DYNAMIC COSMOLOGICAL "CONSTANT" IN BRANS DICKE THEORY

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A new class of homogeneous and isotropic cosmological model with variable cosmological term Λ in the Brans Dicke theory has been obtained. The effect of cosmological term during accelerated expansion of the universe is investigated in the flat, open and closed FRW models of the universe. Various forms of phenomenological decay law for dynamic cosmological "constant" are considered for obtaining exact cosmological solutions. Physical behaviour of the models have also been discussed.
Key words: Cosmological constant, Brans-Dicke theory, FRW models.

1. INTRODUCTION

Since last few decades there is a growing interest in alternative theories of gravitation, especially scalar-tensor theories of gravity, which are very useful tools in understanding early universe models. The Brans-Dicke theory [1] of gravity is most promising one among all existing alternative theories of gravitation. It was shown that inflationary model [2], extended inflationary model [3], hyper extended inflationary model [4], chaotic inflation [5], are based on Brans-Dicke scalar tensor theory. In this theory gravitational constant is replaced by reciprocal of a massless scalar field ϕ. It has been suggested that large value of coupling parameter $(\omega \geq 500)$ makes the results of BD theory practically indistinguishable from Einstein general theory of relativity [6]. A number of authors [7–23] studied cosmological models in Brans-Dicke theory to investigate various aspects of expanding models of the universe.

The end of twentieth century has witness various changes in the theories of cosmic evolution of the universe. The measurements of the luminosity-redshift relations observed for 50 newly discovered type Ia supernovae with redshift $z > 0.35$ [24–25] and WMAP observations [26] predicted accelerated expansion of the universe [27–28]. These observations indicated that present constituent of the universe is dominated by some kind of energy with negative pressure, commonly known as *dark energy*, which constitutes about three fourths of the whole matter of our universe. The simplest and the most favoured candidate of dark energy is a cosmological "constant" which on one hand provide enough negative pressure to account this acceleration and on other the hand contribute an energy density of same order of magnitude than the energy density of the matter [27]. Observational data indicates that Cosmological constant $\Lambda = 10^{-55}\,\mathrm{cm}^{-2}$ while theoretical prediction for Λ is greater than this value by a factor of order 10^{120}. This discrepancy usually called cosmological "constant" problem, which is one of the puzzling problems in standard cosmology. The dynamical Λ was invoked to study the phenomenological decay of Λ so that it might be large at early epochs and reducing to a small value at the present epoch. A number of authors constructed models of more phenomenological character in which specific decay laws are postulated within the framework of general relativity. One of the very promising model among such models has the relation $\Lambda \propto H^2$ [29–33].

The effect of cosmological 'constant' has been extensively studied in the literature within the framework of general relativity and its alternative theories. Singh and Singh [9] investigated a cosmological model in Brans-Dicke theory by considering cosmological "constant" as function of scalar field ϕ. Pimentel [10] obtained exact cosmological solutions in Brans-Dicke theory with uniform cosmological "constant". A class of flat FRW cosmological models with cosmological "constant" in Brans-Dicke theory have also been obtained by Azar and Riazi [12]. The age of the universe from a view point of the nucleosynthesis with Λ term in Brans-Dicke theory was investigated by Etoh *et al* [13]. Azad and Islam [28] extended the idea of Singh and Singh [9] to study cosmological constant in Bianchi type I modified Brans-Dicke cosmology. Recently Qiang *et al* [34] discussed cosmic acceleration in five dimensional Brans-Dicke theory using interacting Higgs and Brans-Dicke fields. Smolyakov [35] investigated a model which provides the necessary value of effective cosmological "constant" at the classical level. Embedding general relativity with varying cosmological term in five dimensional Brans-Dicke theory of gravity in vacuum has been discussed by Reyes *et al* [36].

Motivated by above studies the investigation of role of dynamic cosmological "constant" has been considered in Brans-Dicke theory.

2. FIELD EQUATIONS

The field equation of Brans Dicke theory in presence of cosmological constant may be written as

$$R_{ij}-\frac{1}{2}Rg_{ij}-\Lambda g_{ij}+\frac{\omega}{\phi^2}\left[\phi_{;i}\phi_{;j}-\frac{1}{2}g_{ij}\phi_{;k}\phi^{;k}\right]+\frac{1}{\phi}\left[\phi_{;i;j}-g_{ij}\Box\phi\right]=\frac{8\pi}{\phi}T_{ij}, \quad (1)$$

$$\Box\phi=\phi^{;i}_{;i}=\frac{8\pi}{2\omega+3}T^{i}_{,i} \quad (2)$$

where ϕ is the scalar field. The energy momentum tensor T_{ij} of cosmic fluid can be defined as

$$T_{ij}=(\rho+p)u_i u_j-g_{ij}p \quad (3)$$

Let us consider a homogeneous and isotropic universe represented by FRW space-time metric

$$ds^2=dt^2-R^2(t)\left[\frac{dr^2}{1-kr^2}+r^2(d\theta^2+\sin^2\theta \;\; d\phi^2)\right], \quad (4)$$

