

TOWARDS A FULL CLASSIFICATION OF CONSISTENT INTERACTION VERTICES IN TOPOLOGICAL BF THEORIES*

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All consistent self-interactions for a topological BF model with a maximal field spectrum in an arbitrary spacetime dimension are computed and classified. We use the elegant method of constructing consistent interactions in gauge field theories based on the calculation of the local BRST cohomology under standard hypotheses. The major result is that there appear three main classes of vertices: one involving only A -type forms, another linear in B -like forms, and a third category bilinear in B -like forms. The precise structure of the vertices for a given BF model strongly depends on the spacetime dimension D . The general results are exemplified in detail for $D = 7$ and $D = 8$. The self-interacting Lagrangian always stops at order one in the coupling constant, but its consistency at all orders is dictated by a set of algebraic and differential equations restricting the functions that parameterize the self-couplings. The interacting model is shown to describe a highly nontrivial gauge theory, whose gauge structure is fully modified with respect to its free limit: gauge transformations, their algebra, reducibility functions, reducibility behaviour.

Key words: topological BF theories, consistent interactions in gauge field theory, local BRST cohomology.

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1. INTRODUCTION

Topological field theories [1] are important in view of the fact that certain interacting, non-Abelian versions are related to a Poisson structure algebra [2] present in various versions of Poisson sigma models [3–5], which are known to be useful at the study of two-dimensional gravity [6]. It is well known that pure three-dimensional gravity is just a BF theory. Moreover, in higher dimensions general relativity and supergravity in Ashtekar formalism may also be formulated as topological BF theories with some extra constraints [7–10].

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The main aim of this paper is to fully compute and classify all consistent self-interactions that can be added to a free topological BF model with a maximal field spectrum in an arbitrary spacetime dimension. The method employed at the construction of self-couplings is based on the deformation of the solution to the classical master equation [11, 12] with the help of cohomological techniques based on the computation of local BRST cohomology [13–15]. The self-interactions among the BF form fields have been obtained under the standard hypotheses from field theory: analyticity in the coupling constant, spacetime locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the preservation of the number of derivatives on each field. This paper extends and generalizes previous results from [16–18].

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2. MAIN RESULTS. CLASSIFICATION OF SELF-INTERACTING VERTICES

The starting point is a free theory in an arbitrary dimension D , described by the Lagrangian action of a topological BF model with a maximal field spectrum that contains two classes of form gauge fields, denoted by $(A, B)_{m=0, I(D)}$

$$S^L[\Phi^{\alpha_0}] = \int d^D x \left(\sum_{m=0}^{I(D)} \frac{1}{m+1} B^{[m+1]\mu_1 \dots \mu_{m+1}} \partial_{[\mu_1} A_{\mu_2 \dots \mu_{m+1}]}^{[m]} \right), \quad (1)$$

where $I(D) = \lfloor \frac{D-1}{2} \rfloor$ and the notation $[a]$ signifies the integer part of a . The over-script between brackets represents the form degree (A is a m -form and B a $(m+1)$ -form). The notation $[\mu_1 \mu_2 \dots \mu_n]$ signifies complete antisymmetry with respect to the indices between brackets, with the conventions that the minimum number of terms is always used and the result is never divided by the number of terms. In this paper we work with the Minkowski metric of ‘mostly plus’ signature

$\sigma_{\mu\nu} = \sigma^{\mu\nu} = \text{diag}(- + \dots +)$ and with the Levi-Civita symbol $\varepsilon^{\mu_1 \dots \mu_D}$ defined according to the convention $\varepsilon^{01 \dots D-1} = -\varepsilon_{01 \dots D-1} = -1$. In (1) we denoted all the fields of the theory by

$$\Phi^{\alpha_0} = \left\{ \left(\begin{array}{c} [m] \\ A_{\mu_1 \dots \mu_m}, \quad [m+1] \\ B^{\mu_1 \dots \mu_{m+1}} \end{array} \right)_{m=0, \overline{I(D)}} \right\}. \quad (2)$$

The above Lagrangian action is invariant under the generating set of gauge transformations

$$\delta_{\Omega^{\alpha_1}}^{[0]} A = 0, \quad \delta_{\Omega^{\alpha_1}}^{[m]} A_{\mu_1 \dots \mu_m} = \partial_{[\mu_1} \epsilon_{(m,0)\mu_2 \dots \mu_m]}^{[m-1]}, \quad m = \overline{1, I(D)}, \quad (3)$$

$$\delta_{\Omega^{\alpha_1}}^{[m+1]} B^{\mu_1 \dots \mu_{m+1}} = -(m+2) \partial_\rho \xi_{(m+1,0)}^{[m+2]\rho\mu_1 \dots \mu_{m+1}}, \quad m = \overline{0, I(D)}, \quad (4)$$

where the gauge parameters have been denoted collectively by Ω^{α_1}

$$\Omega^{\alpha_1} = \left\{ \left(\begin{array}{c} [m-1] \\ \epsilon_{(m,0)\mu_1 \dots \mu_{m-1}} \end{array} \right)_{m=1, \overline{I(D)}}, \left(\begin{array}{c} [m+2] \\ \xi_{(m+1,0)}^{\mu_1 \dots \mu_{m+2}} \end{array} \right)_{m=0, \overline{I(D)}} \right\}. \quad (5)$$

All the gauge parameters are bosonic and completely antisymmetric (where applicable). The overscript represents the form degree, while the other two lower indices between parentheses signify the form field to which a certain gauge parameter is associated with and respectively the reducibility level. The above gauge transformations are Abelian and off-shell $(D-2)$ -order reducible.

It has been shown in the literature [11, 12] that it is possible to reformulate the problem of constructing consistent interactions as a deformation problem of the solution to the classical master equation corresponding to a given free theory. This procedure can be solved with the help of the local BRST cohomology [13–15].

