SPECIAL INTERACTIONS BETWEEN A DFLG IN TERMS OF A MIXED SYMMETRY TENSOR FIELD (k,1) AND A TOPOLOGICAL BF MODEL*

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All consistent couplings in $D = k+3$ spacetime dimensions between a topological BF model with a maximal field spectrum and a dual formulation of linearized gravity (DFLG) in terms of a massless tensor field with the mixed symmetry $(k,1)$ have been generated using the method of deforming the solution to the master equation by specific cohomological techniques. All consistent cross-couplings have been computed and classified. As a result of the deformation procedure all the ingredients of the emerging coupled theory are strongly modified compared to their free limit.

Key words: consistent interactions in gauge field theory, local BRST cohomology, topological BF models, mixed symmetry-type tensor fields.

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1. INTRODUCTION

Topological BF field theories [1] are important in view of the fact that pure three-dimensional gravity is just a BF theory and, moreover, in higher dimensions general relativity and supergravity in Ashtekar formalism may also be formulated as topological BF theories with some extra constraints. On the other hand, tensor fields in "exotic" representations of the Lorentz group, characterized by a mixed Young symmetry type [2–5], held the attention lately on some important issues, like the dual formulation of field theories of spin two or higher [6–9], the impossibility of consistent cross-interactions in the dual formulation of linearised gravity [10], or the derivation of some exotic gravitational interactions [11, 12].

The purpose of this paper is to report and classify all consistent cross-couplings in $D = k+3$ between a dual formulation of linearised gravity (DFLG) in terms of a massless tensor gauge field with the mixed symmetry $(k,1)$ and an Abelian BF model with a maximal field spectrum. The method employed at the construction of

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interactions is based on the deformation of the solution to the classical master equation [13, 14] with the help of cohomological techniques based on the computation of local BRST cohomology [15–17]. All couplings have been obtained under some general hypotheses from field theory: analyticity in the coupling constant, space-time locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the preservation of the number of derivatives on each field. This paper generalizes our previous results from [18, 19]. Under the same hypotheses it has been shown that the massless tensor field with the mixed symmetry $(k, 1)$ allows no self-interactions in $D = k+3$ [20]. All consistent self-interactions in an arbitrary dimension D that can be added to a topological BF model with a maximal field spectrum have been presented in [21]. The general procedure was exemplified on two models: $k = 4$ (also considered in [19]) and $k = 5$, approached for the first time in this work. The method from [13,14] has been widely used in the literature at the construction of various interacting models, such as BF models [22], tensor fields of degree two [23], or $D = 11$ SUGRA [24].

The main characteristics of the interacting model can be synthesized as follows: (1) the Lagrangian action (and its gauge symmetries) ends at order two in the coupling constant λ; (2) all cross-couplings lie at order one in λ; (3) there appear new self-interactions among the BF fields at order two in λ, strictly linked to the presence of the DFLG sector; (4) the consistency of the entire gauge structure of the coupled theory is dictated by a set of algebraic and differential equations that involve all the functions (of the undifferentiated scalar field from the BF sector) that parametrize the self-interactions and cross-couplings at order one in λ, called consistency equations; (5) the gauge structure of the coupled models strongly depends on the precise nontrivial solutions to the consistency equations. The new major results regard the full classification of all cross-couplings into three categories, performed in terms of the number of form gauge fields from the BF sector of strictly positive form degree.

2. MAIN RESULT. CLASSIFICATION OF CROSS-COUPLING VERTICES

The starting point is a free theory in $D = k+3$, $k \geq 2$, with the Lagrangian action written as the sum between the Lagrangian action of a topological BF model with a maximal field spectrum that contains two sorts of fields, $(\overset{[m]}{A}, \overset{[m+1]}{B})_{m=\overline{0,I(k)}}$, and the Lagrangian action of a free, massless tensor field with the mixed symmetry $(k, 1)$ $t_{\mu_1\ldots\mu_k|\alpha}$ (meaning it is antisymmetric in its first k indices and fulfils the identity $t_{[\mu_1\ldots\mu_k|\alpha]} \equiv 0$)

$$S^{\mathrm{L}}\left[\Phi^{\alpha_0}\right] = \int d^{k+3}x \left[\sum_{m=0}^{I(k)} \tfrac{1}{m+1} \overset{[m+1]}{B}{}^{\mu_1\cdots\mu_{m+1}} \partial_{[\mu_1} \overset{[m]}{A}_{\mu_2\ldots\mu_{m+1}]}\right.$$

$$- \tfrac{1}{2\cdot(k+1)!}\left(F_{\mu_1\ldots\mu_{k+1}|\alpha}F^{\mu_1\cdots\mu_{k+1}|\alpha}-(k+1)F_{\mu_1\ldots\mu_k}F^{\mu_1\cdots\mu_k}\right)\Bigg]$$
$$\equiv S^{\mathrm{L,BF}}[\overset{[m]}{A}_{\mu_1\ldots\mu_m},\overset{[m+1]}{B}{}^{\mu_1\cdots\mu_{m+1}}]+S^{\mathrm{L,t}}[t_{\mu_1\ldots\mu_k|\alpha}]. \tag{1}$$

The field spectrum, denoted by Φ^{α_0}, contains two types of p-forms from the BF sector and the tensor field with the mixed symmetry $(k,1)$

$$\Phi^{\alpha_0}=\left\{\left(\overset{[m]}{A}_{\mu_1\ldots\mu_m},\overset{[m+1]}{B}{}^{\mu_1\cdots\mu_{m+1}}\right)_{m=\overline{0,I(k)}},t_{\mu_1\ldots\mu_k|\alpha}\right\}, \tag{2}$$

where $I(k)$ the maximum value of m, equal to $I(k)=[k/2]+1$, with $[k/2]$ the integer part of $k/2$. For the fields from the BF sector we use an overscript which represents the form degree ($\overset{[m]}{A}$ is a m-form and $\overset{[m+1]}{B}$ a $(m+1)$-form). The notation $[\mu_1\mu_2\ldots\mu_{k+1}]$ signifies complete antisymmetry with respect to the indices between brackets, with the convention that the minimum number of terms is always used. In this paper we work with the Minkowski metric of 'mostly plus' signature $\sigma_{\mu\nu}=\sigma^{\mu\nu}=\mathrm{diag}\,(-+\cdots+)$ and with the Levi–Civita symbol $\varepsilon^{\mu_1\cdots\mu_{k+3}}$ defined according to the convention $\varepsilon^{01\ldots k+2}=-\varepsilon_{01\ldots k+2}=-1$. The tensor field $F_{\mu_1\ldots\mu_{k+1}|\alpha}$ from (1) displays the mixed symmetry $(k+1,1)$ and reads as

$$F_{\mu_1\cdots\mu_{k+1}|\alpha}=\partial_{[\mu_1}t_{\mu_2\ldots\mu_{k+1}]|\alpha},\;\;F_{\mu_1\cdots\mu_k}=\sigma^{\mu_{k+1}\alpha}F_{\mu_1\cdots\mu_{k+1}|\alpha}. \tag{3}$$

