad I > 5. \tag{61}$$

Looking at the definitions (35)–(37) involving the transformed antifields (32) and taking into account formulae (25)–(30) with respect to the $(2,2)$ sector, we can organize the non-trivial representatives of $H_I(\delta|d)$ (at pure ghost number equal to zero) and $H_I^{\rm inv}(\delta|d)$ with $I \geq 2$ in the following table.

Table 1

Non-trivial representatives spanning $H_I(\delta|d)$ and $H_I^{\rm inv}(\delta|d)$

agh	$H_I(\delta\|d)$, $H_I^{\rm inv}(\delta\|d)$
$I > 5$	none
$I = 5$	$\overset{(4)}{\eta}{}^{*\mu_1}$
$I = 4$	$\overset{(3)}{C'}{}^{*\mu_1\|\|\alpha}$
$I = 3$	$\overset{(2)}{C'}{}^{*\mu_1\mu_2\|\|\alpha}, \mathcal{C}^{*\mu_1\mu_2}$
$I = 2$	$\overset{(1)}{C'}{}^{*\mu_1\mu_2\mu_3\|\|\alpha}, \mathcal{C}^{*\mu_1\mu_2\|\alpha_1}$

We remark that there is no non-trivial element in $(H_I(\delta|d))_{I\geq 2}$ or $\left(H_I^{\rm inv}(\delta|d)\right)_{I\geq 2}$ that effectively involves the curvature tensors and/or their derivatives, and the same stands for the quantities that are more than linear in the antifields and/or depend on their derivatives. In contrast to the groups $(H_I(\delta|d))_{I\geq 2}$ and $\left(H_I^{\rm inv}(\delta|d)\right)_{I\geq 2}$, which are finite-dimensional, the cohomology $H_1(\delta|d)$ at pure ghost number zero, that is related to global symmetries and ordinary conservation laws, is infinite-dimensional since the theory is free.

The previous results on $H(\delta|d)$ and $H^{\rm inv}(\delta|d)$ at strictly positive antighost numbers are important because they control the obstructions to removing the antifields from the first-order deformation. Indeed, due to (61), it follows that we can successively eliminate all the pieces with $I > 5$ from the non-integrated density of the first-order deformation by adding only trivial terms, so we can take, without loss of non-trivial objects, the condition $I \leq 5$ in the decomposition (47). The last representative is of the form (57), where the invariant polynomial is necessarily a non-trivial object from $H_I^{\mathrm{inv}}(\delta|d)$ for $I = \overline{2,5}$ and respectively from $H_1(\delta|d)$ for $I = 1$.

6. BASIC RESULTS — IDENTIFICATION OF THE COUPLED MODEL

Now, we have at hand all the necessary ingredients for computing the general form of the first-order deformation of the solution to the master equation as solution to Eq. (46). In view of this, we decompose the first-order deformation like

$$a = a^{\mathrm{t}} + a^{\mathrm{r}} + a^{\mathrm{t-r}}, \tag{62}$$

where a^{t} denotes the part responsible for the self-interactions of the field $t_{\mu_1\mu_2\mu_3\mu_4|\alpha_1}$, a^{r} is related to the self-interactions of the field $r_{\mu_1\mu_2|\alpha_1\alpha_2}$, and $a^{\mathrm{t-r}}$ signifies the component that describes only the cross-couplings between $t_{\mu_1\mu_2\mu_3\mu_4|\alpha_1}$ and $r_{\mu_1\mu_2|\alpha_1\alpha_2}$. Obviously, Eq. (46) becomes equivalent with three equations, one for each component

$$sa^{\mathrm{t}} = \partial_\mu m^\mu_{\mathrm{t}}, \qquad sa^{\mathrm{r}} = \partial_\mu m^\mu_{\mathrm{r}}, \qquad sa^{\mathrm{t-r}} = \partial_\mu m^\mu_{\mathrm{t-r}}. \tag{63}$$