Using the above technique we completely computed the first-order deformation for the free BF theory under study, which complies with all the working hypotheses, S_1 . It is parametrized in terms of some functions denoted by Z and W that may

depend at most on the undifferentiated scalar field $\overset{[0]}{A} \equiv \varphi$. By computing (S_1, S_1) we have shown that all the higher-order deformations are trivial and the parametrizing functions are restricted to satisfy a set of algebraic and differential equations, called consistency equations. Once we have computed the fully deformed solution to the master equation, we are able to extract from it the entire gauge formulation of the self-coupled BF model.

The piece with antighost zero from the fully deformed solution to the master equation is nothing but the Lagrangian action of the interacting gauge theory and has

the general form

$$\begin{aligned}
\bar{S}^L[\Phi^{\alpha_0}] &= S^L[\Phi^{\alpha_0}] + \lambda \int d^D x \left(M(\varphi) \right. \\
&+ \sum_{m=0}^{I(D)} \sum_{N=1}^{V_{m+1}} \sum_{m_1, m_2, \dots, m_N \in \mathbb{N}^*} W_{(m_1, m_2, \dots, m_N)} \times \\
&\times \frac{[m_1]}{A} \frac{[m_2]}{A} \cdots \frac{[m_N]}{A} \frac{[m+1]^{\mu_1 \dots \mu_{m+1}}}{B} + \\
&+ \sum_{\bar{N}=3}^{V_D} \sum_{\bar{m}_1, \bar{m}_2, \dots, \bar{m}_{\bar{N}} \in \mathbb{N}^*} \left(\prod_{i=1}^{\bar{N}} \bar{m}_i! \right) \left[\left(\sum_{i=1}^{\bar{N}} \bar{m}_i \right)! \right]^{-1} Z_{(\bar{m}_1, \dots, \bar{m}_{\bar{N}})} \times \quad (6) \\
&\times \varepsilon_{\mu_1 \dots \mu_D} \frac{[\bar{m}_1]^{\mu_1} [\bar{m}_2]^{\mu_2} \cdots [\bar{m}_{\bar{N}}]^{\mu_D}}{A} \\
&+ \delta_D^{4m} \frac{1}{2} \varepsilon_{\mu_1 \dots \mu_{4m}} Z_1 \frac{[2m]^{\mu_1 \dots \mu_{2m}} [2m]^{\mu_{2m+1} \dots \mu_{4m}}}{B} \\
&+ \delta_D^{2m+1} \varepsilon_{\mu_1 \dots \mu_{2m+1}} Z_2 \frac{[m]^{\mu_1 \dots \mu_m} [m+1]^{\mu_{m+1} \dots \mu_{2m+1}}}{B} \\
&+ \delta_D^{4m+1} \frac{1}{2} \varepsilon_{\mu_1 \dots \mu_{4m+1}} Z_3 A_\lambda \frac{[2m+1]^{\lambda \mu_1 \dots \mu_{2m}} [2m+1]^{\mu_{2m+1} \dots \mu_{4m+1}}}{B} \left. \right).
\end{aligned}$$

Here, λ stands for the coupling constant (deformation parameter) and N, \bar{N} count the number of A 's in each interaction vertex. In the second term from (6) N starts from 2 for the maximum value of m because the field spectrum does not contain an A -type form that may couple to the highest B -like form. The limits V_{m+1} and V_D from (6) read as

$$V_{m+1} = \left[\frac{m+1}{2} \right] + (m+1) \bmod 2, \quad V_D = \left[\frac{D}{2} \right] + D \bmod 2 \quad (7)$$

and the sums after $(m_1, m_2, \dots, m_N \in \mathbb{N}^*)$ and $(\bar{m}_1, \bar{m}_2, \dots, \bar{m}_{\bar{N}} \in \mathbb{N}^*)$ are restricted to satisfy the conditions

$$\begin{aligned}
m_1 + m_2 + \dots + m_N &= m + 1, \\
1 \leq m_1 \leq m_2 \leq \dots \leq m_N &\leq I(D), \\
m_i = m_j &\Leftrightarrow m_i = 2n, \quad (8)
\end{aligned}$$

with $m = \overline{0, I(D)}$ and $N = \overline{1, V_{m+1}}$, and respectively

$$\begin{aligned}
\bar{m}_1 + \bar{m}_2 + \dots + \bar{m}_{\bar{N}} &= D, \\
1 \leq \bar{m}_1 \leq \bar{m}_2 \leq \dots \leq \bar{m}_{\bar{N}} &\leq I(D), \\
\bar{m}_i = \bar{m}_j &\Leftrightarrow \bar{m}_i = 2n, \quad (9)
\end{aligned}$$

with $\bar{N} = \overline{3, V_D}$.

We remark that there are four types of interaction vertices, parametrized by the smooth functions $W_{(m_1, m_2, \dots, m_N)}$, $Z_{(\bar{m}_1, \dots, \bar{m}_{\bar{N}})}$, $(Z_i)_{i=\overline{1,3}}$, which depend only on the undifferentiated scalar field. As we can see, only the first type of couplings is PT-invariant, the other couplings breaking this invariance. Recall that these functions are not arbitrary, but subject to the consistency equations. Analysing (6), we can classify all self-interaction vertices according to the number of B -like forms: (1) with more than two B s there is none; (2) with two B s there appear two kinds — (2a) one class of vertices that couple two B s and one A field only in $D = 4m + 1, m \geq 1$, and (2b) two kinds of vertices that couple precisely two B s, namely, one in $D = 4m$ and the other in $D = 2m + 1, m \geq 1$; (3) with a single B -type form and at least one A field; (4) with no B s and at least three A s.

3. EXAMPLE. RESULTS FOR $D = 7$

In this section we particularize the general results from the previous section to $D = 7$. Since $I(D) = 3$, the field spectrum consists of one scalar field $\overset{[0]}{A} \equiv \varphi$, two types of one-forms, two kinds of two-forms, two sorts of three-forms, and one four-form.

