BIANCHI TYPE-I VISCOUS FLUID COSMOLOGICAL MODELS WITH VARIABLE DECELERATION PARAMETER

CHANCHAL CHAWLA[1], R. K. MISHRA[2]

[1,2]Department of Mathematics, Sant Longowal Institute of Engineering and Technology,
Longowal-148106, Punjab, India
E-mail[1]: c.chawla137@gmail.com
E-mail[2]: ravkmishra@yahoo.co.in

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Exact solutions of Einstein's field equations are obtained in a spatially homogeneous and anisotropic Bianchi type-I space-time in presence of a dissipative fluid by considering time dependent deceleration parameter (DP). Following the technique (Pradhan *et al.* in Astrophys. Space Sci 337 :401 2012), we consider two types of scale factors (i) $a(t) = \sinh(\alpha t)$ and (ii) $a(t) = te^t$, which yield time dependent DP. To get the deterministic solution, a barotropic equation of state together with a linear relation between shear viscosity and expansion scalar is also assumed. It is observed that the initial nature of singularity is not changed due to the presence of viscous fluid. The basic equations of thermodynamics have been deduced and the thermodynamic aspects of the models have been discussed.

Key words: Bianchi type-I universe, Exact solution, Alternative gravitation theory, Variable deceleration parameter.

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1. INTRODUCTION AND MOTIVATION

The astronomical observations have revealed that our present universe is homogeneous and isotropic on sufficiently large scales and it is usually described by Friedman-Lemaitre-Robertson-Walker (FLRW) cosmology. But it is believed that, the FLRW cosmology does not give a correct matter description of the early universe. Moreover, the theoretical argument [1] and the modern experimental data from cosmic microwave background (CMB) and the large structure observations, support the existence of an anisotropic phase which turns into an isotropic one. Therefore, models having anisotropic background that approach to isotropy at late times are more appropriate for the description of entire evolution of the universe. Also, Bianchi-I space-time provide such a framework. In the literature, Bianchi type models have been discussed by several researchers (Koivisto and Mota [2, 3], Akarsu and Kilinc [4], Ellis [5], Pradhan et al. [6-8], Bali *et al.* [9], Saha [10], Yadav *et al.* [11], Visinescu and Saha [12]) (but not limited to) in different physical context.

The effects of dissipation including both the bulk and shear viscosity play an important role in the early phase evolution of the universe. This is supported by the fact that when neutrino decoupling occurred, the matter behaved like viscous fluid in early stages of the universe. To describe the relativistic theory of viscosity, the first attempt was made by Misner [1], Eckart [13] and, Landau and Lifshitz [14]. Weinberg [15] derived general formulae for bulk and shear viscosity to evaluate the rate of cosmological entropy production. Grøn [16] studied the role of viscosity in the evolution of Bianchi type-I models and deduced that viscosity plays significant role in the process of isotropization of the universe. Several authors (Singh and Kumar [17, 18], Kremer and Devecchi [19], Debnath *et al.* [20], Yadav [21], Pradhan [22, 23], Pradhan *et al.* [24], Singh *et al.* [25], Bali and Pradhan [26]) (but not limited to) have investigated viscous fluid cosmological models in different physical context.

In literature, it is common to use a constant DP (Akarsu and Kilinc [27, 28], Amirhashchi *et al.* [29], Pradhan *et al.* [30]) as it suggests a power law for metric function or corresponding quantity. But recent observations of Type Ia supernova (Riess *et al.* [31, 32], Perlmutter *et al.* [33], Tonry *et al.* [34], Clocchiatti *et al.* [35]) and CMB anisotropies (Bennett *et al.* [36], de Bernardis *et al.* [37], Hanany *et al.* [38]) revealed that the current universe is not only expanding but also accelerating. Transition of the universe from the decelerated phase to the present accelerating phase motivate us to consider variable DP. Several researchers (Pradhan *et al.* [39], Amirhashchi *et al.* [40], Akarsu and Dereli [41]) have discussed the evolution of the universe with variable DP.

Motivated from the studies outlined above, in this paper, we have investigated some physically realistic and anisotropic Bianchi type-I models by considering a dissipative fluid with variable DP. The paper is organized as follows: In Sect. 2, the metric and the field equations are described. Sect. 3 deals with the solutions of the field equations by considering physically relevant assumptions. Further, thermodynamic aspects and physical behavior of the models is also discussed under Sect. 3. Finally, conclusions are summarized in the last Sect. 4.

