SOME BULK VISCOUS MAGNETIZED LRS BIANCHI TYPE-I STRING COSMOLOGICAL MODELS IN LYRA'S GEOMETRY

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Some anisotropic locally rotationally symmetric (LRS) bulk viscous magnetized Bianchi type-I string cosmological models are studied in context of Lyra's geometry. The modified Einstein's field equations have been solved by taking a physically valuable condition that shear scalar (σ) is proportional to the scalar expansion (θ) which leads to $A = aB^n$. This general solution in terms of metric potential (B) describe the characteristic of string Universe in presence of bulk viscosity and magnetic field. The study reveals that the coefficient of viscosity decreases uniformly with cosmic expansion during evolution of universe. In absence of magnetic field, the model can exit during the a span of time and the energy conditions can be fulfilled for a finite interval of time due to presence of bulk viscosity. It has been found that the displacement vector (β) is a decreasing function of time and it approaches to small positive value at late time, which is collaborated with Halford (Aust. J. Phys. 23, 863, 1970) as well as recent observations of SN Ia. The physical behavior of derived models are also described.

Key words: Cosmology, String theory, LRS Bianchi type-I, Lyra's geometry.

1. INTRODUCTION

It is generally assumed that after the big bang, the universe may have undergone a series of phase transitions as its temperature lowered down below some critical temperature as predicted by grand unified theories [1-6]. It is belived that during phase transition the symmetry of the universe is broken spontaneously which give rise to topologically stable defects such as domain walls, strings and monopoles. In all these three cosmological structures, cosmic strings have excited the most interesting consequence [7] because they are believed to give rise to density perturbations which lead to formation of galaxies [4,8]. The strings are free to vibrate and their different vibrational modes present different types of particles carrying the force of gravitation. The general relativistic treatment of strings has been initially given by Letelier [9,10] and Stachel [11]. Letelier [10] obtained massive string cosmological models in Bianchi type-I and Kantowski-Sachs space-times. Benerjee *et al.* [12] have investigated an axially symmetric Bianchi type-I string dust cosmological model in presence and absence of magnetic field.

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On the other hand, introduction of viscosity in the cosmic fluid content plays a significant role in analysis of many important physical aspects of the dynamics of universe. Bulk viscosity is associated with Grand Unified theory (GUT) phase transition and string creation. At an early stage of the universe, when neutrino decoupling occurs during radiation era and decoupling of radiation with matter takes place during recombination era, the matter behaves like a viscous fluid. The coefficient of viscosity is known to decrease as the universe expands. The effect of viscosity on the evolution of cosmological models and the role of viscosity in avoiding the initial big bang singularity has been studied by several authors (Maartens [13], Misner [14], Weinberg [15], Murphy [16]).

In addition, the occurrence of magnetic field on galactic scale is a well established fact today and anisotropic magnetic field models have significant contribution in the evolution of galaxies and stellar objects. Zeldovich *et al.* [17] have pointed out the importance of magnetic field in various astronomical phenomena. Also, Harrison [18] has suggested that magnetic field could have a cosmological origin. Melvin [19] has described that during the evolution of Universe, the matter is in highly ionized state and due to smooth coupling with field it form neutral matter as a result of Universe expansion. Strong magnetic field can be created due to adiabatic compression in clusters of galaxies. Large-scale magnetic field gives rise to anisotropies in the universe. Therefore, the presence of magnetic field in anisotropic string universe is not unrealistic. Bali and Upadhaya [20] have presented LRS Bianchi type-I string dust magnetized cosmological models in general relativity. Wang [21,22] has investigated LRS Bianchi I string cosmological models in general relativity in presence of bulk viscosity and electromagnetic field where constant coefficient of bulk viscosity is considered.