The Lagrangian action (1) is invariant under a generating set of gauge symmetries of the form

$$\delta_{\Omega^{\alpha_1}}\overset{[0]}{A}=0,\quad \delta_{\Omega^{\alpha_1}}\overset{[m]}{A}_{\mu_1\ldots\mu_m}=\partial_{[\mu_1}\overset{[m-1]}{\epsilon}_{(m,0)\mu_2\ldots\mu_m]},\quad m=\overline{1,I(k)}, \tag{4}$$

$$\delta_{\Omega^{\alpha_1}}\overset{[m+1]}{B}{}^{\mu_1\cdots\mu_{m+1}}=-(m+2)\,\partial_\rho\overset{[m+2]}{\xi}{}^{\rho\mu_1\cdots\mu_{m+1}}_{(m+1,0)},\qquad m=\overline{0,I(k)}, \tag{5}$$

$$\delta_{\Omega^{\alpha_1}}t_{\mu_1\ldots\mu_k|\alpha}=\partial_{[\mu_1}\chi_{\mu_2\ldots\mu_k]|\alpha}+\partial_{[\mu_1}\theta_{\mu_2\ldots\mu_k]\alpha}+(-)^{k+1}k\partial_\alpha\theta_{\mu_1\ldots\mu_k}. \tag{6}$$

The gauge parameters from the BF sector are denoted by ϵ's and ξ's while for the mixed symmetry tensor field we used χ and θ, and we collectively denoted all the gauge parameters by Ω^{α_1}. All the gauge parameters are bosonic and completely antisymmetric (where applicable), excepting $\chi_{\mu_1\ldots\mu_{k-1}|\alpha}$, which displays the mixed symmetry $(k-1,1)$. Related to the BF sector, the over-script represents the form degree, while the other two lower indices between parentheses signify the form field to which a certain gauge parameter is associated with and respectively the reducibility level. The above gauge transformations are Abelian and off-shell, $(k+1)$-order reducible. The Lagrangian formulations of a free BF model with a maximal field spectrum in an arbitrary space-time dimension D and respectively of a massless tensor field with the mixed symmetry $(k,1)$ can be found in detail in [21,25].

It has been shown in the literature [13, 14] that it is possible to reformulate the problem of constructing consistent interactions as a deformation problem of the solution to the classical master equation corresponding to a given free theory. This procedure can be solved with the help of the local BRST cohomology [15–17]. Using the above technique we completely computed the fully deformed solution of the master equation, which complies with all the working hypotheses. It stops at order two in the coupling constant and contains all possible interactions vertices: direct self-interactions in the BF sector (at order one), cross-couplings between BF and DFLG theories (at order one), and indirect self-interactions in the BF sector (at order two), generated only by the presence of DFLG. There appear no self-interactions in the $(k,1)$ sector [20] since on the one hand they require $k = 2m$, $D = 4m$, and, on the other hand, the DFLG setting imposes the restriction $D = k+3 \equiv 2m+3$, so they cannot overlap. The detailed structure of direct self-interactions in the BF sector (at order one) can be found in [21]. They are parametrized by some functions denoted by Z and W that may depend at most on the undifferentiated scalar field $\overset{[0]}{A} \equiv \varphi$ from the BF sector. The cross-couplings between BF and DFLG together with the indirect self-interactions in the BF sector are parametrized in terms of some other functions, denoted by U and V, also depending at most of φ. We have shown that the consistency of the deformed solution to the master equation to all orders in the coupling constant require that all these parametrizing functions satisfy a set of algebraic and differential equations, called consistency equations. Once we have computed the fully deformed solution to the master equation, we are able to extract from it the entire gauge formulation of the model that couples a topological BF model to a $(k,1)$-type dual formulation of linearized gravity in $D = k+3$.

The Lagrangian action of this coupled model can be written as

$$\bar{S}^{\mathrm{L}}[\Phi^{\alpha_0}] = S^{\mathrm{L}}[\Phi^{\alpha_0}] + \int d^{k+3}x \left[\lambda \left(a_0^{\mathrm{BF}} + a_0^{\mathrm{int}}\right) + \lambda^2 b_0\right], \tag{7}$$

where $S^{\mathrm{L}}[\Phi^{\alpha_0}]$ is the free action (1) and a_0^{BF} provides the BF self-interactions at order one in the coupling constant λ [see [21], formulas (6)–(9)].

The piece a_0^{int} contains all the cross-couplings at order one in λ and reads as

$$a_0^{\mathrm{int}} = M_{\mu_1\ldots\mu_k} F^{\mu_1\ldots\mu_k}, \tag{8}$$

where $F^{\mu_1\cdots\mu_k}$ is given in (3). The compact notation $M_{\mu_1\ldots\mu_k}$ signifies

$$M_{\mu_1\ldots\mu_k} = (-)^k \varepsilon_{\mu_1\ldots\mu_k\mu_{k+1}\mu_{k+2}\mu_{k+3}} \Bigg(VB^{\mu_{k+1}\mu_{k+2}\mu_{k+3}} + \sum_{m=3}^{I(k)} \sum_{N=1}^{V_{m-2}} \sum_{\substack{n_1,n_2,\\ \ldots,n_N\in\mathbb{N}^*}} V_{(n_1,n_2,\ldots,n_N)} \overset{[n_1]}{A}{}_{[\rho_1} \overset{[n_2]}{A} \cdots \overset{[n_N]}{A}{}_{\ldots\rho_{m-2}]} \overset{[m+1]}{B}{}^{\rho_1\cdots\rho_{m-2}\mu_{k+1}\mu_{k+2}\mu_{k+3}} \Bigg) + \sum_{N=1}^{V_k} \sum_{\bar{n}_1,\bar{n}_2,\ldots,\bar{n}_{\bar{N}}\in\mathbb{N}^*} U_{(\bar{n}_1,\bar{n}_2,\ldots,\bar{n}_{\bar{N}})} \overset{[\bar{n}_1]}{A}{}_{[\mu_1} \overset{[\bar{n}_2]}{A} \cdots \overset{[\bar{n}_{\bar{N}}]}{A}{}_{\ldots\mu_k]}. \tag{9}$$