2. THE METRIC AND FIELD EQUATIONS

We consider totally anisotropic Bianchi type-I metric in the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + A^2\mathrm{d}x^2 + B^2\mathrm{d}y^2 + C^2\mathrm{d}z^2, \tag{1}$$

where the metric potentials A, B and C are functions of cosmic time t alone. This ensures that the model is spatially homogeneous.
The Einstein's field equations (in gravitational units $8\pi G = c = 1$) are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij}, \tag{2}$$

where T_{ij} is the stress energy tensor of matter, which in case of viscous fluid has the form [14]

$$T_{ij}=(\rho+\bar{p})u_iu_j+\bar{p}g_{ij}-\eta\mu_{ij} \tag{3}$$

with

$$\bar{p}=p-(3\xi-2\eta)H. \tag{4}$$

Here p is the isotropic pressure; $\bar{p}$ is the effective pressure; ρ is the matter density; ξ and η stand for the bulk and shear viscosity coefficients respectively and u^i is the four-velocity vector satisfying $u^iu_i=-1$. In a co-moving coordinate system, where $u^i=\delta^i_0$, the field equations (2) and the energy conservation equation $T^{ij}_{;j}=0$, for the anisotropic Bianchi type-I space-time (1) and viscous fluid distribution (3) yield

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=-\bar{p}+2\eta\frac{\dot{A}}{A}, \tag{5}$$

$$\frac{\ddot{C}}{C}+\frac{\ddot{A}}{A}+\frac{\dot{C}\dot{A}}{CA}=-\bar{p}+2\eta\frac{\dot{B}}{B}, \tag{6}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=-\bar{p}+2\eta\frac{\dot{C}}{C}, \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}=\rho \tag{8}$$

and

$$\dot{\rho}=-(p+\rho)\theta+\xi\theta^2+4\eta\sigma^2. \tag{9}$$

Here dot indicates ordinary differentiation with respect to t. We have a system of four independent equations (5)$-$(8) and seven unknown variables namely A, B, C, p, ρ, ξ and η. So for detail study of the system, we are required three appropriate relations among these variables, accordingly considered and presented in the following section to solve the field equations.

3. SOLUTION OF FIELD EQUATIONS

We follow the approach of Saha [10] to solve the field equations (5)$-$(8). Subtracting (5) from (6), (5) from (7), (6) from (7) and taking second integral of each, we get the following three relations respectively:

$$\frac{A}{B}=d_1\exp\left(x_1\int a^{-3}e^{-2\int\eta\mathrm{d}t}\mathrm{d}t\right), \tag{10}$$

$$\frac{A}{C}=d_2\exp\left(x_2\int a^{-3}e^{-2\int\eta\mathrm{d}t}\mathrm{d}t\right), \tag{11}$$

$$\frac{B}{C}=d_3\exp\left(x_3\int a^{-3}e^{-2\int\eta\mathrm{d}t}\mathrm{d}t\right), \tag{12}$$

where d_1, x_1, d_2, x_2, d_3 and x_3 are constants of integration. From Eqs. (10)−(12), the metric functions can be explicitly written as

$$A(t) = a_1 a \exp\left(b_1 \int a^{-3} e^{-2\int \eta \mathrm{d}t} \mathrm{d}t\right), \tag{13}$$

$$B(t) = a_2 a \exp\left(b_2 \int a^{-3} e^{-2\int \eta \mathrm{d}t} \mathrm{d}t\right), \tag{14}$$

$$C(t) = a_3 a \exp\left(b_3 \int a^{-3} e^{-2\int \eta \mathrm{d}t} \mathrm{d}t\right), \tag{15}$$

where

$$a_1 = \sqrt[3]{d_1 d_2}, \quad a_2 = \sqrt[3]{d_1^{-1} d_3}, \quad a_3 = \sqrt[3]{(d_2 d_3)^{-1}},$$
$$b_1 = \tfrac{x_1+x_2}{3}, \quad b_2 = \tfrac{x_3-x_1}{3}, \quad b_3 = \tfrac{-(x_2+x_3)}{3}$$

with $a_1 a_2 a_3 = 1$ and $b_1 + b_2 + b_3 = 0$. Thus the metric functions are found explicitly in terms of average scale factor a.