One of the most intriguing modifications of general relativity is that proposed by Weyl [23], invented to unify gravitation and electromagnetism by means of fundamental changes in Riemannian geometry. Unfortunately the Weyl theory suffers from non integrability of length and is, therefore physically unacceptable. However being interesting from mathematical point of view, it may still have the germs of a future fruitful theory. Later, Lyra [24] modified Riemannian geometry and removed non-integrability of length transfer by introducing a gauge function into the structure-less manifold as a result of which a displacement field arise naturally. Subsequently, Sen *et al.* [25,26] proposed a new scalar tensor theory of gravitation. They constructed an analog of the Einstein Field Equation based on Lyra's geometry given by equation (8). Halford [27, 28] showed that the scalar-tensor treatment based on Lyra's geometry predicts some effects within observational limits, as in Einstein's theory.

In this paper, we present some bulk viscous LRS Bianchi type-I string magnetized cosmological models in frame work of Lyra's geometry. The magnetic field is due to an electric current produced along x-axis with infinite electrical conductivity.

2. METRIC AND FIELD EQUATIONS

We consider the LRS spatially homogeneous Bianchi type I metric of the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + A^2(t)\mathrm{d}x^2 + B^2(t)(\mathrm{d}y^2 + \mathrm{d}z^2) \tag{1}$$

where A and B are the functions of time t alone.
The energy momentum tensor for a cloud of string with magnetic field in co-moving coordinate system is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \zeta\theta(u_i u_j - g_{ij}) + E_{ij} \tag{2}$$

where the vector u_i describes the cloud four-velocity and x_i represents a direction of anisotropy, *i.e.* the string, satisfy the relations

$$u^i u_i = -x^i x_i = -1 \qquad u^i x_i = 0 \tag{3}$$

and in comoving coordinate system,

$$T_1^1 = T_2^2 = 0,\ T_3^3 = \lambda,\ T_4^4 = \rho,\ \ T_j^i = 0 \ \text{ for } \ i \neq j. \tag{4}$$

Here ρ is the rest energy of the cloud of strings with massive particles attached to them. It is given by $\rho = \rho_p + \lambda$, ρ_p being the rest energy density of particles attached to the strings and λ the density of tension that characterizes the strings. The energy momentum for magnetic field is

$$E_{ij} = \frac{1}{4\pi}\left(F_{ik}F_{jl}g^{kl} - \frac{1}{4}g_{ij}F^{kl}F_{kl}\right), \tag{5}$$

where F_{ij} is the electromagnetic field tensor which satisfies the Maxwell's equations

$$F_{[ij;k]} = 0, \qquad \left(F^{ij}\sqrt{-g}\right); j = 0. \tag{6}$$

In comoving coordinates, the incident magnetic field is taken along x-axis, with the help of Maxwell equations (6), the only non vanishing component of F_{ij} is

$$F_{23} = Constant.$$

The general equation equation of state can be considered as

$$\rho = \alpha\lambda. \tag{7}$$

The field equation in the normal gauge for Lyra's manifold, as obtained by Sen are

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -8\pi G T_{ij}, \tag{8}$$

where ϕ_i is the displacement field vector defined as

$$\phi_i = (0,0,0,\beta(t)) \tag{9}$$

and other symbols have their usual meaning as in Riemannian geometry. For metric (1), the field equation (8) with the equations (2)-(7) take the form

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}=\alpha\lambda+\frac{F_{23}^2}{8\pi B^4}+\frac{3}{4}\beta^2 \tag{10}$$

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}=\lambda+\zeta\theta+\frac{F_{23}^2}{8\pi B^4}-\frac{3}{4}\beta^2 \tag{11}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=\zeta\theta-\frac{F_{23}^2}{8\pi B^4}-\frac{3}{4}\beta^2 \tag{12}$$

The quantities with dots overhead refer to their partial derivatives with respect to time co-ordinate and we choose units such that, $8\pi G=1$.