The two sums after $(n_1,n_2,\ldots,n_N\in\mathbb{N}^*)$ and respectively $(\bar{n}_1,\bar{n}_2,\ldots,\bar{n}_{\bar{N}}\in\mathbb{N}^*)$ from (9) are restricted to satisfy the conditions

$$\begin{gathered} n_1+n_2+\ldots+n_N = m-2, \\ 1\leq n_1\leq n_2\leq\ldots\leq n_N\leq I(k), \qquad n_i=n_j \Leftrightarrow n_i=2p, \end{gathered} \tag{10}$$

where $m=\overline{3,I(k)}$, $N=\overline{1,V_{m-2}}$, and respectively

$$\begin{gathered} \bar{n}_1+\bar{n}_2+\ldots+\bar{n}_{\bar{N}} = k, \\ 1\leq \bar{n}_1\leq \bar{n}_2\leq\ldots\leq \bar{n}_{\bar{N}}\leq I(k), \qquad \bar{n}_i=\bar{n}_j \Leftrightarrow \bar{n}_i=2p, \end{gathered} \tag{11}$$

with $\bar{N}=\overline{1,V_k}$. The sum limits V_k and V_{m-2} are respectively defined by

$$V_k = \left[\tfrac{k}{2}\right] + k \bmod 2, \qquad V_{m-2} = \left[\tfrac{m}{2}\right] - 1 + m \bmod 2. \tag{12}$$

The component b_0 from (7) is nothing but the Lagrangian density of the coupled model at order two in the coupling constant. Its existence is dictated by the BRST deformation procedure and follows from the consistency of the first-order deformation of the solution to the master equation at order two in λ. We have proved by specific cohomological computations that this consistency issue of order two is completely equivalent to a set of algebraic and differential equations that must be satisfied by all the parametrizing functions mentioned in the above (of the type Z, W, U, and V), to be called consistency equations. Assuming these equations possess solutions, b_0 can be expressed only in terms of (9) like

$$b_0 = \frac{k!3}{4} M_{\mu_1\ldots\mu_k} M^{\mu_1\cdots\mu_k}. \tag{13}$$

It is clear from (9) that (13) indeed describes only BF self-interactions, as announced earlier. Let us argue why these self-interactions are strictly implemented by the presence of the cross-couplings to the $(k,1)$ sector. Indeed, the vanishing of (8) is equivalent to the vanishing of (9), which further implies the annihilation of (13). On the other hand, the condition $M_{\mu_1\ldots\mu_k}=0$ is completely equivalent to the vanishing of all Us and Vs or, in other words, of all functions that parametrize the cross-couplings.

The fact that the fully deformed solution to the master equation ends at order two in λ and hence 7 behaves in the same manner follows from the result that all the deformations of order three or higher of the solution to the master equation are trivial provided the consistency equations are fulfilled and therefore all components of order three or higher in λ from (7) can be taken to vanish under the same assumption.

We remark, from (8) and (9), that there are three types of cross-couplings between a topological BF model and a particular dual linearised gravity in $D = k+3$ dimension, parametrized by the smooth functions V, $V_{(n_1,n_2,\ldots,n_N)}$, and $U_{(\bar{n}_1,\bar{n}_2,\ldots,\bar{n}_{\bar{N}})}$ of the scalar field φ. As we can see, only the last type of couplings is PT-invariant, the other two breaking this invariance. Recall that these functions are not arbitrary, but together with the functions of the type Z and W that parametrize the BF self-interactions are subject to the consistency equations.

Analysing (7) and then (8) we observe that all cross-couplings lie at order one in λ, are linear in the tensor $F^{\mu_1\cdots\mu_k}$ from the $(k,1)$ sector, and can be appropriately classified according to the number of B- and A-type BF forms as follows: (1) none with more then one B; (2) a single kind with one B and no As, present in any $D \geq 5$ ($k \geq 2$) and containing only the three-form B, (3) at least one vertex with a single B and at least one A, present earliest in $D = 6$ ($k = 3$), and (4) without Bs, but al least one A. The last type of couplings enables a single A-type form only in $D = 5$ ($k = 2$), otherwise (for higher values of k), at least two As are required.

Finally, we mention that the gauge structure of the interacting theory is highly deformed with respect to that of the starting Abelian model: the gauge algebra becomes open and the reducibility relations only hold on-shell. Their concrete expressions cannot be output in the general setting discussed here since they strongly depend on the solutions to the consistency equations.

3. EXAMPLES

In this section we particularize the general results from the previous section to $k = 4$ and $k = 5$. The topological BF model involved has in both cases the same maximal field spectrum, namely one scalar field , two types of one-forms, two kinds of two-forms , two sorts of three-forms, and one four-form. The BF self-interactions in $D = 7$ and respectively $D = 8$ are given in detail in [21]. In what follows we analyse only the cross-couplings and associated BF self-interactions (at order two in the coupling constant). The main task is to find all the distinct solutions to conditions (10) and (11) in order to obtain the concrete form of (9) for each of these situations.

3.1. RESULTS FOR $k = 4$, ALIAS $D = 7$

In this situation $I(4) = 3$ and m involved in conditions (10) is single-valued, $m = 3$. On behalf of (12), it follows $V_1 = 1$, and hence $N = 1$, such that (10) possess a unique solution: $n_1 = 1$. Accordingly, there is a single parametrizing function, $V_{(1)}$. Related to conditions (11), we have that $\bar{N}$ takes values within the range $\overline{2, V_4}$. As the first relation in (12) gives $V_4 = 2$, it follows that $\bar{N} = 2$. There are two distinct solutions to (11): $\bar{n}_1 = 1$, $\bar{n}_2 = 3$ and respectively $\bar{n}_1 = \bar{n}_2 = 2$. The parametrizing functions will consequently be denoted by $U_{(1,3)}$ and $U_{(2,2)}$, but for notational simplicity we switch to $U_{(1,3)} \equiv U_1$, $U_{(2,2)} \equiv U_2$. This particular model has been exposed also in [19]. Under these considerations, (9) and the coupled Lagrangian action for $k = 4$ become respectively

$$M_{\mu_1\mu_2\mu_3\mu_4} = \varepsilon_{\mu_1\mu_2\mu_3\mu_4\mu_5\mu_6\mu_7}\left(V\overset{[3]}{B}{}^{\mu_5\mu_6\mu_7} + V_1\overset{[1]}{A}_\rho\overset{[4]}{B}{}^{\rho\mu_5\mu_6\mu_7}\right) + U_1 A_{[\mu_1}A_{\mu_2\mu_3\mu_4]} + U_2 A_{[\mu_1\mu_2}A_{\mu_3\mu_4]}, \tag{14}$$