The Lagrangian description for the interacting theory in $D = 7$ stems from all the distinct solutions to equations (8) and (9). By solving (8) for $m = \overline{0, 3}$ we obtain all the couplings between A s and B s, while from (9) we determine the interaction vertices that contain only A s (see (10)). On behalf of (7) and since for a given m the number of A s is $N = \overline{1, V_{m+1}}$, it follows that (8) possesses six distinct solutions.

For $m = 0$ and $m = 1$ we get a single solution, namely $m_1 = 1$ and respectively $m_1 = 2$. For $m = 2$ conditions (8) exhibit two distinct solutions: $m_1 = 3$ and $m_1 = 1, m_2 = 2$. Finally, for $m = 3$ there are two solutions, both for $N = 2$: $m_1 = 1, m_2 = 3$ and $m_1 = m_2 = 2$. We remark that in the last case the solution $m_1 = 4$ for $N = 1$ is excluded as the field spectrum does not contain a four-form of the type A . Therefore, the functions that parametrize these couplings for this example reduce to $W_{(1)}$, $W_{(2)}$, $W_{(3)}$, $W_{(1,2)}$, $W_{(1,3)}$ and $W_{(2,2)}$. For simplicity, we redenote them by $(W_{(i)})_{i=\overline{1,3}} \equiv (W_i)_{i=\overline{1,3}}$, $W_{(1,2)} \equiv W_4$, $W_{(1,3)} \equiv W_5$, $W_{(2,2)} \equiv W_6$.

Along the same line we analyse the solutions to conditions (9). In this case we have $\bar{N} = 3, 4$ ($\bar{N} = \overline{3, V_D}$, and $V_7 = 4$, see (7)), so there are two distinct solutions to (9): $\bar{m}_1 = \bar{m}_2 = 2, \bar{m}_3 = 3$ and $\bar{m}_1 = 1, \bar{m}_2 = \bar{m}_3 = \bar{m}_4 = 2$. The functions that parametrize the couplings will consequently be $Z_{(2,2,3)}$ and $Z_{(1,2,2,2)}$, to be redenoted by $Z_{(2,2,3)} \equiv Y_2$, $Z_{(1,2,2,2)} \equiv Y_3$.

In conclusion, the concrete expression of the Lagrangian action for the coupled

theory given in (6) in $D = 7$ reads

$$\begin{aligned}
\bar{S}^L[\Phi^{\alpha_0}] = S^L[\Phi^{\alpha_0}] + \lambda \int d^7x \left[M(\varphi) + W_1 A_{\mu_1}^{[1]} B^{[1]^{\mu_1}} + W_2 A_{\mu_1 \mu_2}^{[2]} B^{[2]^{\mu_1 \mu_2}} \right. \\
+ W_3 A_{\mu_1 \mu_2 \mu_3}^{[3]} B^{[3]^{\mu_1 \mu_2 \mu_3}} + W_4 A_{[\mu_1}^{[1]} A_{\mu_2 \mu_3]}^{[2]} B^{[3]^{\mu_1 \mu_2 \mu_3}} \\
+ W_5 A_{[\mu_1 \mu_2}^{[2]} A_{\mu_3 \mu_4]}^{[2]} B^{[4]^{\mu_1 \mu_2 \mu_3 \mu_4}} + W_6 A_{[\mu_1}^{[1]} A_{\mu_2 \mu_3 \mu_4]}^{[3]} B^{[4]^{\mu_1 \mu_2 \mu_3 \mu_4}} \\
+ \varepsilon_{\mu_1 \dots \mu_7} \left(Z_2 B^{[3]^{\mu_1 \mu_2 \mu_3}} B^{[4]^{\mu_4 \mu_5 \mu_6 \mu_7}} + \frac{1}{2} Y_2 A^{[2]^{\mu_1 \mu_2}} A^{[2]^{\mu_3 \mu_4}} A^{[3]^{\mu_5 \mu_6 \mu_7}} \right. \\
\left. \left. + \frac{1}{3!} Y_3 A^{[1]^{\mu_1}} A^{[2]^{\mu_2 \mu_3}} A^{[2]^{\mu_4 \mu_5}} A^{[2]^{\mu_6 \mu_7}} \right) \right]. \quad (10)
\end{aligned}$$

We remark that in this case the couplings parametrized by Z_1 and Z_3 that appear in (10) are not permitted due to the value of the spacetime dimension. Also, the number of vertices that couple the A s to the B s is higher than the number of vertices that couple only A s among themselves. The self-interactions are parametrized in $D = 7$ by nine functions $M(\varphi)$, $(W_i(\varphi))_{i=1,6}$, $(Y_j(\varphi))_{j=2,3}$, and $Z_2(\varphi)$. The consistency equations mentioned in the previous section take in this case the form:

$$W_1 \frac{dW_2}{d\varphi} + 3W_4 W_2 = 0, \quad W_1 \frac{dW_3}{d\varphi} + (4W_6 - 3W_4)W_3 = 0, \quad (11)$$

$$W_1 \frac{dZ_2}{d\varphi} - (3W_4 + 4W_6)Z_2 = 0, \quad W_1 \frac{dY_2}{d\varphi} + 2(3W_4 + 2W_6)Y_2 - W_3 Y_3 = 0, \quad (12)$$

$$W_1 \frac{dW_5}{d\varphi} + 2(3W_4 - 2W_6)W_5 + 24Z_2 Y_3 = 0, \quad W_1 \frac{dM}{d\varphi} = 0, \quad (13)$$

$$W_2 W_4 + W_3 W_5 - 24Z_2 Y_2 = 0, \quad 2W_2 W_6 + 3(W_3 W_5 + 24Z_2 Y_2) = 0, \quad (14)$$

$$W_2 Y_3 + 9W_5 Y_2 = 0, \quad W_1 W_2 = 0, \quad W_2 W_3 = 0, \quad W_2 Z_2 = 0, \quad W_3 Z_2 = 0. \quad (15)$$

