CONSISTENT INTERACTIONS BETWEEN DUAL FORMULATIONS OF LINEARISED GRAVITY IN TERMS OF MASSLESS TENSOR FIELDS WITH MIXED SYMMETRIES $(k,1)$ AND $(2,2)$ *

C. BIZDADEA, M.T. MIAUTA, S.O. SALIU, M. TOMA

Department of Physics, University of Craiova
13 Al. I. Cuza Str., Craiova 200585, Romania
E-mail: bizdadea@central.ucv.ro, mtudristioiu@central.ucv.ro,
osaliu@central.ucv.ro, mtoma@central.ucv.ro

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Two particular types of consistent interactions of a single massless tensor field with the mixed symmetry corresponding to a two-column Young diagram $(k,1)$, dual to linearised gravity in $D = k+3$, are considered: (a) self-interactions, and (b) cross-interactions with another dual formulation of linearised gravity in terms of a massless tensor field with the mixed symmetry of the linearised Riemann tensor. Under the standard hypotheses from gauge field theories, it is shown that: (1) there appear consistent self-interactions of a single massless tensor field with the mixed symmetry corresponding to a two-column Young diagram $(k,1)$ only if k is even, $k = 2m$ and precisely in $D = 4m$ spacetime dimensions; (2) there are allowed consistent cross-couplings to a single massless tensor field with the mixed symmetry of the linearised Riemann tensor, but only in $D = k+3$. In both cases the coupled models exhibit deformed gauge transformations, but their algebra remains Abelian, like that of the free limit theories. In the latter case the first-order reducibility of the gauge transformations in the $(k,1)$ sector is also changed.

Key words: interacting field theory, local BRST cohomology, Young tableaux, mixed symmetry-type tensor fields, dual formulations of linearised gravity.

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1. INTRODUCTION

There is a revived interest in the construction of dual gravity theories, which led to several new results, *viz.* a dual formulation of linearised gravity in first order tetrad formalism in arbitrary dimensions within the path integral framework [1] or a reformulation of non-linear Einstein gravity in terms of the dual graviton together with the ordinary metric and a shift gauge field [2]. There exist in fact three

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different dual formulations of linearised gravity in D dimensions: the Pauli–Fierz description [3, 4], the version based on a massless tensor field with the mixed symmetry $(D-3,1)$ [5–7], and the formulation in terms of a massless tensor field with the mixed symmetry $(D-3,D-3)$ [8, 9]. The last two versions are obtained by dualizing on one and respectively on both indices the Pauli–Fierz field [10].

An important matter related to the dual formulations of linearised gravity is the study of their consistent interactions, among themselves as well as with other gauge theories. In this paper two particular types of consistent interactions of a single massless tensor field with the mixed symmetry corresponding to a two-column Young diagram $(k,1)$, dual to linearised gravity in $D=k+3$, are considered: (a) self-interactions, and (b) cross-interactions with another dual formulation of linearised gravity in terms of a massless tensor field with the mixed symmetry of the linearised Riemann tensor. Our analysis relies on the deformation of the solution to the master equation [11] by means of cohomological techniques with the help of the local BRST cohomology [12–14], whose component in the $(k,1)$ sector can be solved along the same line like in [15] or [16] and in the $(2,2)$ sector has been investigated in [17, 18]. It is also interesting to mention the developments concerning the dual formulations of linearised gravity from the perspective of M-theory [19–21]. On the other hand, the massless tensor field with the mixed symmetry $(2,2)$ displays all the algebraic properties of the Riemann tensor, describes purely spin-two particles, and also provides a dual formulation of linearised gravity in $D=5$.

The method from [11] has been widely used in the literature at the construction of various interacting models, such as BF models [22], tensor fields of degree two [23], or $D=11$ SUGRA [24].