$$\begin{aligned}\bar{S}^{\mathrm{L}}\left[\Phi^{\alpha_0}\right] = {}& S^{\mathrm{L}}\left[\Phi^{\alpha_0}\right] + \lambda\int d^7x\Big[M(\varphi) + W_1\overset{[1]}{A}_{\mu_1}\overset{[1]}{B}{}^{\mu_1} + W_2\overset{[2]}{A}_{\mu_1\mu_2}\overset{[2]}{B}{}^{\mu_1\mu_2} + W_3\overset{[3]}{A}_{\mu_1\mu_2\mu_2}\overset{[3]}{B}{}^{\mu_1\mu_2\mu_3} \\ & + W_4\overset{[1]}{A}_{[\mu_1}\overset{[2]}{A}_{\mu_2\mu_3]}\overset{[3]}{B}{}^{\mu_1\mu_2\mu_3} + \left(W_5\overset{[2]}{A}_{[\mu_1\mu_2}\overset{[2]}{A}_{\mu_3\mu_4]} + W_6\overset{[1]}{A}_{[\mu_1}\overset{[3]}{A}_{\mu_2\mu_3\mu_4]}\right)\overset{[4]}{B}{}^{\mu_1\mu_2\mu_3\mu_4} \\ & + \varepsilon_{\mu_1\ldots\mu_7}\left(Z_2\overset{[3]}{B}{}^{\mu_1\mu_2\mu_3}\overset{[4]}{B}{}^{\mu_4\mu_5\mu_6\mu_7} + \tfrac{1}{2}Y_2\overset{[2]}{A}{}^{\mu_1\mu_2}\overset{[2]}{A}{}^{\mu_3\mu_4}\overset{[3]}{A}{}^{\mu_5\mu_6\mu_7} + \tfrac{1}{3!}Y_3\overset{[1]}{A}{}^{\mu_1}\overset{[2]}{A}{}^{\mu_2\mu_3}\overset{[2]}{A}{}^{\mu_4\mu_5}\overset{[2]}{A}{}^{\mu_6\mu_7}\right) \\ & + M_{\mu_1\mu_2\mu_3\mu_4}F^{\mu_1\mu_2\mu_3\mu_4}\Big] + 18\lambda^2\int d^7x M_{\mu_1\mu_2\mu_3\mu_4}M^{\mu_1\mu_2\mu_3\mu_4}.\end{aligned} \tag{15}$$

After long and tedious computations we inferred the consistency conditions, mentioned in general in the previous section, as:

$$W_1\frac{dW_2}{d\varphi} + 3W_4W_2 = 0, \quad W_1\frac{dW_3}{d\varphi} + (4W_6 - 3W_4)W_3 = 0, \tag{16}$$

$$W_1\frac{dZ_2}{d\varphi} - (3W_4 + 4W_6)Z_2 = 0, \quad W_1\frac{dY_2}{d\varphi} + 2(3W_4 + 2W_6)Y_2 - W_3Y_3 = 0, \tag{17}$$

$$W_1\frac{dW_5}{d\varphi} + 2(3W_4 - 2W_6)W_5 + 24Z_2Y_3 = 0, \quad W_1\frac{dM}{d\varphi} = 0, \tag{18}$$

$$W_2W_4 + W_3W_5 - 24Z_2Y_2 = 0, \quad 2W_2W_6 + 3(W_3W_5 + 24Z_2Y_2) = 0, \tag{19}$$

$$W_2Y_3 + 9W_5Y_2 = 0, \quad W_1W_2 = 0, \quad W_2W_3 = 0, \quad W_2Z_2 = 0, \quad W_3Z_2 = 0, \tag{20}$$

$$W_1\frac{dU_2}{d\varphi} + 2[-2W_5U_1 + 3W_4U_2 + 12\left(Y_3V - Y_2V_1\right)] = 0, \tag{21}$$

$$W_1\frac{dV}{d\varphi} - 3W_4V + W_3V_1 - 4Z_2U_1 = 0, \quad 2W_2U_1 + 3W_3U_2 + 72Y_2V = 0, \tag{22}$$

$$W_2V_1 + 6(W_5V - Z_2U_2) = 0, \quad W_2V = 0, \tag{23}$$

where all the functions of the type W, Z, Y, or M are specific to first-order BF self-interactions. They are far more complex than the consistency equations satisfied in the case of a pure BF model, which follow from the above ones if we set zero all the Us and Vs.

The deformed generating set of gauge transformations follows from the components of anti-ghost number one present in the deformed solution to the master equation and takes the concrete form

$$\bar{\delta}_{\Omega^{\alpha_1}}\varphi = \lambda\bar{\delta}^{\mathrm{self}}_{\Omega^{\alpha_1}}\varphi, \tag{24}$$

$$\bar{\delta}_{\Omega^{\alpha_1}}\overset{[1]}{A}_{\mu_1} = \partial_{\mu_1}\overset{[0]}{\epsilon}_{(1,0)} + \lambda\bar{\delta}^{\mathrm{self}}_{\Omega^{\alpha_1}}\overset{[1]}{A}_{\mu_1}, \tag{25}$$

$$\bar{\delta}_{\Omega^{\alpha_1}}\overset{[2]}{A}_{\mu_1\mu_2} = \partial_{[\mu_1}\overset{[1]}{\epsilon}_{(2,0)\mu_2]} + \lambda\bar{\delta}^{\mathrm{self}}_{\Omega^{\alpha_1}}\overset{[2]}{A}_{\mu_1\mu_2} + \tfrac{12}{5}\lambda V\varepsilon_{\mu_1\mu_2\mu_3\ldots\mu_7}\partial^{[\mu_3}\theta^{\mu_4\ldots\mu_7]}, \tag{26}$$

$$\begin{aligned}\bar{\delta}_{\Omega^{\alpha_1}}\overset{[3]}{A}_{\mu_1\mu_2\mu_3} &= \partial_{[\mu_1}\overset{[2]}{\epsilon}_{(3,0)\mu_2\mu_3]} + \lambda\bar{\delta}^{\mathrm{self}}_{\Omega^{\alpha_1}}\overset{[3]}{A}_{\mu_1\mu_2\mu_3} - 4\lambda V_1\varepsilon_{\mu_1\mu_2\mu_3\mu_4\ldots\mu_7}A_\rho\partial^{[\rho}\theta^{\mu_4\ldots\mu_7]}\\ &- \lambda\varepsilon_{\mu_1\mu_2\mu_3\mu_4\ldots\mu_7}V_1\left(F^{\mu_4\ldots\mu_7} + 36\lambda M^{\mu_4\ldots\mu_7}\right)\overset{[0]}{\epsilon}_{(1,0)},\end{aligned} \tag{27}$$