The deformed generating set of gauge transformations follows from the components of antighost number one present in the deformed solution to the master equation and takes the concrete form

$$\bar{\delta}_{\Omega^{\alpha_1}} \varphi = -\lambda W_1 \epsilon_{(1,0)}^{[0]}, \quad (16)$$

$$\bar{\delta}_{\Omega^{\alpha_1}} A_{\mu_1}^{[1]} = \partial_{\mu_1} \epsilon_{(1,0)}^{[0]} - 2\lambda W_2 \epsilon_{(2,0)\mu_1}^{[1]}, \quad (17)$$

$$\begin{aligned} \bar{\delta}_{\Omega^{\alpha_1}}^{[2]} A_{\mu_1 \mu_2} &= \partial_{[\mu_1}^{[1]} \epsilon^{(2,0)\mu_2]} + 3\lambda \left[W_4 \left(-A_{\mu_1 \mu_2}^{[2]} \epsilon^{(1,0)} + A_{[\mu_1}^{[1]} \epsilon^{(2,0)\mu_2]} \right) \right. \\ &\quad \left. - W_3^{[2]} \epsilon^{(3,0)\mu_1 \mu_2} + \varepsilon_{\mu_1 \mu_2 \mu_3 \dots \mu_7} Z_2^{[5]^{\mu_3 \dots \mu_7}} \xi^{(4,0)} \right], \end{aligned} \quad (18)$$

$$\begin{aligned} \bar{\delta}_{\Omega^{\alpha_1}}^{[3]} A_{\mu_1 \mu_2 \mu_3} &= \partial_{[\mu_1}^{[2]} \epsilon^{(3,0)\mu_2 \mu_3]} - 4\lambda \left(W_6^{[3]} A_{\mu_1 \mu_2 \mu_3} \epsilon^{(1,0)} \right. \\ &\quad \left. + W_5^{[2]} A_{[\mu_1 \mu_2}^{[1]} \epsilon^{(2,0)\mu_3]} - W_6^{[1]} A_{[\mu_1}^{[2]} \epsilon^{(3,0)\mu_2 \mu_3]} + \varepsilon_{\mu_1 \mu_2 \mu_3 \mu_4 \dots \mu_7} Z_2^{[4]^{\mu_4 \dots \mu_7}} \xi^{(3,0)} \right), \end{aligned} \quad (19)$$

$$\begin{aligned} \bar{\delta}_{\Omega^{\alpha_1}}^{[1]^{\mu_1}} B &= -2\partial_{\rho}^{[2]^{\rho \mu_1}} \xi^{(1,0)} + \lambda \left(\frac{dW_1^{[1]^{\mu_1}}}{d\varphi} B + 3 \frac{dW_4^{[2]} A_{\mu_2 \mu_3}^{[3]^{\mu_1 \mu_2 \mu_3}}}{d\varphi} B \right. \\ &\quad \left. + 4 \frac{dW_6^{[3]} A_{\mu_2 \mu_3 \mu_4}^{[4]^{\mu_1 \mu_2 \mu_3 \mu_4}}}{d\varphi} B + \frac{1}{6} \frac{dY_3}{d\varphi} \varepsilon^{\mu_1 \mu_2 \mu_3 \dots \mu_7} A_{\mu_2 \mu_3}^{[2]} A_{\mu_4 \mu_5}^{[2]} A_{\mu_6 \mu_7}^{[2]} \right) \epsilon^{(1,0)} \\ &\quad + \lambda \left[2 \frac{dW_2^{[2]^{\mu_1 \rho}}}{d\varphi} B - 6 \frac{dW_4^{[1]} A_{\mu_2}^{[3]^{\mu_1 \mu_2 \rho}}}{d\varphi} B + 12 \frac{dW_5^{[2]} A_{\mu_2 \mu_3}^{[4]^{\mu_1 \mu_2 \mu_3 \rho}}}{d\varphi} B \right. \\ &\quad \left. - \varepsilon^{\mu_1 \mu_2 \mu_3 \dots \mu_6 \rho} \left(2 \frac{dY_2^{[2]} A_{\mu_2 \mu_3}^{[3]} A_{\mu_4 \mu_5 \mu_6}^{[2]} + dY_3^{[1]} A_{\mu_2}^{[2]} A_{\mu_3 \mu_4}^{[2]} A_{\mu_5 \mu_6}^{[2]} \right) \right] \epsilon^{(1,0)} \\ &\quad + 3\lambda \left(\frac{dW_3^{[3]^{\mu_1 \mu_2 \mu_3}}}{d\varphi} B - 4 \frac{dW_6^{[1]} A_{\mu_4}^{[4]^{\mu_1 \mu_2 \mu_3 \mu_4}}}{d\varphi} B \right. \\ &\quad \left. + \frac{1}{2} \frac{dY_2}{d\varphi} \varepsilon^{\mu_1 \mu_2 \mu_3 \dots \mu_7} A_{\mu_4 \mu_5}^{[2]} A_{\mu_6 \mu_7}^{[2]} \right) \epsilon^{(3,0)\mu_2 \mu_3} \\ &\quad - 2\lambda \frac{dW_1^{[1]} A_{\mu_2}^{[2]^{\mu_1 \mu_2}}}{d\varphi} \xi^{(1,0)} - 3\lambda \frac{dW_2^{[2]} A_{\mu_2 \mu_3}^{[3]^{\mu_1 \mu_2 \mu_3}}}{d\varphi} \xi^{(2,0)} \\ &\quad - 4\lambda \left(\frac{dW_3^{[3]} A_{\mu_2 \mu_3 \mu_4}^{[4]^{\mu_1 \mu_2 \mu_3 \mu_4}}}{d\varphi} + 3 \frac{dW_4^{[1]} A_{\mu_2}^{[2]} A_{\mu_3 \mu_4}^{[2]} \right) \xi^{(3,0)} \\ &\quad + \lambda \frac{dZ_2}{d\varphi} \varepsilon_{\nu_1 \nu_2 \nu_3 \dots \nu_7} \left(4B^{[4]^{\mu_1 \nu_1 \nu_2 \nu_3} [4]^{\nu_4 \dots \nu_7}} \xi^{(3,0)} - 3B^{[3]^{\mu_1 \nu_1 \nu_2} [5]^{\nu_3 \dots \nu_7}} \xi^{(4,0)} \right) \\ &\quad - 5\lambda \left(\frac{dW_5^{[2]} A_{[\mu_2 \mu_3}^{[2]} A_{\mu_4 \mu_5]}^{[2]} + dW_6^{[1]} A_{[\mu_2}^{[3]} A_{\mu_3 \mu_4 \mu_5]}^{[3]} \right) \xi^{(4,0)}, \end{aligned} \quad (20)$$