Under the standard hypotheses from gauge field theories, it follows that: (1) there appear consistent self-interactions of a single massless tensor field with the mixed symmetry corresponding to a two-column Young diagram $(k,1)$ only if k is even, $k=2m$ and precisely in $D=4m$ spacetime dimensions; (2) there are allowed consistent cross-couplings to a single massless tensor field with the mixed symmetry of the linearised Riemann tensor, but only in $D=k+3$. In both cases the coupled models exhibit deformed gauge transformations, but their algebra remains Abelian, like that of the free limit theories. In the latter case the self-couplings are (analytical) series in the coupling constant, also the gauge transformations and first-order reducibility of the gauge transformations in the $(k,1)$ sector are modified only at order one in the coupling constant. The results related to the cross-couplings between the $(k,1)$ and $(2,2)$ sectors generalize the previous findings from [25] and respectively [26] between a massless tensor field with the mixed symmetry $(3,1)$ and respectively $(4,1)$ and one with the mixed symmetry of the Riemann tensor.

These result are particularly important for at least two reasons. This is the first time when mixed symmetry-type tensor fields with a gauge freedom inherit consistent self-couplings, even though they reduce to mixing-component terms of various orders in the coupling constant. Meanwhile, this is one of the very few situations where consistent cross-couplings to another dual formulation of linearised gravity are granted and such couplings occur precisely in the spacetime dimension in which the $(k,1)$ tensor field is dual to the Pauli–Fierz model.

2. BRIEF REVIEW OF BRST DEFORMATION THEORY

There are three main types of consistent interactions that can be added to a given gauge theory: *(i)* the first type deforms only the Lagrangian action, but not its gauge transformations, *(ii)* the second kind modifies both the action and its transformations, but not the gauge algebra, and *(iii)* the third, and certainly most interesting category, changes everything, namely, the action, its gauge symmetries and the accompanying algebra.

The reformulation of the problem of consistent deformations of a given action and of its gauge symmetries in the antifield-BRST setting is based on the observation that if a deformation of the classical theory can be consistently constructed, then the solution to the master equation for the initial theory can be deformed into the solution of the master equation for the interacting theory

$$\bar{S} = S + \lambda S_1 + \lambda^2 S_2 + O\left(\lambda^3\right), \qquad \varepsilon\left(\bar{S}\right) = 0, \qquad \mathrm{gh}\left(\bar{S}\right) = 0, \tag{1}$$

such that

$$\left(\bar{S}, \bar{S}\right) = 0. \tag{2}$$

Here and in the sequel $\varepsilon(F)$ denotes the Grassmann parity of F. The projection of (2) on the various powers of the coupling constant induces the following tower of equations:

$$\lambda^0 : (S,S) = 0, \tag{3}$$

$$\lambda^1 : (S_1,S) = 0, \tag{4}$$

$$\lambda^2 : \tfrac{1}{2}(S_1,S_1) + (S_2,S) = 0, \tag{5}$$

$$\lambda^3 : (S_1,S_2) + (S_3,S) = 0, \tag{6}$$

$$\vdots$$

The first equation is satisfied by hypothesis. The second one governs the first-order deformation of the solution to the master equation, S_1, and it expresses the fact that S_1 is a BRST co-cycle, $sS_1 = 0$, and hence it exists and is local. The remaining equations are responsible for the higher-order deformations of the solution to the master equation. No obstructions arise in finding solutions to them as long as no further restrictions, such as spatio-temporal locality, are imposed. Obviously, only non-trivial first-order deformations should be considered, since trivial ones ($S_1 = sB$) lead to trivial deformations of the initial theory, and can be eliminated by convenient redefinitions of the fields. Ignoring the trivial deformations, it follows that S_1 is a non-trivial BRST-observable, $S_1 \in H^0(s)$ (where $H^0(s)$ denotes the cohomology space of the BRST differential at ghost number zero). Once the deformation equations ((4)–(6), etc.) have been solved by means of specific cohomological techniques, from the consistent non-trivial deformed solution to the master equation one can extract all the information on the gauge structure of the resulting interacting theory.