$$\begin{aligned}\bar{\delta}_{\Omega^{\alpha_1}}\overset{[1]}{B}{}^{\mu_1} &= -2\partial_\rho\overset{[2]}{\xi}{}^{\rho\mu_1}_{(1,0)} + \lambda\bar{\delta}^{\mathrm{self}}_{\Omega^{\alpha_1}}\overset{[1]}{B}{}^{\mu_1}\\ &-4\lambda\left(\frac{dU_1}{d\varphi}\overset{[1]}{A}_{[\mu_2}\overset{[3]}{A}_{\mu_3\mu_4\mu_5]} + \frac{dU_2}{d\varphi}\overset{[2]}{A}_{[\mu_2\mu_3}\overset{[2]}{A}_{\mu_4\mu_5]}\right)\partial^{[\mu_1}\theta^{\mu_2\mu_3\mu_4\mu_5]}\\ &+\frac{12}{5}\lambda\varepsilon_{\nu_1\nu_2\ldots\nu_7}\left(-\frac{dV}{d\varphi}\overset{[3]}{B}{}^{\mu_1\nu_1\nu_2} + \frac{dV_1}{d\varphi}\overset{[1]}{A}_{\mu_2}\overset{[4]}{B}{}^{\mu_1\mu_2\nu_1\nu_2}\right)\partial^{[\nu_3}\theta^{\nu_4\ldots\nu_7]}\\ &+\lambda\left(F^{\mu_1\mu_2\mu_3\mu_4} + 36\lambda M^{\mu_1\mu_2\mu_3\mu_4}\right)\left[\left(4\frac{dU_1}{d\varphi}\overset{[3]}{A}_{\mu_2\mu_3\mu_4} - \frac{dV_1}{d\varphi}\varepsilon_{\mu_2\ldots\mu_8}\overset{[4]}{B}{}^{\mu_5\ldots\mu_8}\right)\overset{[0]}{\epsilon}_{(1,0)}\right.\\ &+4\left(\frac{dU_2}{d\varphi}\overset{[2]}{A}_{[\mu_2\mu_3}\overset{[1]}{\epsilon}_{(2,0)\mu_4]} - \frac{dU_1}{d\varphi}\overset{[1]}{A}_{[\mu_2}\overset{[2]}{\epsilon}_{(3,0)\mu_3\mu_4]} + \frac{dV}{d\varphi}\varepsilon_{\mu_2\ldots\mu_8}\overset{[4]}{\xi}{}^{\mu_5\ldots\mu_8}_{(3,0)}\right)\\ &\left.-\frac{dV_1}{d\varphi}\overset{[1]}{A}_{[\mu_2}\varepsilon_{\mu_3\mu_4]\nu_1\ldots\nu_5}\overset{[5]}{\xi}{}^{\nu_1\ldots\nu_5}_{(4,0)}\right],\end{aligned} \tag{28}$$

$$\begin{aligned}\bar{\delta}_{\Omega^{\alpha_1}}\overset{[2]}{B}{}^{\mu_1\mu_2} &= -3\partial_\rho\overset{[3]}{\xi}{}^{\rho\mu_1\mu_2}_{(2,0)} + \lambda\bar{\delta}^{\mathrm{self}}_{\Omega^{\alpha_1}}\overset{[2]}{B}{}^{\mu_1\mu_2} - 4\lambda\Big(4U_1\overset{[3]}{A}_{\mu_3\mu_4\mu_5}\partial^{[\mu_1}\theta^{\mu_2\mu_3\mu_4\mu_5]}\\ &-\frac{3}{5}V_1\varepsilon_{\nu_1\nu_2\ldots\nu_7}\overset{[4]}{B}{}^{\mu_1\mu_2\nu_1\nu_2}\partial^{[\nu_3}\theta^{\nu_4\ldots\nu_7]}\Big) - 3\lambda\left(F^{\mu_1\mu_2\mu_3\mu_4} + 36\lambda M^{\mu_1\mu_2\mu_3\mu_4}\right)\\ &\times\left(4U_1\overset{[2]}{\epsilon}_{(3,0)\mu_3\mu_4} + V_1\varepsilon_{\mu_3\mu_4\nu_1\ldots\nu_5}\overset{[5]}{\xi}{}^{\nu_1\ldots\nu_5}_{(4,0)}\right),\end{aligned} \tag{29}$$

$$\bar{\delta}_{\Omega^{\alpha_1}} \overset{[3]}{B}{}^{\mu_1\mu_2\mu_3} = -4\partial_\rho \overset{[4]}{\xi}{}^{\rho\mu_1\mu_2\mu_3}_{(3,0)} + \lambda \bar{\delta}^{\mathrm{self}}_{\Omega^{\alpha_1}} \overset{[3]}{B}{}^{\mu_1\mu_2\mu_3} - 24\lambda U_2 \overset{[2]}{A}_{\mu_4\mu_5} \partial^{[\mu_1} \theta^{\mu_2\mu_3\mu_4\mu_5]} + 12\lambda U_2 \left(F^{\mu_1\mu_2\mu_3\mu_4} + 36\lambda M^{\mu_1\mu_2\mu_3\mu_4}\right) \overset{[1]}{\epsilon}_{(2,0)\mu_4}, \tag{30}$$

$$\bar{\delta}_{\Omega^{\alpha_1}} \overset{[4]}{B}{}^{\mu_1\mu_2\mu_3\mu_4} = -5\partial_\rho \overset{[5]}{\xi}{}^{\rho\mu_1\mu_2\mu_3\mu_4}_{(4,0)} + \lambda \bar{\delta}^{\mathrm{self}}_{\Omega^{\alpha_1}} \overset{[4]}{B}{}^{\mu_1\mu_2\mu_3\mu_4} + 16\lambda U_1 \overset{[1]}{A}_{\mu_5} \partial^{[\mu_1} \theta^{\mu_2\mu_3\mu_4\mu_5]} + 4\lambda U_1 \left(F^{\mu_1\mu_2\mu_3\mu_4} + 36\lambda M^{\mu_1\mu_2\mu_3\mu_4}\right) \overset{[0]}{\epsilon}_{(1,0)}, \tag{31}$$

$$\begin{aligned} \delta_{\Omega^{\alpha_1}} t_{\mu_1\mu_2\mu_3\mu_4|\alpha} &= \partial_{[\mu_1} \chi_{\mu_2\mu_3\mu_4]|\alpha} + \partial_{[\mu_1} \theta_{\mu_2\mu_3\mu_4]\alpha} - 4\partial_\alpha \theta_{\mu_1\mu_2\mu_3\mu_4} \\ &+ 12\lambda \sigma_{\alpha[\mu_1} \left[U_1 \left(2\overset{[3]}{A}_{\mu_2\mu_3\mu_4]} \overset{[0]}{\epsilon}_{(1,0)} - \overset{[1]}{A}_{\mu_2} \overset{[2]}{\epsilon}_{(3,0)\mu_3\mu_4]} \right) + U_2 \overset{[2]}{A}_{\mu_2\mu_3} \overset{[1]}{\epsilon}_{(2,0)\mu_4]} \right] \\ &+ 3\lambda \sigma_{\alpha[\mu_1} \varepsilon_{\mu_2\mu_3\mu_4]\nu_1\nu_2\nu_3\nu_4} \left(4V \overset{[4]}{\xi}{}^{\nu_1\nu_2\nu_3\nu_4}_{(3,0)} - V_1 \overset{[3]}{B}{}^{\nu_1\nu_2\nu_3\nu_4} \overset{[1]}{\epsilon}_{(1,0)} - 5V_1 \overset{[1]}{A}_\rho \overset{[5]}{\xi}{}^{\rho\nu_1\nu_2\nu_3\nu_4}_{(4,0)} \right). \end{aligned} \tag{32}$$

All components carrying the index "self" stem from BF self-interactions at order one in λ. Their structure is detailed in [21]. An important remark is that that also the gauge transformations of the $(k,1)$ tensor field are deformed with respect to the initial ones.