$$\begin{aligned}
\bar{\delta}_{\Omega^{\alpha_1}}^{[2]^{\mu_1\mu_2}} B &= -3\partial_\rho \xi_{(2,0)}^{[3]^\rho\mu_1\mu_2} \\
&- \lambda \left[\left(6W_4 B^{[3]^{\mu_1\mu_2\rho}} + Y_3 \varepsilon^{\mu_1\mu_2\mu_3\dots\mu_6\rho} A_{\mu_3\mu_4}^{[2]} A_{\mu_5\mu_6}^{[2]} \right) \epsilon_{(2,0)\rho}^{[1]} \right. \\
&+ 2 \left(6W_6 B^{[4]^{\mu_1\mu_2\mu_3\mu_4}} \epsilon_{(3,0)\mu_3\mu_4}^{[2]} + W_1 \xi_{(1,0)}^{[2]^{\mu_1\mu_2}} \right. \\
&\left. \left. + 6W_4 A_{\mu_3\mu_4}^{[2]} \xi_{(3,0)}^{[4]^{\mu_1\mu_2\mu_3\mu_4}} + 10W_6 A_{\mu_3\mu_4\mu_5}^{[3]} \xi_{(4,0)}^{[5]^{\mu_1\mu_2\mu_3\mu_4\mu_5}} \right) \right], \quad (21)
\end{aligned}$$

$$\begin{aligned}
\bar{\delta}_{\Omega^{\alpha_1}}^{[3]^{\mu_1\mu_2\mu_3}} B &= -4\partial_\rho \xi_{(3,0)}^{[4]^\rho\mu_1\mu_2\mu_3} \\
&+ 3\lambda \left(W_4 B^{[3]^{\mu_1\mu_2\mu_3}} + \frac{1}{6} Y_3 \varepsilon^{\mu_1\mu_2\mu_3\mu_4\dots\mu_7} A_{\mu_4\mu_5}^{[2]} A_{\mu_6\mu_7}^{[2]} \right) \epsilon_{(1,0)}^{[0]} \\
&+ 2\lambda \left[6W_5 B^{[4]^{\mu_1\mu_2\mu_3\rho}} - \varepsilon^{\mu_1\mu_2\mu_3\mu_4\mu_5\mu_6\rho} \left(Y_2 A_{\mu_4\mu_5\mu_6}^{[3]} \right. \right. \\
&\left. \left. + Y_3 A_{\mu_4}^{[1]} A_{\mu_5\mu_6}^{[2]} \right) \right] \epsilon_{(2,0)\rho}^{[1]} + 3\lambda \left[Y_2 \varepsilon^{\mu_1\mu_2\mu_3\mu_4\dots\mu_7} A_{\mu_4\mu_5}^{[2]} \epsilon_{(3,0)\mu_6\mu_7}^{[2]} \right. \\
&\left. - W_2 \xi_{(2,0)}^{[3]^{\mu_1\mu_2\mu_3}} - 4W_4 A_{\mu_4}^{[1]} \xi_{(3,0)}^{[4]^{\mu_1\mu_2\mu_3\mu_4}} - 10W_5 A_{\mu_4\mu_5}^{[2]} \xi_{(4,0)}^{[5]^{\mu_1\mu_2\mu_3\mu_4\mu_5}} \right], \quad (22)
\end{aligned}$$

$$\begin{aligned}
\bar{\delta}_{\Omega^{\alpha_1}}^{[4]^{\mu_1\mu_2\mu_3\mu_4}} B &= -5\partial_\rho \xi_{(4,0)}^{[5]^\rho\mu_1\mu_2\mu_3\mu_4} \\
&+ 2\lambda \left(2W_6 B^{[4]^{\mu_1\mu_2\mu_3\mu_4}} \epsilon_{(1,0)}^{[0]} - Y_2 \varepsilon^{\mu_1\mu_2\mu_3\mu_4\mu_5\mu_6\mu_7} A_{\mu_5\mu_6}^{[2]} \epsilon_{(2,0)\mu_7}^{[1]} \right) \\
&+ 4\lambda \left(-W_3 \xi_{(3,0)}^{[4]^{\mu_1\mu_2\mu_3\mu_4}} + 5W_6 A_{\mu_5}^{[1]} \xi_{(4,0)}^{[5]^{\mu_1\mu_2\mu_3\mu_4\mu_5}} \right). \quad (23)
\end{aligned}$$

All the gauge transformations are deformed with respect to the original ones. Since the entire gauge structure of the interacting model is controlled by the parametrizing functions, subject to equations (11)–(15), it results that our procedure is correct provided these equations possess non-trivial solutions. One class of solutions to these equations, suggestively resembling to Liouville-type field theories, is given by $W_2 = W_3 = Y_2 = Y_3 = 0$, $W_4 = \alpha W_1$, $W_5 = k_2 \exp[-2(3\alpha - 2\beta)\varphi]$, $W_6 = \beta W_1$, $M = k_1$, $Z_2 = k_3 \exp[(3\alpha + 4\beta)\varphi]$, where W_1 is an arbitrary functions and α , β , k_1 , k_2 , and k_3 , are some arbitrary, non-vanishing real constants. Substituting the above

solutions into (10) and (16)–(23), we determine the final output for the corresponding self-interacting theory.