3. FREE MASSLESS TENSOR FIELD WITH THE MIXED SYMMETRY $(k, 1)$. CONSISTENT SELF-INTERACTIONS

We begin with the Lagrangian action describing a free massless tensor with the mixed symmetry $(k, 1)$

$$S^{\mathrm{L}}\left[t_{\mu_1\cdots\mu_k|\alpha}\right] = -\tfrac{1}{2\cdot k!}\int d^D x\Big[\left(\partial_\mu t_{\mu_1\cdots\mu_k|\alpha}\right)\partial^\mu t^{\mu_1\cdots\mu_k|\alpha} - \left(\partial^\alpha t_{\mu_1\cdots\mu_k|\alpha}\right)\partial_\beta t^{\mu_1\cdots\mu_k|\beta} - k\left(\partial^\lambda t_{\lambda\mu_1\cdots\mu_{k-1}|\alpha}\right)\partial_\rho t^{\rho\mu_1\cdots\mu_{k-1}|\alpha} - k\left(\partial_\mu t_{\mu_1\cdots\mu_{k-1}}\right)\partial^\mu t^{\mu_1\cdots\mu_{k-1}} + 2(-)^{k+1}k\left(\partial^\alpha t_{\mu_1\cdots\mu_k|\alpha}\right)\partial^{\mu_1} t^{\mu_2\cdots\mu_k} + k(k-1)\left(\partial^\lambda t_{\lambda\mu_1\cdots\mu_{k-2}}\right)\partial_\rho t^{\rho\mu_1\cdots\mu_{k-2}}\Big], \tag{7}$$

which is invariant under the (infinitesimal) gauge transformations

$$\delta_{\theta,\epsilon} t_{\mu_1\cdots\mu_k|\alpha} = \partial_{[\mu_1}\overset{(1)}{\theta}_{\mu_2\ldots\mu_k]|\alpha} + \partial_{[\mu_1}\overset{(1)}{\epsilon}_{\mu_2\ldots\mu_k\alpha]} + (-)^{k+1}(k+1)\,\partial_\alpha\overset{(1)}{\epsilon}_{\mu_1\ldots\mu_k}. \tag{8}$$

Action (7) may be completely rewritten in terms of the tensor field

$$F_{\mu_1\cdots\mu_{k+1}|\alpha} = \partial_{[\mu_1} t_{\mu_2\cdots\mu_{k+1}]|\alpha}, \tag{9}$$

with the mixed symmetry $(k+1, 1)$ as

$$S^{\mathrm{L}}\left[t_{\mu_1\cdots\mu_k|\alpha}\right] = -\tfrac{1}{2\cdot(k+1)!}\int d^D x\Big(F_{\mu_1\cdots\mu_{k+1}|\alpha}F^{\mu_1\cdots\mu_{k+1}|\alpha} - (k+1)\,F_{\mu_1\cdots\mu_k}F^{\mu_1\cdots\mu_k}\Big), \tag{10}$$

where

$$F_{\mu_1\cdots\mu_k} = F_{\mu_1\cdots\mu_{k+1}|\alpha}\sigma^{\mu_{k+1}\alpha} \tag{11}$$

represents the trace of (9), which is a completely antisymmetric tensor. Tensor (9) is invariant under the gauge transformations from (8) involving the parameters $\overset{(1)}{\theta}_{\mu_1\ldots\mu_{k-1}|\alpha}$, but not under the part containing $\overset{(1)}{\epsilon}_{\mu_1\ldots\mu_k}$:

$$\delta_{\overset{(1)}{\theta},\overset{(1)}{\epsilon}} F_{\mu_1\cdots\mu_{k+1}|\alpha} = (-)^{k+1}k\,\partial_\alpha\partial_{[\mu_1}\overset{(1)}{\epsilon}_{\mu_2\ldots\mu_{k+1}]}, \tag{12}$$

while its trace exhibits the gauge transformations

$$\delta_{\overset{(1)}{\theta},\overset{(1)}{\epsilon}} F_{\mu_1\cdots\mu_k} = -k\partial^\alpha \partial_{[\alpha} \overset{(1)}{\epsilon}_{\mu_1\ldots\mu_k]}. \tag{13}$$