where $R(t)$ is the scale factor, $k=1,\ 0,\ -1$ for spaces of positive, vanishing and negative curvature which represents closed, flat and open model of the universe respectively.
The FRW metric (4) and energy momentum tensor (3) along with Brans-Dicke field equations yield the following equations

$$\frac{3\dot{R}^2}{R^2}+3\frac{\dot{R}\dot{\phi}}{R\phi}-\frac{\omega\dot{\phi}^2}{2\phi^2}+3\frac{k}{R^2}=\frac{8\pi}{\phi}\rho+\Lambda, \quad (5)$$

$$2\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}+\frac{\ddot{\phi}}{\phi}+\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2}+2\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi}+\frac{k}{R^2}=\frac{-8\pi}{\phi}p+\Lambda, \quad (6)$$

$$\frac{\ddot{\phi}}{\phi}+3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi}=\frac{8\pi}{\phi}\frac{\rho-3p}{3+2\omega}+\frac{2\Lambda}{3+2\omega}. \quad (7)$$

The geometrical quantities of observational interest Hubble parameter H and deceleration parameter q are defined by

$$H=\frac{\dot{R}}{R}, \quad (8)$$

$$q=-\frac{\left(\dot{H}+H^2\right)}{H^2} \quad (9)$$

In order to find exact solutions of basic field equations (5)-(7), one must ensures that set of equations should be closed. Thus two more physically reasonable relations are required amongst the variables.
Now considering a well accepted power law relation [10–12] between scale factor $R(t)$ and scalar field ϕ of the form

$$\phi = \phi_0 R^{\alpha}, \tag{10}$$

the set of field equations (5)-(7), may be written as

$$\left(\frac{6+6\alpha-\omega\alpha^2}{2}\right)\frac{\dot{R}^2}{R^2}+\frac{3k}{R^2}=\frac{8\pi}{\phi_0 R^{\alpha}}\rho+\Lambda, \tag{11}$$

$$(2+\alpha)\frac{\ddot{R}}{R}+\frac{2+2\alpha+2\alpha^2+\omega\alpha^2}{2}\frac{\dot{R}^2}{R^2}+\frac{k}{R^2}=\frac{-8\pi}{\phi_0 R^{\alpha}}p+\Lambda, \tag{12}$$

$$\left[\alpha\frac{\ddot{R}}{R}+\alpha(\alpha+2)\frac{\dot{R}^2}{R^2}\right](3+2\omega)=\frac{8\pi}{\phi_0 R^{\alpha}}(\rho-3p)+2\Lambda, \tag{13}$$

A combination of equations (11)-(13) leads to

$$2(3-\omega\alpha)\frac{\ddot{R}}{R}+\left(6-4\omega\alpha-\omega\alpha^2\right)\frac{\dot{R}^2}{R^2}+6\frac{k}{R^2}=2\Lambda. \tag{14}$$

This equation is playing an important role in obtaining various cosmological solutions.

3. COSMOLOGICAL MODELS FOR FLAT FRW SPACE-TIME

It has been presented in the literature [15] that the findings of BOOMERANG experiment [37] strongly suggest the possibility of a flat universe. For flat model of the universe represented by FRW space-time (k = 0), equation (14) reduces to

$$2(3-\omega\alpha)\frac{\ddot{R}}{R}+\left(6-4\omega\alpha-\omega\alpha^2\right)\frac{\dot{R}^2}{R^2}=2\Lambda. \tag{15}$$

Now various phenomenological models of the dynamical cosmological "constant" Λ will be considered. In the absence of Λ term equation (15) reduces to the case already discussed by Johri and Kalyani [11].

3.1. CASE I: MODEL WITH $\Lambda \propto H^2$

Considering the commonly used relation between the cosmological "constant" and the Hubble parameter (H) ([33] and references there in) as

$$\Lambda = \beta H^2 , \tag{16}$$

equation (15) assumes the form

$$2(3-\omega\alpha)\dot{H} + \left(12 - 6\omega\alpha - \omega\alpha^2 - 2\beta\right) H^2 = 0 , \tag{17}$$

which may be written as $-\dfrac{\dot{H}}{H^2} = \dfrac{\left(12 - 6\omega\alpha - \omega\alpha^2 - 2\beta\right)}{2(3-\omega\alpha)} = a$ (say), (18)

where a is a constant.
On integration (18) yields the solution

$$R = \left(at+b\right)^{\frac{1}{a}} \quad \text{for} \quad a \neq 0, \tag{19}$$

$$R = c_1 e^{H_0 t} \quad \text{for} \quad a = 0. \tag{20}$$

Here c_1, b and H_0 are constants of integration.