The entire gauge structure of the interacting model being controlled by the functions $M(\varphi)$, $(W_i(\varphi))_{i=\overline{1,6}}$, $Z_2(\varphi)$, and $(Y_j(\varphi))_{j=2,3}$, which are restricted to satisfy the consistency equations (16)–(23), our procedure is valid provided these equations possess solutions. We list below one class of solutions, suggestively resembling to Liouville-type field theories: $W_2 = W_3 = Y_2 = Y_3 = 0$, $W_4 = \alpha W_1$, $W_5 = k_2 \exp[-2(3\alpha - 2\beta)\varphi]$, $W_6 = \beta W_1$, $M = k_1$, $Z_2 = k_3 \exp[(3\alpha + 4\beta)\varphi]$, $U_1 = \theta W_1$, $U_2 = k_2 \exp(-6\alpha\varphi)\left(\frac{\vartheta}{k_3} + \frac{\theta}{\beta}\exp 4\beta\varphi\right)$, $V = \frac{k_3}{k_2} U_2 \exp(9\alpha\varphi)$. In the above W_1 and V_1 are arbitrary functions and α, β, θ, k_1, k_2, k_3, and ϑ are some arbitrary, non-vanishing real constants. Replacing back the above solution into action (15) and its gauge transformations (24)–(32), we are then able to determine the entire structure of this particular kind of BF model coupled to DFLG in $D = 7$.

3.2. RESULTS FOR $k = 5$, ALIAS $D = 8$

This situation amounts to $I(5) = 3$ and m is again single valued, $m = 3$, like for $k = 4$. Conditions (10) exhibit exactly the same solutions, $m_1 = 1$. This observation holds in general, for any two models with consecutive values k and $(k+1)$ if k is even, in which case $I(k)$ and $I(k+1)$ are equal, so both models present the same structure of the cross-coupling vertices that break the PT invariance (they may differ by a phase factor). Analysing now conditions (11), we find that $\bar{N} = \overline{2, V_5}$, while (12) yields $V_5 = 3$. For each allowed value of $\bar{N}$, the set (11) inherits a unique solution: $\bar{n}_1 = 2$, $\bar{n}_2 = 3$ for $\bar{N} = 2$; $\bar{n}_1 = 1$, $\bar{n}_2 = \bar{n}_3 = 2$ for $\bar{N} = 3$. The associated parametrizing functions are $U_{(2,3)}$ and $U_{(1,2,2)}$, to be turned into $U_{(2,3)} \equiv U_3$ and respectively $U_{(1,2,2)} \equiv U_4$ for notational simplicity purposes.

Consequently, (9) and the coupled Lagrangian action display the forms

$$M_{\mu_1\mu_2\mu_3\mu_4\mu_5} = -\varepsilon_{\mu_1\mu_2\mu_3\mu_4\mu_5\mu_6\mu_7\mu_8}\left(V\overset{[3]}{B}{}^{\mu_6\mu_7\mu_8} + V_1\overset{[1]}{A}_\rho\overset{[4]}{B}{}^{\rho\mu_6\mu_7\mu_8}\right) + U_3\overset{[2]}{A}_{[\mu_1\mu_2}\overset{[3]}{A}_{\mu_3\mu_4\mu_5]} + U_4\overset{[1]}{A}_{[\mu_1}\overset{[2]}{A}_{\mu_2\mu_3}\overset{[2]}{A}_{\mu_4\mu_5]}, \tag{33}$$

$$\begin{aligned}\bar{S}^{\mathrm{L}}\left[\Phi^{\alpha_0}\right] = & S^{\mathrm{L}}\left[\Phi^{\alpha_0}\right] + \lambda\int d^8x\Bigg[M(\varphi) + W_1\overset{[1]}{A}_{\mu_1}\overset{[1]}{B}{}^{\mu_1} + W_2\overset{[2]}{A}_{\mu_1\mu_2}\overset{[2]}{B}{}^{\mu_1\mu_2}\\ & + \left(W_3\overset{[3]}{A}_{\mu_1\mu_2\mu_2} + W_4\overset{[1]}{A}_{[\mu_1}\overset{[2]}{A}_{\mu_2\mu_3]}\right)\overset{[3]}{B}{}^{\mu_1\mu_2\mu_3} + \left(W_5\overset{[2]}{A}_{[\mu_1\mu_2}\overset{[2]}{A}_{\mu_3\mu_4]}\right.\\ & \left. + W_6\overset{[1]}{A}_{[\mu_1}\overset{[3]}{A}_{\mu_2\mu_3\mu_4]}\right)\overset{[4]}{B}{}^{\mu_1\mu_2\mu_3\mu_4} + \varepsilon_{\mu_1\ldots\mu_8}\left(\tfrac{1}{2}Z_1\overset{[4]}{B}{}^{\mu_1\cdots\mu_4}\overset{[4]}{B}{}^{\mu_5\cdots\mu_8}\right.\\ & \left. + \tfrac{1}{2}R_1\overset{[1]}{A}{}^{\mu_1}\overset{[2]}{A}{}^{\mu_2\mu_3}\overset{[3]}{A}{}^{\mu_4\mu_5}\overset{[3]}{A}{}^{\mu_6\mu_7\mu_8} + \tfrac{1}{4!}R_2\overset{[2]}{A}{}^{\mu_1\mu_2}\overset{[2]}{A}{}^{\mu_3\mu_4}\overset{[2]}{A}{}^{\mu_5\mu_6}\overset{[2]}{A}{}^{\mu_7\mu_8}\right)\\ & + M_{\mu_1\ldots\mu_5}F^{\mu_1\cdots\mu_5}\Bigg] + 90\lambda^2\int d^8x M_{\mu_1\ldots\mu_5}M^{\mu_1\cdots\mu_5}.\end{aligned} \tag{34}$$

The key point of our procedure — the consistency conditions, have been computed and are listed below:

$$W_1\frac{dW_2}{d\varphi} + 3W_4W_2 = 0, \quad W_1\frac{dW_3}{d\varphi} + (4W_6 - 3W_4)W_3 = 0, \tag{35}$$

$$W_1\frac{dZ_1}{d\varphi} - 8W_6Z_1 = 0, \quad W_1\frac{dR_2}{d\varphi} + 12(W_4R_2 - 3W_5R_1) = 0, \tag{36}$$

$$W_1\frac{dW_5}{d\varphi} + 2[(3W_4 - 2W_6)W_5 + 48Z_1R_1] = 0, \quad W_1\frac{dM}{d\varphi} = 0, \tag{37}$$

$$W_2W_4 + W_3W_5 = 0, \quad 2W_2W_6 + 3W_3W_5 = 0, \quad 3W_2R_1 + W_3R_2 = 0, \tag{38}$$

$$W_1W_2 = 0, \quad W_2W_3 = 0, \quad W_3Z_1 = 0, \tag{39}$$

$$W_1\frac{dU_3}{d\varphi} + (3W_4 + 4W_6)U_3 + 3(-W_3U_4 + 24R_1V) = 0, \tag{40}$$