The generating set (8) of gauge transformations of action (7) is Abelian and off-shell, $(k-1)$-order reducible. The detailed structure of the reducibility can be found in [16]. Here we focus only on the first-order reducibility. If in (8) we perform the simultaneous transformations

$$\overset{(1)}{\theta}_{\mu_1\ldots\mu_{k-1}|\alpha} = \partial_{[\mu_1} \overset{(2)}{\theta}_{\mu_2\ldots\mu_{k-1}]|\alpha} + \partial_{[\mu_1} \overset{(2)}{\epsilon}_{\mu_2\ldots\mu_{k-1}\alpha]} + (-)^k k\partial_\alpha \overset{(2)}{\epsilon}_{\mu_1\ldots\mu_{k-1}}, \tag{14}$$

$$\overset{(1)}{\epsilon}_{\mu_1\ldots\mu_k} = \tfrac{k-1}{k+1}\partial_{[\mu_1} \overset{(2)}{\epsilon}_{\mu_2\ldots\mu_k]}, \tag{15}$$

with $\overset{(2)}{\theta}_{\mu_1\ldots\mu_{k-2}|\alpha}$ and $\overset{(2)}{\epsilon}_{\mu_1\ldots\mu_{k-1}}$ some arbitrary tensor fields on $\mathcal{M}$ displaying the mixed symmetry $(k-2,1)$ and respectively $(k-1,0)$ (fully antisymmetric), then the gauge transformations of the tensor field vanish identically, and therefore we get the first-order reducibility relations

$$\delta_{\overset{(1)}{\theta}\left(\overset{(2)}{\theta},\overset{(2)}{\epsilon}\right),\overset{(1)}{\epsilon}\left(\overset{(2)}{\epsilon}\right)} t_{\mu_1\cdots\mu_k|\alpha} = 0. \tag{16}$$

It can be checked that the functions defining the field equations associated with action (7), the gauge generators, as well as all the reducibility functions, satisfy the general regularity assumptions from [27], such that the model under discussion is described by a normal gauge theory of Cauchy order equal to $(k+1)$.

The consistent self-interactions are constructed under the general hypotheses of analyticity in the coupling constant, spacetime locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the requirement that the interaction vertices contain at most two spatio-temporal derivatives of the fields. Solving the deformation Eqs. (4)–(6), etc. in the presence of these working hypotheses by means of the local BRST cohomology corresponding to the free model under consideration, we determine the fully deformed solution $\bar{S}$ (1) to the master equation (2). Its main features can be synthesized as follows:

- it is non-trivial iff k is even, $k = 2m$, and the spacetime dimension is valued as $D = 2k = 4m$;
- it is an infinite series in the coupling constant λ, reducing to an analytic function in this parameter;
- the components of strictly positive antighost number reduce to terms linear in λ and, moreover, are linear in the antifield of the tensor field $t_{\mu_1\cdots\mu_k|\alpha}$;

- the entire dependence of $t_{\mu_1 \cdots \mu_k | \alpha}$ is realized via the dependence of the trace (11) of the tensor field (9).

The computation of the fully deformed solution to the master equation identifies the entire gauge structure of the self-coupled model. Consequently, we get that the Lagrangian action of a self-interacting massless tensor field with the mixed symmetry $(k, 1)$ reads

$$\begin{aligned} \bar{S}^{\mathrm{L}}\left[t_{\mu_1 \cdots \mu_{2m} | \alpha}\right] &= S^{\mathrm{L}}\left[t_{\mu_1 \cdots \mu_{2m} | \alpha}\right] \\ &- \int d^{4m}x \Big[\tfrac{2m-1}{2(2m)^2 \cdot (2m-1)!} \tfrac{\lambda}{1+[\lambda \cdot (2m)!]^2} \varepsilon^{\mu_1 \ldots \mu_{4m}} F_{\mu_1 \ldots \mu_{2m}} F_{\mu_{2k+1} \cdots \mu_{4m}} \\ &+ \tfrac{(2m-1) \cdot (2m-1)!}{2} \tfrac{\lambda^2}{1+[\lambda \cdot (2m)!]^2} F_{\mu_1 \ldots \mu_{2m}} F^{\mu_1 \ldots \mu_{2m}} \Big] \end{aligned} \tag{17}$$

and a generating set of gauge transformations for this action can be taken as

$$\begin{aligned} \bar{\delta}_{\theta, \epsilon} t_{\mu_1 \ldots \mu_{2m|\alpha}} = & \partial_{[\mu_1} \overset{(1)}{\theta}_{\mu_2 \ldots \mu_{2m}]|\alpha} + \partial_{[\mu_1} \overset{(1)}{\varepsilon}_{\mu_2 \ldots \mu_{2m}]\alpha} - 2m \partial_\alpha \overset{(1)}{\varepsilon}_{\mu_1 \ldots \mu_{2m}} + \\ & + \lambda \varepsilon_{\mu_1 \ldots \mu_{2m} \mu_{2m+1} \cdots \mu_{4m}} \partial^{[\beta} \overset{(1)}{\varepsilon}{}^{\mu_{2m+1} \cdots \mu_{4m}]} \sigma_{\beta\alpha}. \end{aligned} \tag{18}$$