3. SOLUTION IN PRESENCE OF BULK VISCOSITY AND MAGNETIC FIELD

The research on exact solutions is based on some physically reasonable restrictions used to simplify the Einstein equations. Equations (10)-(12) are three independent equations connecting five unknowns (A, B, λ, ζ and β), for the complete determinacy of the system, we need two extra conditions. Firstly, we assume that the coefficient of bulk viscosity (ζ) is inversely proportional to the expansion θ. This condition leads to

$$\zeta\theta=K(constant) \tag{13}$$

and secondly, we consider the expansion θ is proportional to the shear σ. This condition leads to

$$A=aB^n, \tag{14}$$

where a and $n>0$ are constants.
From equations (10)-(12), with the help of equations (13) and (14), eliminating λ and β, we have

$$\frac{\ddot{B}}{B}+\mu\frac{\dot{B}^2}{B^2}=\nu K-\frac{lM}{B^4} \tag{15}$$

where $\mu=\frac{[(n+1)^2-\alpha(1-n^2)]}{[\alpha(1-n)-(n+1)]}$, $\nu=\frac{2\alpha}{[\alpha(1-n)-(n+1)]}$, $l=\frac{(\alpha+1)}{[\alpha(1-n)-(n+1)]}$ and $M=\frac{F_{23}^2}{8\pi}$
To solve equation (15), let us assume that $\dot{B}=f(B)$. Thus, $\ddot{B}=f\frac{\mathrm{d}f}{\mathrm{d}B}$. Accordingly equation (15) leads to

$$2f\frac{\mathrm{d}f}{\mathrm{d}B}+2\mu\frac{f^2}{B}=2\nu KB-2lMB^{-3} \tag{16}$$

This is a first order linear differential equation which can be written as

$$\frac{\mathrm{d}}{\mathrm{d}B}(f^2B^{2\mu})=2\nu KB^{2\mu+1}-2lMB^{2\mu-3} \tag{17}$$

On integrating equation (17), we obtain

$$t = \int \frac{1}{\sqrt{\left[\frac{m}{B^{2\mu}} + \frac{\nu K B^2}{(\mu+1)} - \frac{lM}{(\mu-1)B^2}\right]}} \mathrm{d}B \tag{18}$$

where m is constant of integration.
For this solution, the geometry of the universe is described by the line-element

$$\mathrm{d}s^2 = -\left[\frac{m}{B^{2\mu}} + \frac{\nu K B^2}{(\mu+1)} - \frac{lM}{(\mu-1)B^2}\right]^{-1} \mathrm{d}B^2 + a^2 B^{2n} \mathrm{d}x^2 + B^2(\mathrm{d}y^2 + \mathrm{d}z^2) \tag{19}$$

Under suitable transformation of coordinates, the metric (19) can be reduced to the form

$$\mathrm{d}s^2 = -\left[\frac{m}{T^{2\mu}} + \frac{\nu K T^2}{(\mu+1)} - \frac{lM}{(\mu-1)T^2}\right]^{-1} \mathrm{d}T^2 + a^2 T^{2n} \mathrm{d}X^2 + T^2(\mathrm{d}Y^2 + \mathrm{d}Z^2) \tag{20}$$

where $\mu \neq \pm 1$ and the cosmic scale $T = B$ can be determined by equation (18).

3.1. SOME PHYSICAL AND GEOMETRIC FEATURES

The expression for the energy density (ρ), the string tension density (λ), particle density (ρ_p), the coefficient of bulk viscosity (ζ) and the displacement vector (β) are given by

$$\rho = \left[\frac{2m\alpha[(n+1)-\mu]}{(1+\alpha)} - M\left(\frac{2l\alpha[(n+1)-\mu]}{(1+\alpha)(1+\mu)} + 2(1+l)\right)T^{-2}\right]T^{-2} + K\left[\frac{2\nu\alpha[n+1-\mu]}{(1+\alpha)(1+\mu)} + (2\nu-1)\right] \tag{21}$$

$$\lambda = \frac{1}{\alpha}\rho \tag{22}$$

$$\rho_p = \rho - \lambda = (1 - \frac{1}{\alpha})\rho \tag{23}$$

$$\zeta = \frac{K}{(n+2)}\left[\frac{m}{T^{2(\mu+1)}} + \frac{\nu K}{(\mu+1)} - \frac{lM}{(\mu-1)T^4}\right]^{-\frac{1}{2}} \tag{24}$$

$$\beta = \frac{2}{\sqrt{3}}\left[m[\mu(n+1)-n^2]T^{-2(\mu+1)} + \left(lM(n+1) - \frac{[\mu(n+1)-n^2]}{(\mu-1)}\right)T^{-4} + K\left(\frac{\nu[\mu(n+1-n^2)]}{(\mu+1)} + 1\right)\right]^{\frac{1}{2}}. \tag{25}$$