The solutions to the first two equations from (63) were investigated in [5] and respectively [7] and read as

$$a^{\mathrm{t}} = 0, \qquad a^{\mathrm{r}} = r. \tag{64}$$

In order to solve the third equation from (63), we decompose $a^{\mathrm{t-r}}$ along the antighost number like in (47) and stop, according to (61), at $I = 4$

$$a^{\mathrm{t-r}} = \sum_{k=0}^{I} a_k^{\mathrm{t-r}}, \qquad I \leq 4, \tag{65}$$

where $a_I^{\mathrm{t-r}}$ can be taken as solution to the equation $\gamma a_I^{\mathrm{t-r}} = 0$, and therefore it is of the form (57). It is possible to show, following a line similar to that employed in [17], that it possible to take $I = 2$ in (65). Under these circumstances, we can completely determine the first-order deformation S_1 and, using Eq. (44), also the second-order deformation, S_2. Moreover, it follows that we can safely take all the remaining higher-order deformations to be trivial, $S_m = 0$, $m > 2$. We skip the proofs and only list the main findings below.

The fully deformed solution to the master equation (41) ends at order two in the coupling constant and is given by

$$\bar{S} = S + \lambda \int d^7x \left[\varepsilon_{\mu_1 \ldots \mu_7} \overset{(1)}{\eta}{}^{*\mu_1 \ldots \mu_4} \partial^{\mu_5} \mathcal{C}^{\mu_6 \mu_7} \right.$$
$$+ \frac{5}{2} t^{*\mu_1 \ldots \mu_4 | \alpha} \varepsilon_{\mu_1 \ldots \mu_7} \left(\partial^{\mu_5} \mathcal{C}^{\mu_6 \mu_7|}{}_{\alpha} - \frac{1}{5} \delta^{\mu_5}_{\alpha} \partial^{[\beta} \mathcal{C}^{\mu_6 \mu_7]|}{}_{\beta} \right)$$
$$\left. + \frac{5}{48} t_{\mu_1 \ldots \mu_4 | \alpha} \varepsilon^{\mu_1 \ldots \mu_7} \left(\partial_\lambda \partial_{\mu_5} r_{\mu_6 \mu_7|}{}^{\alpha\lambda} + \frac{2}{5} \delta^{\alpha}_{\mu_5} \partial^\lambda \partial_{\mu_6} r_{\mu_7 \lambda} \right) + r \right]$$
$$+ \frac{5\lambda^2}{2} \int d^7x \left[\left(\partial_{[\mu_1} r_{\mu_2 \mu_3]|}{}^{\alpha_1 \alpha_2} \right) \partial^{[\mu_1} r^{\mu_2 \mu_3]|}{}_{\alpha_1 \alpha_2} \right.$$
$$\left. - \left(\partial_{[\mu_1} r_{\mu_2 \mu_3]|}{}^{\mu_1 \alpha_1} \right) \partial^{[\nu_1} r^{\mu_2 \mu_3]|}{}_{\nu_1 \alpha_1} \right]. \qquad (66)$$

We recall that r denotes the double trace of $r_{\mu_1\mu_2|\alpha_1\alpha_2}$. We observe that this solution 'lives' in a seven-dimensional spacetime. From (66) we read all the information on the gauge structure of the coupled theory. The terms of antighost number zero in (66) provide the Lagrangian action. They can be organized as