Analyzing formulas (17) and (18) we conclude the following:

- the self-couplings contain only mixing-component terms of any order (series) in the coupling constant, which are nevertheless analytical in λ, as required by the working hypotheses;
- the gauge transformations of the tensor field $t_{\mu_1 \ldots \mu_{2m} | \alpha}$ are deformed only at order one in the coupling constant;
- since $\bar{S}$ contains no terms quadratic in the antifield of the field $t_{\mu_1 \ldots \mu_{2m} | \alpha}$, it follows that the gauge algebra of the coupled model is the same with the original, Abelian one from the free limit;
- since there are no terms of antighost number strictly great that one in $\bar{S}$ involving the coupling constant that are simultaneously linear in the ghosts ant in the ghost antifields, it results that all the reducibility relations remain unaffected by the deformation procedure;
- if we add the supplementary restriction that the cross-couplings are PT-invariant, then we obtain no deformation at all;
- there are no cross-couplings possible for an odd value of k even though we do not impose the PT-invariance.

4. CONSISTENT CROSS-COUPLINGS WITH A MASSLESS TENSOR FIELD WITH THE MIXED SYMMETRY OF THE RIEMANN TENSOR

We begin with a Lagrangian action describing a free massless tensor field with the mixed symmetry $(k,1)$ $(k>4)$ and one with the mixed symmetry of the Riemann tensor, $r_{\mu_1\mu_2|\alpha_1\alpha_2}$

$$S^{\mathrm{L}}\left[t_{\mu_1\cdots\mu_k|\alpha}, r_{\mu_1\mu_2|\alpha_1\alpha_2}\right] = S^{\mathrm{L}}\left[t_{\mu_1\cdots\mu_k|\alpha}\right] + S^{\mathrm{L}}\left[r_{\mu_1\mu_2|\alpha_1\alpha_2}\right], \tag{19}$$

in $D \geq k+2$ spacetime dimensions, where $S^{\mathrm{L}}\left[t_{\mu_1\cdots\mu_k|\alpha}\right]$ is given in (7) (or equivalently in (10)) and

$$\begin{aligned} S^{\mathrm{L}}\left[r_{\mu_1\mu_2|\alpha_1\alpha_2}\right] = \int d^D x \Big[& \tfrac{1}{8}\left(\partial^\lambda r^{\mu_1\mu_2|\alpha_1\alpha_2}\right)\left(\partial_\lambda r_{\mu_1\mu_2|\alpha_1\alpha_2}\right) \\ & - \tfrac{1}{2}\left(\partial_{\mu_1} r^{\mu_1\mu_2|\alpha_1\alpha_2}\right)\left(\partial^{\nu_1} r_{\nu_1\mu_2|\alpha_1\alpha_2}\right) \\ & - \left(\partial_{\mu_1} r^{\mu_1\mu_2|\alpha_1\alpha_2}\right)\left(\partial_{\alpha_2} r_{\mu_2\alpha_1}\right) - \tfrac{1}{2}\left(\partial^{\mu_1} r^{\mu_2\alpha_1}\right)\left(\partial_{\mu_1} r_{\mu_2\alpha_1}\right) \\ & + \left(\partial_{\mu_1} r^{\mu_1\alpha_1}\right)\left(\partial^{\nu_1} r_{\nu_1\alpha_1}\right) - \tfrac{1}{2}\left(\partial_{\mu_1} r^{\mu_1\alpha_1}\right)\left(\partial_{\alpha_1} r\right) + \tfrac{1}{8}\left(\partial^{\mu_1} r\right)\left(\partial_{\mu_1} r\right)\Big]. \end{aligned} \tag{20}$$

