THE ROLE OF THE NON-ZERO INITIAL MOMENTUM ANDMODIFIED IONIZATION POTENTIAL IN THE CORRECTED AMMOSOV-DELONE-KRAINOV THEORY

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In this paper the dependence of corrected Ammosov-Delone-Krainov (cADK) ionization probability on ion net charge Z, on modified ionization potential E_{mod} and on non-zero initial momentum p in the case of linearly polarized laser field is examined. The physical system in question is the potassium atom irradiated by CO_2 laser, with intensities that ranged from $10^{14}\,\mathrm{W/cm^2}$ to $5\times10^{16}\,\mathrm{W/cm^2}$. It turns out that dominant ionization probability, for a given intensity of laser field, depends on Z and p, *i.e.* of an electron that awaits next step in sequential process of ionization. The influence of modified ionization potential on ionization probability is most readily apparent in the case of high laser intensities that vary from $10^{15}\,\mathrm{W/cm^2}$ to $10^{16}\,\mathrm{W/cm^2}$.

*Key words***:** tunneling ionization, corrected ADK theory, non-zero momentum, ionization potential.
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1. INTRODUCTION

In Keldysh's famous paper [1] the transition probability between an initial state of bound electron and final state of a free electron (represented by Volkov state) was calculated, while all intermediate states were neglected. This transition was due to the atom interaction with intense electromagnetic radiation. Aforementioned paper has motivated numerous theoretical and experimental studies of atom ionization by strong laser fields in the regime that ranges from multiphoton to tunneling ionization [2, 3]. One of the main accomplishments of Keldysh theory [1] was the introduction of adiabatic (Keldysh) parameter that was defined in the following manner: $\gamma=\omega\sqrt{2E_{\mathrm{i}}}/F$, where ω represents frequency of laser field, F is strength of laser field, and E_{i} is ionization potential.

The type of ionization is determined by value of Keldysh parameter γ: multiphoton ionization occurs for $\gamma >> 1$, while tunneling ionization happens when $\gamma << 1$.

Perelomov, Popov and Terentev [4] developed a method for calculating the tunneling ionization probability of a bound state under the action of an alternating field (so-called *PPT theory*). When condition $\gamma << 1$ is fulfilled, the electron is faced with practically stationary barrier: $\omega << \omega_t$ ($\omega_t = F/\sqrt{2E_i}$ is a tunneling frequency), so it can be assumed that changes in laser field are much slower than the tunneling time.

Afterwards, Ammosov, Delone and Krainov [5] further extended the results of PPT theory, and obtained a following formula for tunneling ionization probability

$$W_{\mathrm{ADK}} = \left(\frac{4\,\mathrm{e}\,Z^3}{F\,n^{*4}}\right)^{2n^*-1} \mathrm{e}^{-\frac{2Z^3}{3F\,n^{*3}}}. \tag{1}$$

where Z is ion net charge, while n^* is the effective principal quantum number.

Their theory was named ADK theory, and it has proved to be one of the main approaches in current theories of strong laser field physics. The basic tenet of that theory is an assumption that substantial ionization occurs within a period of time that is only a fraction of an optical cycle, so that the laser field can be regarded as quasi-static.

Throughout this paper the atomic unit system $e = \hbar = m_e = 1$ will be used.

2. THE CORRECTED ADK IONIZATION PROBABILITY

In original paper [5], the Coulomb interaction was neglected in process of calculating the turning point. The corrected form of ADK ionization probability (with that interaction included) was obtained in paper [6], and corresponding formula is as follows

$$W_{\mathrm{cADK}} = \left[\frac{4\,\mathrm{e}\,Z^3}{F\,n^{*4}} \frac{1}{1 + \frac{2ZF}{(p^2 + 2E_i)^2} + \frac{Z^2F^2}{2E_i(p^2 + 2E_i)^3}}\right]^{2n^*-1} \mathrm{e}^{-\frac{2Z^3}{3F\,n^{*3}}}, \tag{2}$$

where $E_i = Z^2/(2\mathrm{n}^2)$ is ionization potential, while p is non-zero initial momentum of ejected electrons.