$$W_1\frac{dV}{d\varphi} - 3W_4V + W_3V_1 = 0, \quad W_2U_4 + 2(W_5U_3 + 4R_2V) = 0, \tag{41}$$

$$W_2V_1 + 2(3W_5V - 4Z_1U_3) = 0, \quad W_2V = 0. \tag{42}$$

The completely deformed solution to the master equation offer the associated gauge symmetries of action (34):

$$\bar{\delta}_{\Omega^{\alpha_1}}\varphi = \lambda\bar{\delta}^{(\mathrm{self})}_{\Omega^{\alpha_1}}\varphi, \tag{43}$$

$$\bar{\delta}_{\Omega^{\alpha_1}}\overset{[1]}{A}_{\mu_1} = \partial_{\mu_1}\overset{[0]}{\epsilon}_{(1,0)} + \lambda\bar{\delta}^{(\mathrm{self})}_{\Omega^{\alpha_1}}\overset{[1]}{A}_{\mu_1}, \tag{44}$$

$$\bar{\delta}_{\Omega^{\alpha_1}}\overset{[2]}{A}_{\mu_1\mu_2} = \partial_{[\mu_1}\overset{[1]}{\epsilon}_{(2,0)\mu_2]} + \lambda\bar{\delta}^{(\mathrm{self})}_{\Omega^{\alpha_1}}\overset{[2]}{A}_{\mu_1\mu_2} + \frac{5}{2}\lambda V\varepsilon_{\mu_1\mu_2\ldots\mu_8}\partial^{[\mu_3}\theta^{\mu_4\ldots\mu_8]}, \tag{45}$$

$$\begin{aligned}\bar{\delta}_{\Omega^{\alpha_1}}\overset{[3]}{A}_{\mu_1\mu_2\mu_3} =& \partial_{[\mu_1}\overset{[2]}{\epsilon}_{(3,0)\mu_2\mu_3]} + \lambda\bar{\delta}^{(\mathrm{self})}_{\Omega^{\alpha_1}}\overset{[3]}{A}_{\mu_1\mu_2\mu_3} - 5\lambda V_1\varepsilon_{\mu_1\mu_2\mu_3\mu_4\ldots\mu_8}\overset{[1]}{A}_{\rho}\partial^{[\rho}\theta^{\mu_4\ldots\mu_8]}\\ &- \lambda\varepsilon_{\mu_1\mu_2\mu_3\mu_4\ldots\mu_8}V_1\left(F^{\mu_4\ldots\mu_8} + 180\lambda M^{\mu_4\ldots\mu_8}\right)\overset{[0]}{\epsilon}_{(1,0)},\end{aligned} \tag{46}$$

$$\begin{aligned}\bar{\delta}_{\Omega^{\alpha_1}}\overset{[1]}{B}{}^{\mu_1} =& -2\partial_{\rho}\overset{[2]}{\xi}{}^{\rho\mu_1}_{(1,0)} + \lambda\bar{\delta}^{(\mathrm{self})}_{\Omega^{\alpha_1}}\overset{[1]}{B}{}^{\mu_1}\\ &- 5\lambda\left(\frac{dU_3}{d\varphi}\overset{[2]}{A}_{[\mu_2\mu_3}\overset{[3]}{A}_{\mu_4\mu_5\mu_6]} + \frac{dU_4}{d\varphi}\overset{[1]}{A}_{[\mu_2}\overset{[2]}{A}_{\mu_3\mu_4}\overset{[2]}{A}_{\mu_5\mu_6]}\right)\partial^{[\mu_1}\theta^{\mu_2\ldots\mu_6]}\\ &+ \frac{5}{2}\lambda\varepsilon_{\nu_1\ldots\nu_8}\left(-\frac{dV}{d\varphi}\overset{[3]}{B}{}^{\mu_1\nu_1\nu_2} + \frac{dV_1}{d\varphi}\overset{[1]}{A}_{\mu_2}\overset{[4]}{B}{}^{\mu_1\mu_2\nu_1\nu_2}\right)\partial^{[\nu_3}\theta^{\nu_4\ldots\nu_8]}\\ &+ 5\lambda\left(F^{\mu_1\mu_2\mu_3\mu_4\mu_5} + 180\lambda M^{\mu_1\mu_2\mu_3\mu_4\mu_5}\right)\left[\left(\frac{dU_4}{d\varphi}\overset{[2]}{A}_{[\mu_2\mu_3}\overset{[2]}{A}_{\mu_4\mu_5]}\right.\right.\\ &\left. - \frac{1}{4}\frac{dV_1}{d\varphi}\varepsilon_{\mu_2\ldots\mu_5\nu_1\nu_2\nu_3\nu_4}\overset{[4]}{B}{}^{\nu_1\nu_2\nu_3\nu_4}\right)\overset{[0]}{\epsilon}_{(1,0)}\\ &- \left(\frac{dU_3}{d\varphi}\overset{[3]}{A}_{[\mu_2\mu_3\mu_4} + \frac{dU_4}{d\varphi}\overset{[1]}{A}_{[\mu_2}\overset{[2]}{A}_{\mu_3\mu_4}\right)\overset{[1]}{\epsilon}_{(2,0)\mu_5]} + \frac{dU_3}{d\varphi}\overset{[2]}{A}_{[\mu_2\mu_3}\overset{[2]}{\epsilon}_{(3,0)\mu_4\mu_5]}\\ &\left. + \frac{dV}{d\varphi}\varepsilon_{\mu_2\ldots\mu_5\nu_1\nu_2\nu_3\nu_4}\overset{[4]}{\xi}{}^{\nu_1\nu_2\nu_3\nu_4}_{(3,0)} + \frac{dV_1}{d\varphi}\overset{[1]}{A}_{\mu_2}\varepsilon_{\mu_3\mu_4\mu_5\nu_1\ldots\nu_5}\overset{[5]}{\xi}{}^{\nu_1\ldots\nu_5}_{(4,0)}\right],\end{aligned} \tag{47}$$

$$\begin{aligned}\bar{\delta}_{\Omega^{\alpha_1}}\overset{[2]}{B}{}^{\mu_1\mu_2} =& -3\partial_{\rho}\overset{[3]}{\xi}{}^{\rho\mu_1\mu_2}_{(2,0)} + \lambda\bar{\delta}^{(\mathrm{self})}_{\Omega^{\alpha_1}}\overset{[2]}{B}{}^{\mu_1\mu_2}\\ &- 5\lambda\left(5U_4\overset{[2]}{A}_{[\mu_3\mu_4}\overset{[2]}{A}_{\mu_5\mu_6]}\partial^{[\mu_1}\theta^{\mu_2\ldots\mu_6]} - \frac{1}{2}V_1\varepsilon_{\nu_1\ldots\nu_8}\overset{[4]}{B}{}^{\mu_1\mu_2\nu_1\nu_2}\partial^{[\nu_3}\theta^{\nu_4\ldots\nu_8]}\right)\\ &- 5\lambda\left(F^{\mu_1\mu_2\mu_3\mu_4\mu_5} + 180\lambda M^{\mu_1\mu_2\mu_3\mu_4\mu_5}\right)\times\\ &\times\left(4U_4\overset{[2]}{A}_{[\mu_3\mu_4}\overset{[1]}{\epsilon}_{(2,0)\mu_5]} - V_1\varepsilon_{\mu_3\mu_4\mu_5\nu_1\ldots\nu_5}\overset{[5]}{\xi}{}^{\nu_1\ldots\nu_5}_{(4,0)}\right),\end{aligned} \tag{48}$$