A generating set of gauge transformations for action (19) can be taken of the form (8) in the $(k,1)$ sector and respectively

$$\delta_\xi r_{\mu_1\mu_2|\alpha_1\alpha_2} = \partial_{\mu_1}\xi_{\alpha_1\alpha_2|\mu_2} - \partial_{\mu_2}\xi_{\alpha_1\alpha_2|\mu_1} + \partial_{\alpha_1}\xi_{\mu_1\mu_2|\alpha_2} - \partial_{\alpha_2}\xi_{\mu_1\mu_2|\alpha_1} \tag{21}$$

in the $(2,2)$ sector. The gauge parameters $\xi_{\mu_1\mu_2|\alpha_1}$ display the mixed symmetry $(2,1)$. The generating set of gauge transformations (8) and (21) is off-shell reducible of order $(k-1)$ and respectively one, the accompanying gauge algebra being obviously Abelian.

Since on the one hand the non-trivial self-interactions of the tensor field $(k,1)$ hold only for $k=2m$ and in $D=2k=4m$ dimensions and, on the other hand, the consistent cross-couplings with the tensor field with the mixed symmetry of the Riemann tensor take place only in $D=k+3$ spacetime dimensions, it follows that they can never survive together. Since here we are primarily interested in cross-couplings, we find that the fully deformed solution $\bar{S}$ of the form (1) to the master equation (2) "lives" precisely in a spacetime dimension $D=k+3$, ends at order two in the coupling constant, displays a component of maximum antighost number equal to two, and contains no self-interactions of the $(k,1)$ sector.

The Lagrangian action of the model that couples the tensor fields $t_{\mu_1\cdots\mu_k|\alpha}$ and

$r_{\mu_1\mu_2|\alpha_1\alpha_2}$ takes the form

$$\bar{S}^{\mathrm{L}}\left[t_{\mu_1\cdots\mu_k|\alpha}, r_{\mu_1\mu_2|\alpha_1\alpha_2}\right] = S^{\mathrm{L}}\left[t_{\mu_1\cdots\mu_k|\alpha}, r_{\mu_1\mu_2|\alpha_1\alpha_2}\right]$$
$$+\lambda\int d^{k+3}x\Big[r + (-)^k \tfrac{k+1}{2\cdot k!} t_{\mu_1\ldots\mu_k|\alpha}\varepsilon^{\mu_1\cdots\mu_{k+3}}\big(\partial_\lambda\partial_{\mu_{k+1}} r_{\mu_{k+2}\mu_{k+3}|}{}^{\alpha\lambda}$$
$$+\tfrac{2}{k+1}\delta^{\alpha}_{\mu_{k+1}}\partial^{\lambda}\partial_{\mu_{k+2}} r_{\mu_{k+3}\lambda}\big)\Big]$$
$$+\tfrac{(k+1)\lambda^2}{24}\int d^{k+3}x\Big[(k+2)\left(\partial_{[\mu_1} r_{\mu_2\mu_3]|}{}^{\alpha_1\alpha_2}\right)\partial^{[\mu_1} r^{\mu_2\mu_3]|}{}_{\alpha_1\alpha_2}$$
$$-6\left(\partial_{[\mu_1} r_{\mu_2\mu_3]|}{}^{\mu_1\alpha_1}\right)\partial^{[\nu_1} r^{\mu_2\mu_3]|}{}_{\nu_1\alpha_1}\Big], \qquad (22)$$

where $S^{\mathrm{L}}\left[t_{\mu_1\cdots\mu_k|\alpha}, r_{\mu_1\mu_2|\alpha_1\alpha_2}\right]$ is the free Lagrangian action (19) in $D = k+3$ spacetime dimensions. In the above r denotes the double trace of $r_{\mu_1\mu_2|\alpha_1\alpha_2}$, $r = r_{\mu_1\mu_2|}{}^{\mu_1\mu_2}$. The above action is invariant under the deformed gauge transformations in the form