As was shown in papers [7, 8], the non-zero momentum that electron possesses when leaving the atom has a significant influence on the ionization probability. Its exact expression was obtained in [7] and has a following form (in parabolic coordinates, that are defined as $\xi = r + z$, $\eta = r - z$, $\phi = \arctan(y/x)$ where $\xi, \eta \in [0, \infty]$ and $\phi \in [0, 2\pi]$) as per [9]

$$p(\eta) = \sqrt{-\frac{1}{4} + \frac{1}{2\eta} + \frac{1}{4\eta^2} + \frac{F\eta}{4}}. \tag{3}$$

Based on [8], as $1/\eta << 1$, the momentum (3) can be expanded into a power series (in order for momentum to have real values), and will have the following form

$$p(\eta) = \frac{1}{2}\left(\sqrt{F\eta - 1} + \frac{1}{\eta\sqrt{F\eta - 1}} + ...\right) \text{ outside potential barrier } \eta > \frac{1}{F}. \tag{4}$$

Ionization potential E_i that figures in (2) will be modified because of the influence of external laser field, which causes the ionization. This will lead to following expression for modified ionization potential, obtained in [10]

$$E_{mod} \approx \frac{Z^2 \left(n + 2F^2 Z^2\right)}{2n^3}. \tag{5}$$

Also, at low frequencies and for high intensities, tunneling becomes the dominant effect. Therefore, including the non-zero initial momentum and modified form of ionization potential E_{mod} will enable the better understanding of these processes within the framework of ADK theory.

After incorporating the determined quantities from Eq. (4) and Eq. (5) into formula for corrected ADK ionization probability (2), the following expression is obtained

$$W_{cADK} = \left\{ \frac{4\mathrm{e}Z^3}{Fn^{*4}} \frac{1}{1 + \frac{2ZF}{\left[p^2(\eta) + 2E_{mod}(F,Z)\right]^2} + \frac{Z^2F^2}{2E_{mod}\left[p^2(\eta) + 2E_{mod}(F,Z)\right]^3}} \right\}^{2n^*-1} \times \mathrm{e}^{-\frac{2Z^3}{3Fn^{*3}} - \frac{p^2(\eta)\gamma^3}{3\omega}} \tag{6}$$

The additional term in exponent determines the kinetic-energy distribution of the ejected electrons during tunneling ionization, and it was first introduced in [11]. The additional term in exponent determines how the ionization probability depends on the initial momentum of an ejected electron.

The goal of this paper is to examine the dependence of ionization probability (6) on ion net charge Z, on modified ionization potential E_{mod} and on non-zero momentum p (that depends on a parabolic coordinate η) in the case of linearly polarized intense field.

3. REMARKS ON THE BEHAVIOUR OF THE IONIZATION PROBABILITY

Before the beginning of our discussion concerning the ionization probability, the value of Keldysh parameter γ will be examined. This parameter is shown at Fig. 1 for each ion net charge Z (ranges from 1 to 5) of potassium atom, at intensities that varies from 10^{12} to 10^{14}. Since CO_2 laser emits photons of wavelength $\lambda = 10.6\ \mu\text{m}$, their energy has value of $\omega = 0.004298$ a.u.

From Figure 1 is quite obvious that for all field intensities the value of γ will be less than 1, which indicates that the ionization occurs exclusively in the tunneling regime.