4. EXAMPLE. RESULTS FOR $D = 8$

We act like in the previous section and determine all the distinct solutions to equations (8) and (9) in $D = 8$. As here we have again that $I(D) = 3$, the field spectrum is the same with that from $D = 7$ ($m = \overline{0,3}$) and the set (8) displays the same solutions as in $D = 7$, meaning that the couplings between the A s and the B s are precisely those from the previous example. The set (9), which depends on D , has also two solutions, like in the previous situation, but they are completely complementary to those present in $D = 7$. Here we have again $\bar{N} = 3, 4$ ($V_8 = 4$), but there is no coupling containing three A s, so (9) exhibits no solution for $\bar{N} = 3$. The two solutions stem from $\bar{N} = 4$: $\bar{m}_1 = 1$, $\bar{m}_2 = \bar{m}_3 = 2$, $\bar{m}_4 = 3$, and respectively $\bar{m}_1 = \bar{m}_2 = \bar{m}_3 = \bar{m}_4 = 2$. As a consequence, the parametrizing functions are $Z_{(1,2,2,3)}$ and $Z_{(2,2,2,2)}$, to be redenoted by R_1 and respectively R_2 .

The coupled Lagrangian action reads as

$$\begin{aligned} \bar{S}^L[\Phi^{\alpha_0}] = S^L[\Phi^{\alpha_0}] + \lambda \int d^8x & \left[M(\varphi) + W_1 A_{\mu_1}^{[1]} B^{[1]\mu_1} + W_2 A_{\mu_1\mu_2}^{[2]} B^{[2]\mu_1\mu_2} \right. \\ & + W_3 A_{\mu_1\mu_2\mu_3}^{[3]} B^{[3]\mu_1\mu_2\mu_3} + W_4 A_{[\mu_1}^{[1]} A_{\mu_2\mu_3]}^{[2]} B^{[3]\mu_1\mu_2\mu_3} \\ & + W_5 A_{[\mu_1\mu_2}^{[2]} A_{\mu_3\mu_4]}^{[2]} B^{[4]\mu_1\mu_2\mu_3\mu_4} + W_6 A_{[\mu_1}^{[1]} A_{\mu_2\mu_3\mu_4]}^{[3]} B^{[4]\mu_1\mu_2\mu_3\mu_4} \\ & + \varepsilon_{\mu_1\dots\mu_8} \left(\frac{1}{2} Z_1 B^{[4]\mu_1\dots\mu_4} B^{[4]\mu_5\dots\mu_8} + \frac{1}{2} R_1 A^{[1]\mu_1} A^{[2]\mu_2\mu_3} A^{[2]\mu_4\mu_5} A^{[3]\mu_6\mu_7\mu_8} \right. \\ & \left. \left. + \frac{1}{4!} R_2 A^{[2]\mu_1\mu_2} A^{[2]\mu_3\mu_4} A^{[2]\mu_5\mu_6} A^{[2]\mu_7\mu_8} \right) \right]. \end{aligned} \quad (24)$$

The consistency equations have been computed and reduce to:

$$W_1 \frac{dW_2}{d\varphi} + 3W_4W_2 = 0, \quad W_1 \frac{dW_3}{d\varphi} + (4W_6 - 3W_4)W_3 = 0, \quad (25)$$

$$W_1 \frac{dZ_1}{d\varphi} - 8W_6Z_1 = 0, \quad W_1 \frac{dR_2}{d\varphi} + 12(W_4R_2 - 3W_5R_1) = 0, \quad (26)$$

$$W_1 \frac{dW_5}{d\varphi} + 2[(3W_4 - 2W_6)W_5 + 48Z_1Z_2] = 0, \quad W_1 \frac{dM}{d\varphi} = 0, \quad (27)$$

$$W_2W_4 + W_3W_5 = 0, \quad 2W_2W_6 + 3W_3W_5 = 0, \quad 3W_2Z_2 + W_3R_2 = 0, \quad (28)$$

$$W_1W_2 = 0, \quad W_2W_3 = 0, \quad W_3Z_1 = 0. \quad (29)$$

The deformed gauge transformations are again identified from the antighost number one terms in the deformed solution to the master equation and read as

$$\bar{\delta}_{\Omega^{\alpha 1}} \varphi = -\lambda W_1 \overset{[0]}{\epsilon}_{(1,0)}, \quad (30)$$

$$\bar{\delta}_{\Omega^{\alpha 1}} \overset{[1]}{A}_{\mu_1} = \partial_{\mu_1} \overset{[0]}{\epsilon}_{(1,0)} - 2\lambda W_2 \overset{[1]}{\epsilon}_{(2,0)\mu_1}, \quad (31)$$

$$\begin{aligned} \bar{\delta}_{\Omega^{\alpha 1}} \overset{[2]}{A}_{\mu_1 \mu_2} = & \partial_{[\mu_1} \overset{[1]}{\epsilon}_{(2,0)\mu_2]} + 3\lambda \left(-W_4 \overset{[2]}{A}_{\mu_1 \mu_2} \overset{[0]}{\epsilon}_{(1,0)} \right. \\ & \left. + W_4 \overset{[1]}{A}_{[\mu_1} \overset{[1]}{\epsilon}_{(2,0)\mu_2]} - W_3 \overset{[2]}{\epsilon}_{(3,0)\mu_1 \mu_2} \right), \end{aligned} \quad (32)$$