3.1. CASE 1: WHEN $a(t) = \sinh(\alpha t)$

As suggested by Pradhan *et al.* [39], firstly we consider the variation of scale factor a with cosmic time t by the relation

$$a(t) = \sinh(\alpha t), \tag{16}$$

where α is an arbitrary constant and the constant of integration is absorbed in t without any loss of generality. Now, Equation (16) yields a time dependent DP as

$$q(t) = -\frac{a\ddot{a}}{\dot{a}^2} = -\tanh^2(\alpha t). \tag{17}$$

Secondly, we assume that the coefficient of shear viscosity (η) is proportional to the expansion scalar (θ) *i.e.* $\eta \propto \theta$, which leads to

$$\eta = \eta_0 \theta \tag{18}$$

with

$$\theta = 3H = 3\alpha \coth(\alpha t). \tag{19}$$

Here η_0 is proportionality constant and $H = \frac{\dot{a}}{a}$ is Hubble's parameter. Such relation (18) has already been proposed in the physical literature as a physically plausible relation [42]. Finally, we assume the perfect gas equation of state expressed as

$$p = \gamma\rho, 0 \leq \gamma \leq 1. \tag{20}$$

Using Eqs. (16), (18) and (19) into (13)−(15), we have the following set of expressions for the scale factors

$$A = a_1 \sinh(\alpha t) \exp\left[b_1 \int \left\{[\sinh(\alpha t)]^{-3(1+2\eta_0)}\right\} \mathrm{d}t\right], \tag{21}$$

$$B = a_2 \sinh(\alpha t) \exp\left[b_2 \int \left\{[\sinh(\alpha t)]^{-3(1+2\eta_0)}\right\} \mathrm{d}t\right], \tag{22}$$

$$C = a_3 \sinh(\alpha t) \exp\left[b_3 \int \left\{[\sinh(\alpha t)]^{-3(1+2\eta_0)}\right\} \mathrm{d}t\right]. \tag{23}$$

The physical quantities of observational interest in cosmology such as directional Hubble's parameters (H_i), spatial volume (V), shear scalar (σ^2) and mean anisotropy parameter (A_m) are respectively given by

$$H_i = \alpha \coth(\alpha t) + b_i [\sinh(\alpha t)]^{-3(1+2\eta_0)}, i = 1, 2, 3, \tag{24}$$

$$V = \sinh^3(\alpha t), \tag{25}$$

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2 - \frac{1}{3}\theta^2\right] = \frac{1}{2}\beta_1 [\sinh(\alpha t)]^{-6(1+2\eta_0)}, \tag{26}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{\beta_1 [\sinh(\alpha t)]^{-6(1+2\eta_0)}}{3\alpha^2 \coth^2(\alpha t)}, \tag{27}$$

where $\beta_1 = b_1^2 + b_2^2 + b_3^2$.
The expressions for energy density, effective pressure, isotropic pressure, bulk viscosity and shear viscosity of the model are given by

$$\rho = 3\alpha^2 \coth^2(\alpha t) + \beta_2 [\sinh(\alpha t)]^{-6(1+2\eta_0)}, \tag{28}$$

$$\bar{p} = \alpha^2 [(6\eta_0 - 1)\coth^2(\alpha t) - 2] - \beta_3 [\sinh(\alpha t)]^{-6(1+2\eta_0)}, \tag{29}$$

$$p = 3\gamma\alpha^2 \coth^2(\alpha t) + \beta_2 \gamma [\sinh(\alpha t)]^{-6(1+2\eta_0)}, \tag{30}$$

$$\xi = \frac{1}{3}\left[(1+3\gamma)\alpha \coth(\alpha t) + \tanh(\alpha t)\left\{2\alpha + \frac{(\beta_2\gamma + \beta_3)}{\alpha[\sinh(\alpha t)]^{6(1+2\eta_0)}}\right\}\right], \tag{31}$$

$$\eta = 3\eta_0 \alpha \coth(\alpha t), \tag{32}$$

where $\beta_2 = b_1 b_2 + b_2 b_3 + b_1 b_3$ and $\beta_3 = b_2^2 + b_3^2 + b_2 b_3$.