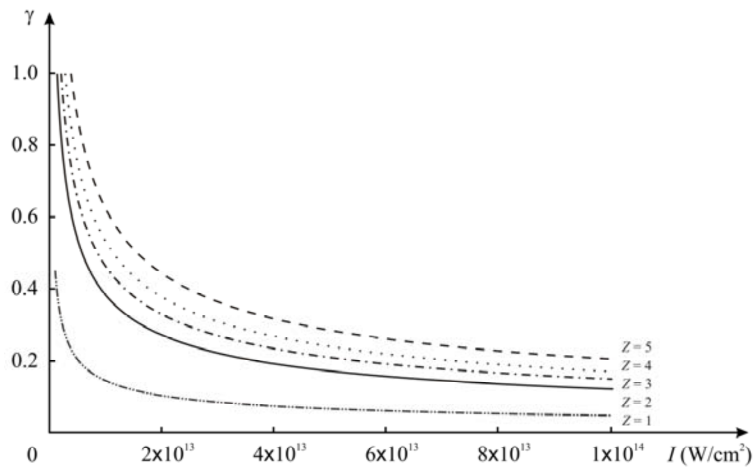

Fig. 1 – Dependence of Keldysh parameter on laser intensity, for different values of Z. Graph shows that values of γ lie within tunneling ionization limits.

One of the basic points of the ADK model is that the ionization probability depends critically on the ionization potential of an atom and on laser intensity [4, 5]. Ionization probabilities (6) are calculated for a potassium atom, for different ion net charges Z, and for non-zero initial momentum $p(\eta)$ in linearly polarized laser field [7]. The parabolic coordinate η has a lower limit value that is dependent on field strength according to expression: $\eta \sim 1/F$ (a. u.), and its values are listed in Table 1 for appropriate laser intensities. It is important to emphasize that the limit values for η were chosen in order for $p(\eta)$ to have real values.

Table 1

Lower limit values for parabolic coordinate η and appropriate laser intensities

$I\,(\mathrm{W/cm^2})$	10^{12}	10^{13}	10^{14}	10^{15}	10^{16}	10^{17}
η	185.455	58.6459	18.5455	5.86459	1.85455	0.586459

The calculation of ionization probability (6) also includes the range of laser intensities from $10^{14}\ \mathrm{W/cm^2}$ to $10^{16}\ \mathrm{W/cm^2}$, and also the different modified ionization potential E_{mod} for each one of available electrons.

4. RESULTS AND DISCUSSION

On the graphs that follow (Figs. 2–5) the ionization probabilities (6) are shown for each of the electrons of potassium atom whose effective quantum number is $n^* = 3$, while E_{mod} and Z vary [12]. Also, the influences of unmodified and modified ionization potential (5) are shown side-by-side for prevailing ionization probability and can be compared for the first time.

It should be remembered that there is only one electron in the open fourth shell of potassium atom, and the case of its ionization $Z = 1$ has been studied in detail and discussed in paper [7]; there, it was noticed that valence electron can be ionized easily even at lower field intensities $\sim 10^{12}$.

So in this paper a special attention will be given to ionization of four electrons that lie in atomic orbital 3p. The first ionized electron belongs to a closed shell of K atom, and because this electron has a higher value of ionization potential compared to one of valence electron in atomic orbital 4s, field intensity needs to be much higher in order to break this stable shell. Therefore, for starting laser intensity is chosen a value $2 \times 10^{14}\ \mathrm{W/cm^2}$, Fig. 2. It is quite obvious that the greatest ionization probability is for the first electron in 3p orbital, Fig. 2a. From the Fig. 2b can be seen that influence of E_{mod} on ionization probability is negligible because the specified laser field intensity has relatively low value.