The spatial volume (V) of Universe is given by

$$V = aT^{(n+2)} \tag{26}$$

The spatial volume $V \to 0$ when $T \to 0$, and $V \to \infty$ when $T \to \infty$.
The physical quantities expansion scalar (θ), and shear scalar (σ) are given by

$$\theta = u^i_{;i} = (n+2)\left[\frac{m}{T^{2(\mu+1)}} + \frac{\nu K}{(\mu+1)} - \frac{lM}{(\mu-1)T^4}\right]^{\frac{1}{2}} \tag{27}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{(n-1)^2}{3}\left[\frac{m}{T^{2(\mu+1)}} + \frac{\nu K}{(\mu+1)} - \frac{lM}{(\mu-1)T^4}\right], \tag{28}$$

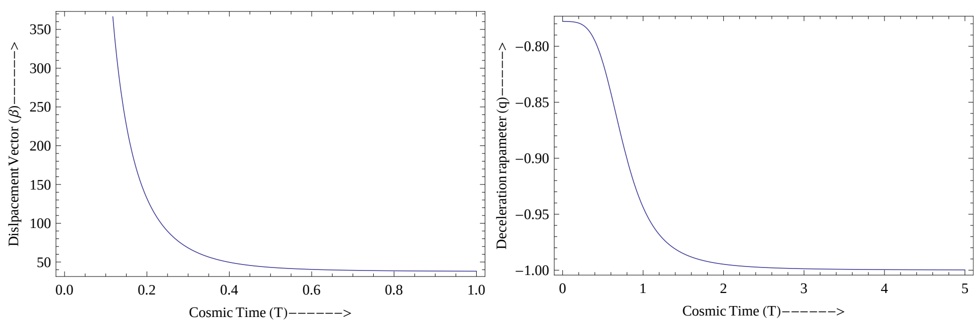
hence

$$\lim_{T\to\infty}\left(\frac{\sigma^2}{\theta^2}\right) = \frac{(n-1)^2}{3(n+2)^2} = constant. \tag{29}$$

Thus the model does not approach isotropy for large value of T.
The Hubble parameter (H) and deceleration parameter (q) are given by

$$H = \frac{(n+2)}{3}\left[\frac{m}{T^{2(\mu+1)}} + \frac{\nu K}{(\mu+1)} - \frac{lM}{(\mu-1)T^4}\right]^{\frac{1}{2}} \tag{30}$$

$$q = -\frac{3}{(n+2)}\left[\left(\nu K T^2 - \frac{lM}{T^2}\right)\left(\frac{m}{T^{2\mu}} + \frac{\nu K T^2}{(\mu+1)} - \frac{lM}{(\mu-1)T^2}\right)^{-1} + \frac{[n-(1+3\mu)]}{3}\right] \tag{31}$$



(a) The plot of 'β' *vs.* 'T' for model-20 with parameters $m = 5.5$, $n = 0.5$, $\mu = 0.7$, $M = 0.5$, $l = 20$, $\nu = 50$, and $K = 40$.

(b) The plot 'q' *vs.* 'T' for model-20 with parameters $m = 0.5$, $n = 25$, $\mu = 0.5$, $\nu = 50$ and $K = 1.5$.

Fig. 1 – The plots of displacement vector β and deceleration parameter 'q' *vs.* cosmic time T for model (20).

It is observed from figure 1(a) that the displacement vector $\beta(t)$ has a very large value at beginning and reduces fast during evolution of Universe analogous to cosmological constant Λ. The negative value of deceleration parameter 'q' implies that our model (20) of universe is accelerating. Figure 1(b) shows that the value of q lies in the range $-1 \leq q < 0$ which is consistent with current observations [30-37].