From Eqs. (19), (26) and (28), we get the following relations:

$$\frac{\sigma^2}{\theta^2} = \frac{\beta_1 \tanh^2(\alpha t)}{18\alpha^2 [\sinh(\alpha t)]^{6(1+2\eta_0)}}, \frac{\rho}{\theta^2} = \frac{1}{3} + \frac{\beta_2 \tanh^2(\alpha t)}{9\alpha^2 [\sinh(\alpha t)]^{6(1+2\eta_0)}}. \tag{33}$$

3.1.1. Thermodynamic Equations

The energy in a co-moving volume is $U = \rho V$. Due to the dissipative effects in a fluid, the equation for production of entropy S is given by

$$T\dot{S} = \dot{U} + p\dot{V} = 3V(3\xi + 2\eta A_m)H^2. \tag{34}$$

In a cosmic fluid where the energy density and pressure of the cosmic fluid are functions of temperature only *i.e.* $\rho = \rho(T)$, $p = p(T)$, and where the cosmic fluid has no net charge, we obtain easily (Grøn [16])

$$S = \frac{V}{T}(1+\gamma)\rho. \tag{35}$$

Let the entropy density be s so that

$$s = \frac{S}{V} = \frac{(1+\gamma)\rho}{T}. \tag{36}$$

The first law of thermodynamics may be written as

$$\mathrm{d}(\rho V) + \gamma\rho \mathrm{d}V = (1+\gamma)T\mathrm{d}\left(\frac{\rho V}{T}\right), \tag{37}$$

which on integration yields

$$T \sim \rho^{\left(\frac{\gamma}{1+\gamma}\right)}. \tag{38}$$

From Eqs. (36) and (38), we get the entropy density as

$$s \sim \rho^{\left(\frac{1}{1+\gamma}\right)}. \tag{39}$$

Using Eqs. (25) and (28), we find the respective temperature (T), entropy density (s) and total entropy (S) from Eqs. (36), (38) and (39) as

$$T = T_0\left[3\alpha^2\coth^2(\alpha t) + \beta_2[\sinh(\alpha t)]^{-6(1+2\eta_0)}\right]^{\frac{\gamma}{(1+\gamma)}}, \tag{40}$$

$$s = s_0\left[3\alpha^2\coth^2(\alpha t) + \beta_2[\sinh(\alpha t)]^{-6(1+2\eta_0)}\right]^{\frac{1}{(1+\gamma)}}, \tag{41}$$

$$S = S_0\sinh^3(\alpha t)\left[3\alpha^2\coth^2(\alpha t) + \beta_2[\sinh(\alpha t)]^{-6(1+2\eta_0)}\right]^{\frac{1}{(1+\gamma)}}, \tag{42}$$

where T_0, s_0 and S_0 are positive constants. From Eqs. (34) and (35), we get the entropy production rate as

$$\frac{\dot{S}}{S} = \frac{3(3\xi + 2\eta A_m)H^2}{(1+\gamma)\rho}. \tag{43}$$

Using Eqs. (27), (28), (31) and (32), Equation (43) yields

$$\frac{\dot{S}}{S} = \frac{3}{(1+\gamma)\left[3\alpha^2\coth^2(\alpha t) + \beta_2\{\sinh(\alpha t)\}^{-6(1+2\eta_0)}\right]}\Bigg[(1+3\gamma)\alpha^3\coth^3(\alpha t) + 2\alpha^3\coth(\alpha t) + \alpha(2\eta_0\beta_1 + \gamma\beta_2 + \beta_3)\coth(\alpha t)\{\sinh(\alpha t)\}^{-6(1+2\eta_0)}\Bigg]. \tag{44}$$

3.2. CASE 2: WHEN $a(t) = te^t$

Following Amirhashchi *et al.* [40], we consider the following scale factor

$$a(t) = te^t, \tag{45}$$

which yields a time dependent DP as

$$q(t) = -\frac{t(t+2)}{(t+1)^2}. \tag{46}$$

Further, expansion scalar

$$\theta = 3\left(\frac{t+1}{t}\right). \tag{47}$$

Using Eqs. (18), (45) and (47) into Eqs. (13)–(15), we have the following set of expressions for the scale factors

$$A = a_1(te^t)\exp\left(b_1\int t^{-3(1+2\eta_0)}e^{-3(1+2\eta_0)t}\mathrm{d}t\right), \tag{48}$$

$$B = a_2(te^t)\exp\left(b_2\int t^{-3(1+2\eta_0)}e^{-3(1+2\eta_0)t}\mathrm{d}t\right), \tag{49}$$

$$C = a_3(te^t)\exp\left(b_3\int t^{-3(1+2\eta_0)}e^{-3(1+2\eta_0)t}\mathrm{d}t\right). \tag{50}$$