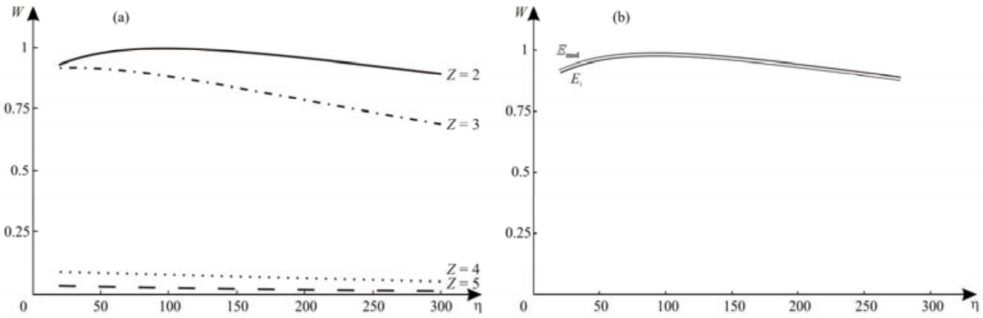

Fig. 2 – (a) Ionization probability dependence on η coordinate at laser intensity of $I = 2\times10^{14}$ W/cm^2, for each of four ionized electrons in orbital 3p . It is obvious that the greatest probability is for $Z = 2$. (b) The influence of both E_i and E_mod on ionization probability for $Z = 2$.

After the first electron from 3p orbital leaves the atom, the ionization of a second electron in this sequential ionization process requires the higher laser intensity of 10^{15} W/cm^2, because it has a greater ionization potential. Consequently, for that intensity this electron has the greatest ionization probability, Fig. 3. The influence of E_mod on ionization probability is similar for $Z = 2$ and $Z = 3$ at lower intensities. However, as η is lower, laser intensity is higher, see Table 1, and dominance of E_mod for $Z = 3$ can readily be seen, Fig. 3b.

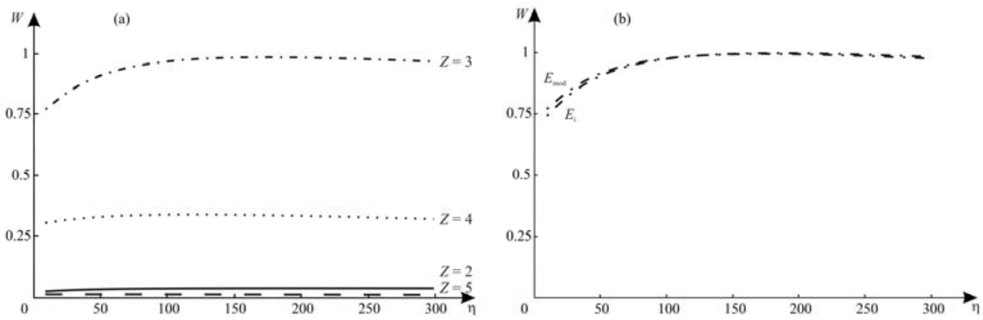

Fig. 3 – (a) Ionization probability dependence on η coordinate at laser intensity of $I = 10^{15}$ W/cm^2 , for each of four ionized electrons in orbital 3p . It is obvious that the greatest probability is for $Z = 3$. (b) The influence of both E_i and E_mod on ionization probability for $Z = 3$.

The situation is similar for other two electrons, whose ionization probabilities are depicted at Figs. 4 and 5. When $I = 5\times10^{15}$ W/cm^2 , the ionization probability is dominant for $Z = 4$, Fig. 4a, while from Fig. 4b is obvious that modified ionization potential exercises much greater influence on the ionization probability then before.

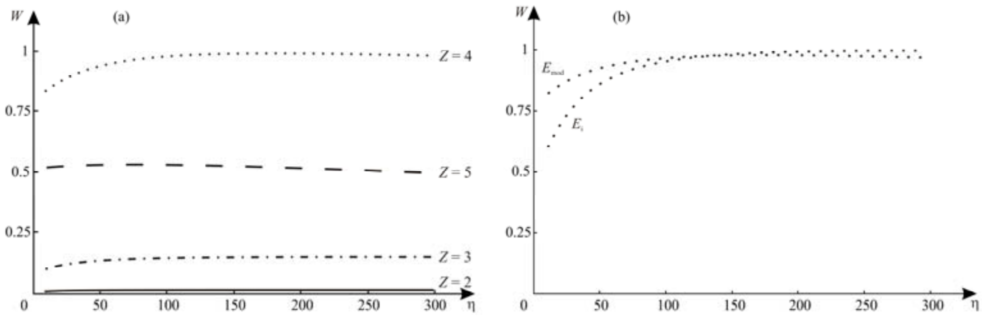

Fig. 4 – (a) Ionization probability dependence on η coordinate at laser intensity of $I = 5\times10^{15}$ W/cm^2, for each of four ionized electrons in orbital 3p. It is obvious that the greatest probability is for $Z = 4$. (b) The influence of both E_i and E_mod on ionization probability for $Z = 4$.