3.2. DISCUSSION

The space-time (20) represents the string magnetized LRS Bianchi type-I Universe with bulk viscosity. From equation (21), we observe that the energy condition $\rho \geq 0$ given by Hawking and Ellis [29] leads to

$$\left[K\left(\frac{\nu}{(1+\mu)} + \frac{(1+\alpha)(2\nu-1)}{2\alpha[n+1-\mu]}\right) + \frac{m}{T^2}\right]T^4 \geq M\left[\frac{l}{(1+\mu)} + \frac{(1+\alpha)(1+l)}{\alpha[n+1-\mu]}\right] \quad (32)$$

Equations (22) and (23) shows that when $\alpha \geq 1$, the particle density $\rho_p \geq 0$ and string tension density $\lambda \geq 0$, however, when $\alpha < 0$, $\rho_p > 0$ and $\lambda < 0$. The energy density ρ is infinite at $T = 0$, and $\rho \to K\left[\frac{2\nu\alpha[n+1-\mu]}{(1+\alpha)(1+\mu)} + (2\nu-1)\right]$ when $T \to \infty$, provided $n+1 > \mu$ and $2\nu - 1 > 0$. The spatial volume V tends to zero when $T \to 0$ and $V \to \infty$ when $T \to \infty$. The scalar expansion θ is infinite at $T = 0$, and $\theta \to \frac{(n+2)]\nu K}{(\mu+1)}$ when $T \to \infty$, provided $\mu+1 > 0$. The figure 1(a) shows that the displacement vector β is a decreasing function of cosmic time analogous to cosmological constant Λ.

Since $\lim\limits_{T\to\infty}\left(\frac{\sigma}{\theta}\right) = constant$, the model does not approach isotropy for large value of T. Further, when $\alpha > 2$ or $\alpha < 0$, we have $\frac{\rho_p}{|\lambda|} > 1$, therefore in this case the massive strings dominate the universe in the process of evolution. However, when $1 < \alpha < 2$, we have $\frac{\rho_p}{|\lambda|} < 1$ and in this case the strings dominate over the particles. Recent observations show that the value of q is confined in the range $-1 \leq q < 0$ and the present day Universe is undergoing an accelerated expansion (Perlmutter *et al.* [30-32], Riess *et al.* [33, 34], Tonry *et al.* [35], John [36] and Knop *et al.* [37]). Figure 1(b) shows that the value of q lies in the range $-1 \leq q < 0$ which is consistent with current observations and its negative value indicates that our proposed model (20) is accelerating. In our derived LRS Bianchi type-I homogeneous model (20) isotropy is achieved for $n = 1$, $A \approx B = T$.

3.3. BULK VISCOUS MODEL IN ABSENCE OF MAGNETIC FIELD

In absence of magnetic field ($M = 0$), we obtain string cosmological model with bulk viscosity and in this case metric (20) reduces to the form

$$\mathrm{d}s^2 = -\left[\frac{m}{T^{2\mu}} + \frac{\nu K T^2}{(\mu+1)}\right]^{-1} \mathrm{d}T^2 + a^2 T^{2n} \mathrm{d}X^2 + T^2(\mathrm{d}Y^2 + \mathrm{d}Z^2). \quad (33)$$

The expression for the energy density (ρ), the string tension density (λ), particle density (ρ_p), the coefficient of bulk viscosity (ζ) and the displacement vector (β) are given by

$$\rho=\left[\frac{2m\alpha[(n+1)-\mu]}{(1+\alpha)}T^{-2}+K\left(\frac{2\nu\alpha[n+1-\mu]}{(1+\alpha)(1+\mu)}+(2\nu-1)\right)\right] \tag{34}$$

$$\lambda=\frac{1}{\alpha}\rho \tag{35}$$

$$\rho_p=\rho-\lambda=(1-\frac{1}{\alpha})\rho \tag{36}$$

$$\zeta=\frac{K}{(n+2)}\left[\frac{m}{T^{2(\mu+1)}}+\frac{\nu K}{(\mu+1)}\right]^{-\frac{1}{2}} \tag{37}$$

$$\beta=\frac{2}{\sqrt{3}}\left[m[\mu(n+1)-n^2]T^{-2(\mu+1)}+\frac{[\mu(n+1)-n^2]}{(\mu-1)}T^{-4}\right.$$
$$\left.+K\left(\frac{\nu[\mu(n+1-n^2)]}{(\mu+1)}+1\right)\right]^{\frac{1}{2}}. \tag{38}$$