$$\bar{\delta}_{\theta,\epsilon,\xi} t_{\mu_1\cdots\mu_k|\alpha} = \partial_{[\mu_1} \overset{(1)}{\theta}_{\mu_2\ldots\mu_k]|\alpha} + \partial_{[\mu_1} \overset{(1)}{\epsilon}_{\mu_2\ldots\mu_k]\alpha} + (-)^{k+1} k\partial_\alpha \overset{(1)}{\epsilon}_{\mu_1\ldots\mu_k}$$
$$+(-)^k \tfrac{(k+1)\lambda}{2}\varepsilon_{\mu_1\ldots\mu_{k+3}}\left(\partial^{\mu_{k+1}}\xi^{\mu_{k+2}\mu_{k+3}|}{}_{\alpha} - \tfrac{1}{k+1}\delta^{\mu_{k+1}}_{\alpha}\partial^{[\beta}\xi^{\mu_{k+2}\mu_{k+3}]|}{}_{\beta}\right), \qquad (23)$$

$$\delta_\xi r_{\mu_1\mu_2|\alpha_1\alpha_2} = \partial_{\mu_1}\xi_{\alpha_1\alpha_2|\mu_2} - \partial_{\mu_2}\xi_{\alpha_1\alpha_2|\mu_1} + \partial_{\alpha_1}\xi_{\mu_1\mu_2|\alpha_2} - \partial_{\alpha_2}\xi_{\mu_1\mu_2|\alpha_1}. \qquad (24)$$

We observe that action (22) contains only mixing-component terms of order one and two in the coupling constant. It is interesting to note that only the gauge transformations of the tensor field $(k,1)$ are modified during the deformation process. This is enforced at order one in the coupling constant by a term linear in the first-order derivatives of the gauge parameters from the $(2,2)$ sector. From the terms of antighost number equal to two involving λ present in the deformed solution to the master equation we learn that only the first-order reducibility functions are modified at order one in the coupling constant, the others coinciding with the original ones. More precisely, if we make the transformations (14) and

$$\overset{(1)}{\epsilon}_{\mu_1\ldots\mu_k} = \tfrac{k-1}{k+1}\partial_{[\mu_1}\overset{(2)}{\epsilon}_{\mu_2\ldots\mu_k]} + \lambda\varepsilon_{\mu_1\ldots\mu_{k+3}}\partial^{\mu_{k+1}}\hat{\xi}^{\mu_{k+2}\mu_{k+3}}, \qquad (25)$$

$$\xi_{\mu_1\mu_2|\alpha} = 2\partial_\alpha\hat{\xi}_{\mu_1\mu_2} - \partial_{[\mu_1}\hat{\xi}_{\mu_2]\alpha}, \qquad (26)$$

in (23), where $\hat{\xi}_{\mu_1\mu_2}$ is an antisymmetric tensor, whose components stand for the first-order reducibility parameters in the $(2,2)$ sector, then we find that

$$\bar{\delta}_{\overset{(1)}{\theta}\left(\overset{(2)}{\theta},\overset{(2)}{\epsilon}\right),\overset{(1)}{\epsilon}\left(\overset{(2)}{\epsilon},\hat{\xi}\right),\xi(\hat{\xi})} t_{\mu_1\cdots\mu_k|\alpha} = 0.$$

Since there are no other terms of antighost number two or higher in the deformed solution to the master equation, we deduce that the gauge algebra of the coupled model is unchanged by the deformation procedure, being the same Abelian one like for the starting free theory. It is easy to see from (22)–(24) that if we impose the PT-invariance at the level of the coupled model, then we obtain no interactions (we must set $\lambda = 0$ in these formulas).

5. CONCLUSIONS

The main conclusion of this paper is that under the standard hypotheses from gauge field theories, it is possible to show, using the powerful and elegant method of constructing consistent interactions in gauge theories based on the computation of the local BRST cohomology, the next non-trivial results:

1. there appear consistent self-interactions of a single massless tensor field with the mixed symmetry corresponding to a two-column Young diagram $(k,1)$ only if k is even, $k = 2m$ and precisely in $D = 4m$ spacetime dimensions;
2. there are allowed consistent cross-couplings to a single massless tensor field with the mixed symmetry of the linearised Riemann tensor, but only in $D = k+3$.

In both cases the coupled models exhibit deformed gauge transformations, but their algebra remains Abelian, like that of the free limit theories. In the latter case the first-order reducibility of the gauge transformations in the $(k,1)$ sector is also changed.

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