In Fig. 5b, because of high intensity $I = 5\times10^{16}$ W/cm^2, E_mod will significantly influence the ionization probability, so there is marked difference between two lines where E_mod dominates over an unmodified ionization potential E_i.

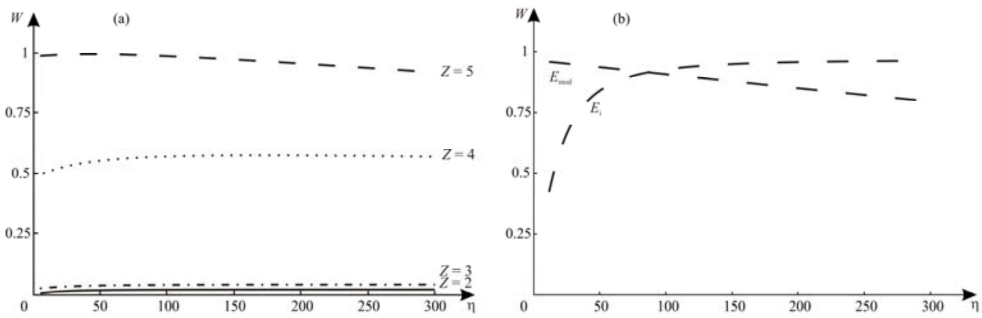

Fig. 5 – (a) Ionization probability dependence on η coordinate at laser intensity of $I = 5\times10^{16}$ W/cm^2, for each of four ionized electrons in orbital 3p. It is obvious that the greatest probability is for $Z = 5$. (b) The influence of both E_i and E_mod on ionization probability for $Z = 5$.

Above results seem logical, since for every succeeding electron modified ionization potential E_mod increases: for $4\mathrm{s}^1$ electron its value is 0.16 (in a.u.), for $3\mathrm{p}^6$ electron it is 1.17, for $3\mathrm{p}^5$ electron it is 1.77, for $3\mathrm{p}^4$ electron it is 3.62, and, finally, for $3\mathrm{p}^3$ electron it is 9.77. These values are obtained from (5), based on the standard values for unmodified ionization potential E_i, taken from [13]. The remaining electrons of potassium were neglected because their ionization would require the field intensities far greater than $10^{18}\ \mathrm{W/cm^2}$, which would belong to the domain of relativistic intensities.

5. CONCLUSION

It is evident that formula for ionization probability has been constantly improved upon, in order to gain a better agreement between current state of theory and recently performed experiments (with constantly increasing laser intensities).

We calculated the ionization probabilities for potassium atom, for different ion net charges Z, and for non-zero initial momentum $p(\eta)$ in linearly polarized laser field. These calculations also included the range of laser intensities that varied from $10^{14}\ \mathrm{W/cm^2}$ to $10^{16}\ \mathrm{W/cm^2}$, and a modified ionization potential E_{mod} for each one of separable electrons in 3p atomic orbital. The lower limit values for parabolic coordinate η in Table 1 were chosen according to the condition: $\eta \sim 1/F$ in order for η to have real values. From Figs. 2-5 it can easily be discerned that a dominant ionization probability, for a given intensity of laser field, depends on Z, *i.e.* of an electron that awaits next step in sequential process of ionization.

The influence of modified ionization potential on ionization probability is most readily apparent in the case of high laser intensities that vary from $10^{15}\ \mathrm{W/cm^2}$ to $10^{16}\ \mathrm{W/cm^2}$.

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