The physical quantities expansion scalar (θ) and shear scalar (σ) are given by

$$\theta=u^i_{;i}=(n+2)\left[\frac{m}{T^{2(\mu+1)}}+\frac{\nu K}{(\mu+1)}\right]^{\frac{1}{2}} \tag{39}$$

$$\sigma^2=\frac{1}{2}\sigma_{ij}\sigma^{ij}=\frac{(n-1)^2}{3}\left[\frac{m}{T^{2(\mu+1)}}+\frac{\nu K}{(\mu+1)}\right] \tag{40}$$

hence

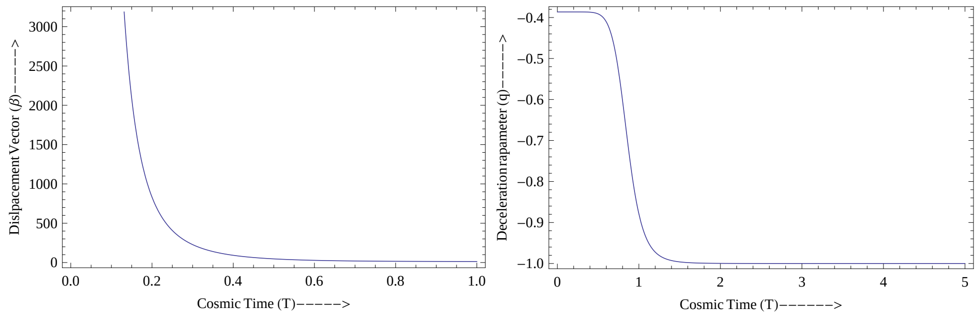
$$\lim_{T\to\infty}\left(\frac{\sigma^2}{\theta^2}\right)=\frac{(n-1)^2}{3(n+2)^2}=constant. \tag{41}$$

Thus the model does not approach isotropy.
The Hubble parameter (H) and deceleration parameter (q) are given by

$$H=\frac{(n+2)}{3}\left[\frac{m}{T^{2(\mu+1)}}+\frac{\nu K}{(\mu+1)}\right]^{\frac{1}{2}} \tag{42}$$

$$q=-\frac{3}{(n+2)}\left[\nu KT^2\left(\frac{m}{T^{2\mu}}+\frac{\nu KT^2}{(\mu+1)}\right)^{-1}+\frac{[n-(1+3\mu)]}{3}\right] \tag{43}$$

The displacement vector $\beta(t)$ is large at beginning of Universe and reduces fast during its evolution analogous to cosmological constant Λ as shown in figure 2(a). Figure 2(b) shows that the value of q confined in the range $-1\leq q<0$ which is consistent with current observations [30-37] and this negative value indicates that our proposed model (33) is accelerating.



(a) The plot of 'β' *vs.* 'T' for model-33 with parameters $m = 5.5$, $\alpha = 1.5$, $n = 1.6$, $\mu = 2.2$, $\nu = 40$ and $K = 50$.

(b) The plot 'q' *vs.* 'T' for model-33 with parameters $m = 10.5$, $n = 20$, $\mu = 3.5$, $\nu = 35$ and $K = 2.5$.

Fig. 2 – The plots of displacement vector β and deceleration parameter 'q' *vs.* cosmic time T for model-33.