The physical parameters as described in the case 1 are expressed as

$$H_i = 1 + \frac{1}{t} + b_i t^{-3(1+2\eta_0)}e^{-3(1+2\eta_0)t}, \quad i = 1,2,3, \tag{51}$$

$$V = t^3 e^{3t}, \tag{52}$$

$$A_m = \frac{1}{3}\beta_1\frac{t^{-4(1+3\eta_0)}e^{-6(1+2\eta_0)t}}{(t+1)^2}, \tag{53}$$

$$\sigma^2 = \frac{1}{2}\beta_1 t^{-6(1+2\eta_0)}e^{-6(1+2\eta_0)t}. \tag{54}$$

The expressions for energy density, effective pressure, isotropic pressure, bulk viscosity and shear viscosity of the model are given by

$$\rho = 3\left(\frac{t+1}{t}\right)^2 + \beta_2 t^{-6(1+2\eta_0)}e^{-6(1+2\eta_0)t}, \tag{55}$$

$$\bar{p} = (6\eta_0 - 1)\left(\frac{t+1}{t}\right)^2 - 2\left(\frac{t+2}{t}\right) - \beta_3 t^{-6(1+2\eta_0)}e^{-6(1+2\eta_0)t}. \tag{56}$$

$$p = 3\gamma\left(\frac{t+1}{t}\right)^2 + \beta_2\gamma t^{-6(1+2\eta_0)}e^{-6(1+2\eta_0)t}, \tag{57}$$

$$\xi = \frac{1}{3}\left[(1+3\gamma)\left(\frac{t+1}{t}\right)+2\left(\frac{t+2}{t+1}\right)+(\beta_2\gamma+\beta_3)\frac{t^{-(5+12\eta_0)}e^{-6(1+2\eta_0)t}}{(t+1)}\right], \quad (58)$$

$$\eta = 3\eta_0\left(\frac{t+1}{t}\right). \quad (59)$$

Further from Eqs. (47), (54) and (55), we get

$$\frac{\sigma^2}{\theta^2} = \frac{1}{18}\beta_1\frac{t^{-4(1+3\eta_0)}e^{-6(1+2\eta_0)t}}{(t+1)^2}, \frac{\rho}{\theta^2} = \frac{1}{3}+\frac{1}{9}\beta_2\frac{t^{-4(1+3\eta_0)}e^{-6(1+2\eta_0)t}}{(t+1)^2}. \quad (60)$$

3.2.1. Thermodynamic Equations

Using Eqs. (52) and (55), we find the respective temperature (T), entropy density (s) and total entropy (S) from Eqs. (36), (38) and (39), as

$$T = T_0\left[3\left(\frac{t+1}{t}\right)^2+\beta_2 t^{-6(1+2\eta_0)}e^{-6(1+2\eta_0)t}\right]^{\frac{\gamma}{1+\gamma}}, \quad (61)$$

$$s = s_0\left[3\left(\frac{t+1}{t}\right)^2+\beta_2 t^{-6(1+2\eta_0)}e^{-6(1+2\eta_0)t}\right]^{\frac{1}{1+\gamma}}, \quad (62)$$

$$S = S_0 t^3 e^{3t}\left[3\left(\frac{t+1}{t}\right)^2+\beta_2 t^{-6(1+2\eta_0)}e^{-6(1+2\eta_0)t}\right]^{\frac{1}{1+\gamma}}. \quad (63)$$

Further, rate of change of entropy is expressed as

$$\frac{\dot{S}}{S} = \frac{3}{(1+\gamma)\left[3\left(\frac{t+1}{t}\right)^2+\beta_2 t^{-6(1+2\eta_0)}e^{-6(1+2\eta_0)t}\right]}\left[(1+3\gamma)\left(\frac{t+1}{t}\right)^3\right.$$
$$\left.+\frac{2(t+1)(t+2)}{t^2}+(\beta_2\gamma+\beta_3+2\eta_0\beta_1)(t+1)t^{-(7+12\eta_0)}e^{-6(1+2\eta_0)t}\right]. \quad (64)$$

3.3. PHYSICAL BEHAVIOR OF THE MODELS

It is observed that the solutions presented in this paper (for both the cases) satisfy the energy conservation equation (9) identically. Therefore, the presented solutions are exact solutions of Einstein's field equations (5)−(8).

Deceleration Parameter: From Fig. 1, it is interesting to notice that q decreases very rapidly and reaches upto value -1 and, then after it remains constant (*i.e.* -1) like de Sitter universe.

Energy Density and Pressure: From Eqs. (28) and (30) for case 1, and Eqs. (55) and (57) for case 2, we conclude that the energy density ρ and isotropic pressure p are always positive and decreasing functions of time. Fig. 2 depicts the same character for energy density ρ.