$$\begin{aligned} \bar{\delta}_{\Omega^{\alpha 1}} \overset{[3]}{A}_{\mu_1 \mu_2 \mu_3} = & \partial_{[\mu_1} \overset{[2]}{\epsilon}_{(3,0)\mu_2 \mu_3]} - 4\lambda \left(W_6 \overset{[3]}{A}_{\mu_1 \mu_2 \mu_3} \overset{[0]}{\epsilon}_{(1,0)} \right. \\ & \left. + W_5 \overset{[2]}{A}_{[\mu_1 \mu_2} \overset{[1]}{\epsilon}_{(2,0)\mu_3]} - W_6 \overset{[1]}{A}_{[\mu_1} \overset{[2]}{\epsilon}_{(3,0)\mu_2 \mu_3]} + \varepsilon_{\mu_1 \mu_2 \mu_3 \mu_4 \dots \mu_8} Z_1 \overset{[5]}{\xi}^{\mu_4 \dots \mu_8} \overset{[4]}{\xi}_{(4,0)} \right), \end{aligned} \quad (33)$$

$$\begin{aligned} \bar{\delta}_{\Omega^{\alpha 1}} \overset{[1]}{B}^{\mu_1} = & -2\partial_{\rho} \overset{[2]}{\xi}_{(1,0)}^{\rho \mu_1} + \lambda \left(\frac{dW_1}{d\varphi} \overset{[1]}{B}^{\mu_1} + 3 \frac{dW_4}{d\varphi} \overset{[3]}{A}_{\mu_2 \mu_3} \overset{[3]}{B}^{\mu_1 \mu_2 \mu_3} \right. \\ & \left. + 4 \frac{dW_6}{d\varphi} \overset{[3]}{A}_{\mu_2 \mu_3 \mu_4} \overset{[4]}{B}^{\mu_1 \mu_2 \mu_3 \mu_4} + \frac{1}{2} \frac{dR_1}{d\varphi} \varepsilon^{\mu_1 \mu_2 \dots \mu_8} \overset{[2]}{A}_{\mu_2 \mu_3} \overset{[2]}{A}_{\mu_4 \mu_5} \overset{[3]}{A}_{\mu_6 \mu_7 \mu_8} \right) \overset{[0]}{\epsilon}_{(1,0)} \\ & + 2\lambda \left[\frac{dW_2}{d\varphi} \overset{[2]}{B}^{\mu_1 \rho} - 3 \frac{dW_4}{d\varphi} \overset{[1]}{A}_{\mu_2} \overset{[3]}{B}^{\mu_1 \mu_2 \rho} + 6 \frac{dW_5}{d\varphi} \overset{[2]}{A}_{\mu_2 \mu_3} \overset{[4]}{B}^{\mu_1 \mu_2 \mu_3 \rho} \right. \\ & \left. + \varepsilon^{\mu_1 \mu_2 \dots \mu_7 \rho} \left(\frac{dR_1}{d\varphi} \overset{[1]}{A}_{\mu_2} \overset{[2]}{A}_{\mu_3 \mu_4} \overset{[3]}{A}_{\mu_5 \mu_6 \mu_7} + \frac{1}{6} \frac{dR_2}{d\varphi} \overset{[2]}{A}_{\mu_2 \mu_3} \overset{[2]}{A}_{\mu_4 \mu_5} \overset{[2]}{A}_{\mu_6 \mu_7} \right) \right] \overset{[1]}{\epsilon}_{(2,0)\rho} \\ & + 3\lambda \left(\frac{dW_3}{d\varphi} \overset{[3]}{B}^{\mu_1 \mu_2 \mu_3} - 4 \frac{dW_6}{d\varphi} \overset{[1]}{A}_{\mu_4} \overset{[4]}{B}^{\mu_1 \mu_2 \mu_3 \mu_4} - \frac{1}{2} \frac{dR_1}{d\varphi} \varepsilon^{\mu_1 \mu_2 \dots \mu_8} \overset{[1]}{A}_{\mu_4} \overset{[2]}{A}_{\mu_5 \mu_6} \overset{[2]}{A}_{\mu_7 \mu_8} \right) \overset{[2]}{\epsilon}_{(3,0)\mu_2 \mu_3} \\ & - 2\lambda \frac{dW_1}{d\varphi} \overset{[1]}{A}_{\mu_2} \overset{[2]}{\xi}_{(1,0)}^{\mu_1 \mu_2} - 3\lambda \frac{dW_2}{d\varphi} \overset{[2]}{A}_{\mu_2 \mu_3} \overset{[3]}{\xi}_{(2,0)}^{\mu_1 \mu_2 \mu_3} - 4\lambda \left(\frac{dW_3}{d\varphi} \overset{[3]}{A}_{\mu_2 \mu_3 \mu_4} \right. \\ & \left. + \frac{dW_4}{d\varphi} \overset{[1]}{A}_{[\mu_2} \overset{[2]}{A}_{\mu_3 \mu_4]} \right) \overset{[4]}{\xi}_{(3,0)}^{\mu_1 \dots \mu_4} + 4\lambda \frac{dZ_1}{d\varphi} \varepsilon_{\nu_1 \dots \nu_8} \overset{[4]}{B}^{\mu_1 \nu_1 \nu_2 \nu_3} \overset{[5]}{\xi}_{(4,0)}^{\nu_4 \dots \nu_8} \\ & - 5\lambda \left(\frac{dW_5}{d\varphi} \overset{[2]}{A}_{[\mu_2 \mu_3} \overset{[2]}{A}_{\mu_4 \mu_5]} + \frac{dW_6}{d\varphi} \overset{[1]}{A}_{[\mu_2} \overset{[3]}{A}_{\mu_3 \mu_4 \mu_5]} \right) \overset{[5]}{\xi}_{(4,0)}^{\mu_1 \dots \mu_5}, \end{aligned} \quad (34)$$