3.4. DISCUSSION

The space-time (33) represents the bulk viscous LRS Bianchi type-I string cosmological model in absence of magnetic field. We observe from equation (34) that the energy condition $\rho \geq 0$ leads to

$$-K\left[\frac{\nu}{(1+\mu)}+\frac{(1+\alpha)(2\nu-1)}{2m\alpha[(n+1)-\mu]}\right]\leq T^{-2}\leq K\left[\frac{\nu}{(1+\mu)}+\frac{(1+\alpha)(2\nu-1)}{2m\alpha[(n+1)-\mu]}\right]. \quad (44)$$

The model (33) can exit during the span of time given by (44). The energy condition can be fulfill for a finite interval of time due to presence of bulk viscosity. Equations (35) and (36) shows that when $\alpha \geq 1$, the particle density $\rho_p \geq 0$ and string tension density $\lambda \geq 0$, however, $\rho_p > 0$ and $\lambda < 0$ when $\alpha < 0$. The energy density ρ is infinite at $T = 0$, and $\rho \to K\left[\frac{2\nu\alpha[n+1-\mu]}{(1+\alpha)(1+\mu)}+(2\nu-1)\right]$ when $T \to \infty$. The spatial volume V tends to zero when $T \to 0$ and $V \to \infty$ when $T \to \infty$. The scalar expansion θ is infinite at $T = 0$, and $\theta \to \left[\frac{(n+2)]\nu K}{(\mu+1)}\right]$ when $T \to \infty$, provided $\mu + 1 > 0$.

Since $\lim_{T\to\infty}\left(\frac{\sigma}{\theta}\right) = constant$ the model does not approach isotropy for large value of T. Further, when $1 < \alpha < 2$, we have $\frac{\rho_p}{|\lambda|} < 1$ and in this case the strings dominate over the particles. However, when $\alpha > 2$ or $\alpha < 0$ we have $\frac{\rho_p}{|\lambda|} > 1$, therefore the massive strings dominate the universe in the process of evolution. This model has singularity at $T = 0$. The figure 2(a) shows that the displacement vector β is a decreasing function of cosmic time (T) which is analogous to the observed values of cosmological constant Λ. The current observations [30-37] show that the value of (q) is confined in the range $-1 \leq q < 0$ and the present day Universe is undergoing an accelerated expansion. Figure 2(b) shows that the value of q lies in the range $-1 \leq q < 0$ which is consistent with current observations. The negative value 'q'

represents that our proposed model (33) is accelerating.

4. CONCLUDING REMARKS

The present paper deals with the study of some LRS Bianchi type I string cosmological models in presence and absence of bulk viscosity and electromagnetic field in frame work of Lyra's manifold. In section 3, The modified Einstein's field equations have been solved for a general case in terms of a scale factor (T), describing the behavior of string Universe in presence of bulk viscosity and magnetic field. The main features of the work are as follows:

- The present study reveals that the coefficient of viscosity decreases with expansion of Universe and become insignificant at a late time. The bulk viscous model can exist in a span of times discussed in section-3.
- The presence of magnetic field affect energy density (ρ), strings particle density (ρ_p), tension density (λ), coefficient of bulk viscosity (ζ), displacement vector (β) and expansion as well as acceleration of proposed Universe. During the evolution of Universe, highly ionized matter smoothly coupled with fields to form neutral matter which causes the expansion of Universe.
- When $1 < \alpha < 2$, we have $\frac{\rho_p}{|\lambda|} < 1$, and in this case, the strings dominate over the particles. However, when $\alpha > 2$ or $\alpha < 0$ we have $\frac{\rho_p}{|\lambda|} > 1$, therefore the massive strings dominate the Universe in the process of evolution.
- It all the singular models, the displacement vector β is large at beginning of the Universe and reduces fast during its evolution [see Figs. 1(a), 2(a)]. The observed value of β is analogous to the value of cosmological constant Λ which is collaborated with Halford as well as recent observations of SN Ia.
- In all the deterministic models, $\frac{\sigma}{\theta} = constant$ implies that the models do not approach isotropy.
- Our Universe is isotropic and homogeneous at present and well described by FRW models. For $n = 1$, $A \approx B$, therefore isotropy is achieved in our derived homogeneous LRS Bianchi type-I models.

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