$$\bar{S}^{\mathrm{L}} \left[t_{\mu_1\mu_2\mu_3\mu_4|\alpha_1}, r_{\mu_1\mu_2|\alpha_1\alpha_2} \right] = S^{\mathrm{L}} \left[t_{\mu_1\mu_2\mu_3\mu_4|\alpha_1}, r_{\mu_1\mu_2|\alpha_1\alpha_2} \right]$$
$$+ \lambda \int d^7x \left[r + \frac{5}{48} t_{\mu_1 \ldots \mu_4 | \alpha} \varepsilon^{\mu_1 \ldots \mu_7} \left(\partial_\lambda \partial_{\mu_5} r_{\mu_6 \mu_7|}{}^{\alpha\lambda} + \frac{2}{5} \delta^{\alpha}_{\mu_5} \partial^\lambda \partial_{\mu_6} r_{\mu_7 \lambda} \right) \right.$$
$$+ \frac{5\lambda}{2} \left(\left(\partial_{[\mu_1} r_{\mu_2 \mu_3]|}{}^{\alpha_1 \alpha_2} \right) \partial^{[\mu_1} r^{\mu_2 \mu_3]|}{}_{\alpha_1 \alpha_2} \right.$$
$$\left.\left. - \left(\partial_{[\mu_1} r_{\mu_2 \mu_3]|}{}^{\mu_1 \alpha_1} \right) \partial^{[\nu_1} r^{\mu_2 \mu_3]|}{}_{\nu_1 \alpha_1} \right) \right], \qquad (67)$$

where $S^{\mathrm{L}} \left[t_{\mu_1\mu_2\mu_3\mu_4|\alpha_1}, r_{\mu_1\mu_2|\alpha_1\alpha_2} \right]$ is the Lagrangian action appearing in (1) in $D = 7$. We observe that action (67) contains only mixing-component terms of order one and two in the coupling constant. The piece of antighost number one appearing in (66) gives the deformed gauge transformations in the form

$$\bar{\delta}_{\chi,\epsilon,\xi} t_{\mu_1\mu_2\mu_3\mu_4|\alpha_1} = \partial_{[\mu_1} \chi_{\mu_2\mu_3\mu_4]|\alpha} + \partial_{[\mu_1} \epsilon_{\mu_2\mu_3\mu_4]\alpha} - 4 \partial_\alpha \epsilon_{\mu_1\mu_2\mu_3\mu_4}$$
$$+ \frac{5\lambda}{2} \varepsilon_{\mu_1 \ldots \mu_7} \left(\partial^{\mu_5} \xi^{\mu_6\mu_7|}{}_{\alpha} - \frac{1}{5} \delta^{\mu_5}_{\alpha} \partial^{[\beta} \xi^{\mu_6\mu_7]|}{}_{\beta} \right), \qquad (68)$$

$$\delta_\xi r_{\mu_1\mu_2|\alpha_1\alpha_2} = \partial_{\mu_1} \xi_{\alpha_1\alpha_2|\mu_2} - \partial_{\mu_2} \xi_{\alpha_1\alpha_2|\mu_1} + \partial_{\alpha_1} \xi_{\mu_1\mu_2|\alpha_2} - \partial_{\alpha_2} \xi_{\mu_1\mu_2|\alpha_1}. \qquad (69)$$

It is interesting to note that only the gauge transformations of the tensor field $(4,1)$ are modified during the deformation process. This is enforced at order one in the coupling constant by a term linear in the first-order derivatives of the gauge parameters from the $(2,2)$ sector. From the terms of antighost number equal to two present in (66) we learn that only the first-order reducibility functions are modified at order one in the coupling constant, the others coinciding with the original ones. Since there are no other terms of antighost number two in (66), it follows that the gauge algebra of the coupled model is unchanged by the deformation procedure, being the same Abelian one like for the starting free theory. It is easy to see from (67)–(69) that if we impose the PT-invariance at the level of the coupled model, then we obtain no interactions (we must set $\lambda = 0$ in these formulae).

It is important to stress that the problem of obtaining consistent interactions strongly depends on the spatiotemporal dimension. For instance, if one starts with action (1) in $D > 7$, then one inexorably gets $\bar{S} = S + \lambda \int d^D x \ r$, so no cross-interaction term can be added to either the original Lagrangian or its gauge transformations. Finally, we note that the results obtained here generalize our previous findings from [17] on the interactions between a tensor with the mixed symmetry $(3, 1)$ and one with the mixed symmetry of the Riemann tensor. Apart from the obvious similarities between the Lagrangian actions and gauge transformations, the most interesting feature is that the first-order deformation ends in both cases at antighost number two.

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