$$\begin{aligned}
\bar{\delta}_{\Omega^{\alpha_1}}^{[2]\mu_1\mu_2} B &= -3\partial_\rho \xi^{[3]\rho\mu_1\mu_2} \\
&- 2\lambda \left(3W_4 B^{[3]\mu_1\mu_2\rho} - R_1 \varepsilon^{\mu_1\mu_2\mu_3\dots\mu_7\rho} A_{\mu_3\mu_4}^{[2]} A_{\mu_5\mu_6\mu_7}^{[3]} \right) \epsilon^{[1]}_{(2,0)\rho} \\
&- 3\lambda \left(4W_6 B^{[4]\mu_1\mu_2\mu_3\mu_4} + \frac{1}{2} R_1 \varepsilon^{\mu_1\mu_2\mu_3\dots\mu_8} A_{\mu_5\mu_6}^{[2]} A_{\mu_7\mu_8}^{[2]} \right) \epsilon^{[2]}_{(3,0)\mu_3\mu_4} \\
&- 2\lambda W_1 \xi^{[2]\mu_1\mu_2}_{(1,0)} - 12\lambda W_4 A_{\mu_3\mu_4}^{[2]} \xi^{[4]\mu_1\mu_2\mu_3\mu_4}_{(3,0)} - 20\lambda W_6 A_{\mu_3\mu_4\mu_5}^{[3]} \xi^{[5]\mu_1\mu_2\mu_3\mu_4\mu_5}_{(4,0)}, \quad (35)
\end{aligned}$$

$$\begin{aligned}
\bar{\delta}_{\Omega^{\alpha_1}}^{[3]\mu_1\mu_2\mu_3} B &= -4\partial_\rho \xi^{[4]\rho\mu_1\mu_2\mu_3} \\
&+ \lambda \left(3W_4 B^{[3]\mu_1\mu_2\mu_3} + R_1 \varepsilon^{\mu_1\mu_2\mu_3\dots\mu_8} A_{\mu_4\mu_5}^{[2]} A_{\mu_6\mu_7\mu_8}^{[3]} \right) \epsilon^{[0]}_{(1,0)} \\
&+ \lambda \left[12W_5 B^{[4]\mu_1\mu_2\mu_3\rho} + \varepsilon^{\mu_1\mu_2\mu_3\dots\mu_7\rho} \left(2R_1 A_{\mu_4}^{[1]} A_{\mu_5\mu_6\mu_7}^{[3]} \right. \right. \\
&\left. \left. + R_2 A_{\mu_4\mu_5}^{[2]} A_{\mu_6\mu_7}^{[2]} \right) \right] \epsilon^{[1]}_{(2,0)\rho} - 3\lambda \left(R_1 \varepsilon^{\mu_1\mu_2\mu_3\dots\mu_8} A_{\mu_4}^{[1]} A_{\mu_5\mu_6}^{[2]} \epsilon^{[2]}_{(3,0)\mu_7\mu_8} \right. \\
&\left. + W_2 \xi^{[3]\mu_1\mu_2\mu_3}_{(2,0)} + 4W_4 A_{\mu_4}^{[1]} \xi^{[4]\mu_1\mu_2\mu_3\mu_4}_{(3,0)} + 10W_5 A_{\mu_4\mu_5}^{[2]} \xi^{[5]\mu_1\mu_2\mu_3\mu_4\mu_5}_{(4,0)} \right), \quad (36)
\end{aligned}$$

$$\begin{aligned}
\bar{\delta}_{\Omega^{\alpha_1}}^{[4]\mu_1\mu_2\mu_3\mu_4} B &= -5\partial_\rho \xi^{[5]\rho\mu_1\mu_2\mu_3\mu_4} \\
&+ \lambda \left(4W_6 B^{[4]\mu_1\mu_2\mu_3\mu_4} + \frac{1}{2} R_1 \varepsilon^{\mu_1\mu_2\mu_3\mu_4\dots\mu_8} A_{\mu_5\mu_6}^{[2]} A_{\mu_7\mu_8}^{[2]} \right) \epsilon^{[0]}_{(1,0)} \\
&- 2\lambda \left(R_1 \varepsilon^{\mu_1\mu_2\mu_3\mu_4\dots\mu_8} A_{\mu_5}^{[1]} A_{\mu_6\mu_7}^{[2]} \epsilon^{[1]}_{(2,0)\mu_8} \right. \\
&\left. + 2W_3 \xi^{[4]\mu_1\mu_2\mu_3\mu_4}_{(3,0)} - 10W_6 A_{\mu_5}^{[1]} \xi^{[5]\mu_1\mu_2\mu_3\mu_4\mu_5}_{(4,0)} \right). \quad (37)
\end{aligned}$$

Exactly like in the previous example the gauge structure of the interacting theory is highly deformed with respect to that of the starting Abelian model: the gauge algebra becomes open and the reducibility relations only hold on-shell.

We have seen that our procedure is non-trivial provided the consistency equations (25)–(29) admit non-trivial solutions. This is indeed the case, one of these solutions being $W_2 = W_3 = 0$, $W_4 = \alpha W_1$, $W_5 = -\frac{48\rho}{3\alpha+2\beta} Z_1$, $W_6 = \beta W_1$, $M = k_1$,

$Z_1 = k_2 \exp(8\beta\varphi)$, $R_1 = \rho W_1$, $R_2 = -\frac{432\rho^2}{(3\alpha+2\beta)^2} Z_1$, where W_1 is an arbitrary function and α , β , ρ , k_1 , and k_2 are some arbitrary, non-vanishing real constants, with $3\alpha + 2\beta \neq 0$. Inserting this solution into (24) and (30)–(37) one finds a possible form of self-interacting BF model in $D = 8$.

5. CONCLUSION

To conclude with, we have completely computed and classified all consistent vertices for a self-interacting topological BF model with a maximal field spectrum in an arbitrary spacetime dimension by means of the local BRST cohomology under standard hypotheses from gauge field theory.

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