$$\bar{\delta}_{\Omega^{\alpha_1}} \overset{[3]}{B}{}^{\mu_1\mu_2\mu_3} = -4\partial_\rho \overset{[4]}{\xi}{}^{\rho\mu_1\mu_2\mu_3}_{(3,0)} + \lambda\bar{\delta}^{(\mathrm{self})}_{\Omega^{\alpha_1}} \overset{[3]}{B}{}^{\mu_1\mu_2\mu_3}$$
$$-50\lambda \left(U_3 \overset{[3]}{A}_{\mu_4\mu_5\mu_6} + U_4 \overset{[1]}{A}_{[\mu_4} \overset{[2]}{A}_{\mu_5\mu_6]} \right) \partial^{[\mu_1}\theta^{\mu_2\mu_3\mu_4\mu_5\mu_6]}$$
$$+30\lambda \left(F^{\mu_1\mu_2\mu_3\mu_4\mu_5} + 180\lambda M^{\mu_1\mu_2\mu_3\mu_4\mu_5} \right) \left(U_4 \overset{[2]}{A}_{\mu_4\mu_5} \overset{[0]}{\epsilon}_{(1,0)} \right.$$
$$\left. -U_4 \overset{[1]}{A}_{[\mu_4} \overset{[1]}{\epsilon}_{(2,0)\mu_5]} + U_3 \overset{[2]}{\epsilon}_{(3,0)\mu_4\mu_5} \right), \tag{49}$$

$$\bar{\delta}_{\Omega^{\alpha_1}} \overset{[4]}{B}{}^{\mu_1\mu_2\mu_3\mu_4} = -5\partial_\rho \overset{[5]}{\xi}{}^{\rho\mu_1\mu_2\mu_3\mu_4}_{(4,0)} + \lambda\bar{\delta}^{(\mathrm{self})}_{\Omega^{\alpha_1}} \overset{[4]}{B}{}^{\mu_1\mu_2\mu_3\mu_4} - 50\lambda U_3 \overset{[2]}{A}_{\mu_5\mu_6} \partial^{[\mu_1}\theta^{\mu_2\mu_3\mu_4\mu_5\mu_6]}$$
$$-20\lambda U_3 \left(F^{\mu_1\mu_2\mu_3\mu_4\mu_5} + 180\lambda M^{\mu_1\mu_2\mu_3\mu_4\mu_5} \right) \overset{[1]}{\epsilon}_{(2,0)\mu_5}, \tag{50}$$

$$\delta_{\Omega^{\alpha_1}} t_{\mu_1\mu_2\mu_3\mu_4\mu_5|\alpha} = \partial_{[\mu_1}\chi_{\mu_2\mu_3\mu_4\mu_5]|\alpha} + \partial_{[\mu_1}\theta_{\mu_2\mu_3\mu_4\mu_5]\alpha} + 5\partial_\alpha\theta_{\mu_1\mu_2\mu_3\mu_4\mu_5}$$
$$+60\lambda\sigma_{\alpha[\mu_1} \left[U_3 \left(\overset{[3]}{A}_{\mu_2\mu_3\mu_4} \overset{[1]}{\epsilon}_{(2,0)\mu_5]} - \overset{[2]}{A}_{\mu_2\mu_3} \overset{[2]}{\epsilon}_{(3,0)\mu_4\mu_5]} \right) \right.$$
$$\left. -U_4 \left(\overset{[2]}{A}_{\mu_2\mu_3} \overset{[2]}{A}_{\mu_4\mu_5]} \overset{[0]}{\epsilon}_{(1,0)} - \overset{[1]}{A}_{\mu_2} \overset{[2]}{A}_{\mu_3\mu_4} \overset{[1]}{\epsilon}_{(2,0)\mu_5]} \right) \right] \tag{51}$$
$$+15\lambda\sigma_{\alpha[\mu_1}\varepsilon_{\mu_2\mu_3\mu_4\mu_5]\nu_1\ldots\nu_4} \left(-4V \overset{[4]}{\xi}{}^{\nu_1\ldots\nu_4}_{(3,0)} + V_1 \overset{[4]}{B}{}^{\nu_1\ldots\nu_4} \overset{[0]}{\epsilon}_{(1,0)} + 5V_1 \overset{[1]}{A}_\rho \overset{[5]}{\xi}{}^{\rho\nu_1\ldots\nu_4}_{(4,0)} \right).$$

Finally, we give several solutions to the consistency equations (35)–(42), which involve all the parameterizing functions. A first class of solutions, suggestively resembling to Liouville-type field theories, is: $W_2 = W_3 = 0$, $W_4 = \alpha W_1$, $W_5 = -\frac{48\rho}{3\alpha+2\beta} Z_1$, $W_6 = \beta W_1$, $M = k_1$, $Z_1 = k_2 \exp(8\beta\varphi)$, $R_1 = \rho W_1$, $R_2 = -\frac{432\rho^2}{(3\alpha+2\beta)^2} Z_1$, $U_3 = -\frac{36\rho}{3\alpha+2\beta} V$, $V = k_3 \exp(3\alpha\varphi)$, where W_1, U_4, and V_1 are arbitrary functions, with α, β, ρ, k_1, k_2, and k_3 are arbitrary, non-vanishing constants satisfying $3\alpha + 2\beta \neq 0$. A second class of solutions is represented by: $W_1 = W_2 = W_5 = Z_1 = R_2 = 0$, $W_4 = \frac{4}{3} W_6$, $U_3 = \alpha W_3$, $U_4 = \frac{8\alpha}{2} W_6 + 24\beta R_1$, $V = \beta W_3$, $V_1 = 4\beta W_6$, with M, W_3, W_6, and R_1 arbitrary functions and α, β non-vanishing constants. The third class reads as: $W_1 = W_2 = W_3 = Z_1 = U_3 = V = 0$, $W_4 = \frac{2}{3} W_6$, $R_1 = \alpha W_6$, $R_2 = \frac{9\alpha}{2} W_5$, where M, W_5, W_6, U_4, and V_1 are arbitrary functions and α, β denote some non-vanishing constants. The fourth and last class of solutions corresponds to the choice: $W_1 = W_2 = W_3 = W_4 = W_5 = W_6 = Z_1 = R_1 = R_2 = 0$, while M and $(V_i)_{i=\overline{1,4}}$ remain arbitrary. Each of the above solutions replaced in (34) and (43)–(51)

produces a different coupled model, with distinct gauge behaviours.

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