Bulk Viscosity: It is self explanatory from Fig. 3 and Fig. 4 that ξ is a positive decreasing function of time and it approaches to a constant quantity which is near to zero. This is in good agreement with the physical behavior of ξ.

Anisotropy Parameter: It is obvious from Eqs. (27) and (53) that the anisotropy parameter decreases faster with time due to the presence of viscosity.

Further from Eqs. (33) and (60): (i) $\frac{\rho}{\theta^2}$ is maximum at $t=\infty$ and the maximum value is $\frac{1}{3}$, (ii)$\frac{\sigma^2}{\theta^2}$ is a decreasing function of time and decays to zero as $t\to\infty$. Hence the model approaches to isotropy for large values of time *i.e.* at present epoch.

Entropy: It is clear from Eqs. (44) and (64) that $\frac{\dot{S}}{S}>0$, which implies that the total entropy goes on increasing as the evolution progresses.

Moreover, we observe that the spatial volume and spatial scale factors are zero and the expansion scalar is infinite at $t=0$, which is big bang scenario. All the physical quantities pressure (p), energy density (ρ), bulk viscosity (ξ), shear viscosity (η), Hubble factor (H) and shear scalar (σ) diverge at $t=0$. As $t\to\infty$, scale factors and volume becomes infinite whereas p, ρ approach to zero.

4. CONCLUDING REMARKS

In the present paper, we have investigated Bianchi type-I space-time in presence of a dissipative fluid by considering time dependent DP. It is observed that, our derived model has accelerated expansion at present epoch which is consistent with recent observations of Type Ia supernova and CMB anisotropies. As shown in §3.3, viscosity played an important role in the process of isotropization of the large scale structure of the universe. We also conclude that the model represents shearing, non-rotating and expanding universe, which starts with a big bang and approaches to isotropy at present epoch.

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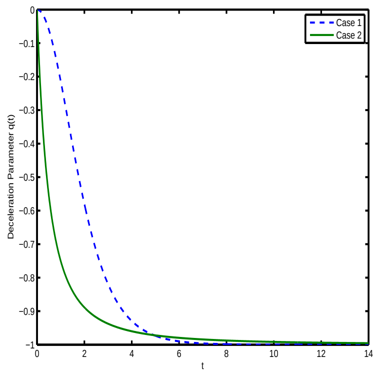

Fig. 1 – Plot of DP q *versus* t for Case 1 and Case 2 with $\alpha = 0.5$.

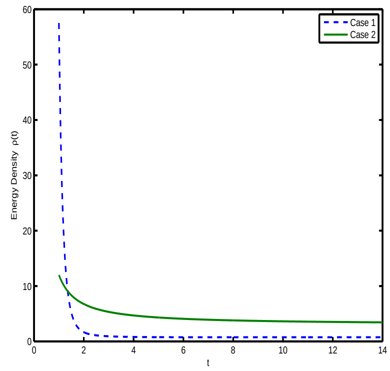

Fig. 2 – Plot of energy density ρ *versus* t for Case 1 and Case 2 with $\beta_2 = 1$, $\alpha = 0.5$ and $\eta_0 = 0.01$.

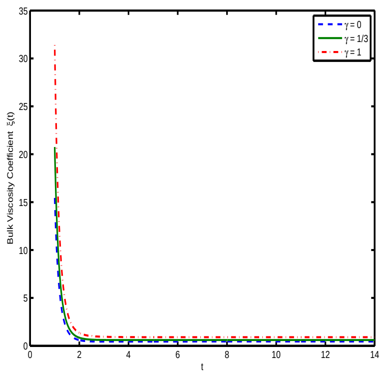

Fig. 3 – Plot of bulk viscosity coefficient ξ *versus* t for Case 1 with $\beta_2 = \beta_3 = 1$, $\alpha = 0.5$ and $\eta_0 = 0.01$.

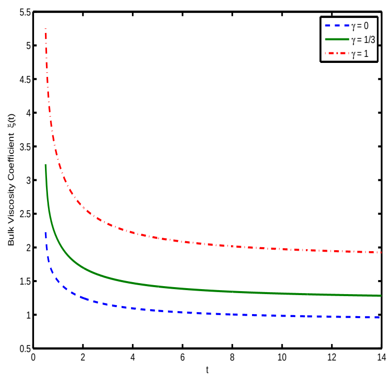

Fig. 4 – Plot of bulk viscosity coefficient ξ *versus* t for Case 2 with $\beta_2 = \beta_3 = 1$ and $\eta_0 = 0.01$.