3.1.1. Subcase I: model with power law solution

In order to obtain nonsingular cosmological model of the universe using power law relation (19) between the scale factor and cosmic time *t*. That gives relations for the scalar field and cosmological term respectively

$$\phi = k_1 \left(at+b\right)^{\frac{\alpha}{a}}, \tag{21}$$

$$\Lambda = \frac{\beta}{\left(at+b\right)^2} . \tag{22}$$

Using these relations one can obtain following expressions for energy density and pressure

$$\rho = \frac{\phi_0}{8\pi} \frac{(6 - \omega\alpha^2 + 6\alpha - 2\beta)}{\left(at+b\right)^{2-\frac{\alpha}{a}}}, \tag{23}$$

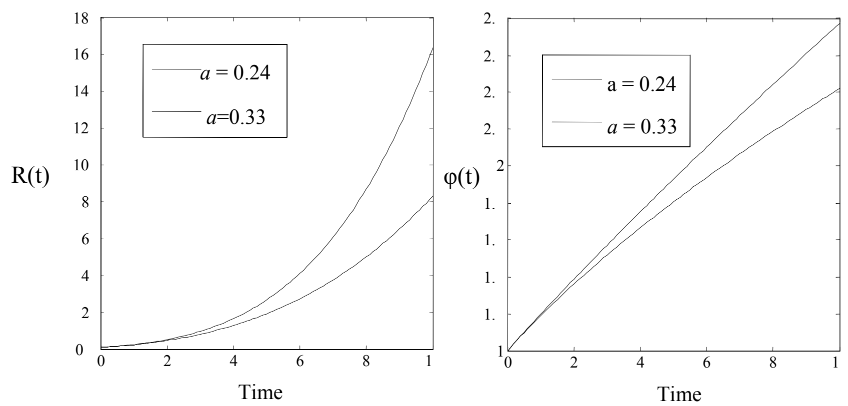
$$p = -\frac{\phi_0}{16\pi} \frac{6 + 4\alpha - 2\beta - 2a(2+\alpha) + \alpha^2(2+\omega)}{\left(at+b\right)^{2-\frac{\alpha}{a}}}. \tag{24}$$

In this case the geometrical quantities of observational interest take the form

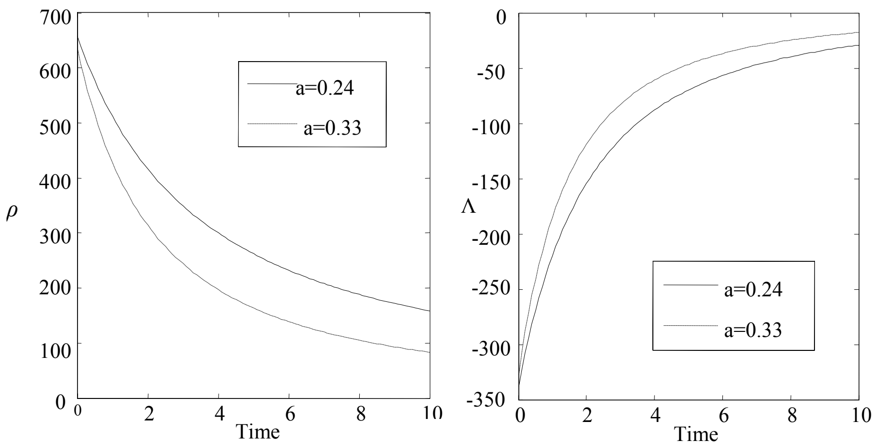
$$H = \frac{1}{at+b} , \tag{25}$$

$$q = -1 + a . \tag{26}$$

The *SNIa* observations [38] suggest the range for deceleration parameter as – 0.715 to ± 0.045 which yields $0.24 \le a \le 0.33$.



Figs. 1 and 2 show the variation of Scale factor and scalar field against cosmic time *t*.



Figs. 3 and 4 show the variation of energy density and cosmological constant against cosmic time t .

In this case $b = 1$, $\omega = 600$, $\alpha = 0.2$, $\frac{\phi_0}{8\pi} = 1$ $a = 0.24$ and 0.33 is considered. For these values of constants, β can be obtained from equation (18) as β= –337.92 and – 327.39 resp. This shows that for all values of *a* in the above range, β< 0 which clearly shows that $\Lambda < 0$.

It can be easily seen that energy density, pressure, expansion scalar are decreasing with evolution of the universe. The deceleration parameter suggests that for all $a < 1$ the model presents accelerating expansion of the universe.

3.1.2. Subcase II: model with exponential solution

Considering expression of scale factor as in (20), the scalar function assumes the form

$$\phi = c_1 e^{H_0 t} \tag{27}$$

In this case the energy density and pressure assumes uniform value.

3.2. CASE II: MODEL WITH $\Lambda \propto R^{-n}$

Several authors ([33] and references there in) considered a familier relation between the scale factor and cosmological constant as $\Lambda \propto R^{-n}$.

Therefore
$$\Lambda = \Lambda_0 R^{-n}. \tag{28}$$

By use of equation (28), one can write equation (15) as

$$2(3-\omega\alpha)\frac{\ddot{R}}{R} + \left(6 - 4\omega\alpha - \omega\alpha^2\right)\frac{\dot{R}^2}{R^2} = \frac{2\Lambda_0}{R^n}. \tag{29}$$

With the change of variable $u = \dot{R}^2$, equation (29) may be written as

$$\frac{du}{dR} + \frac{\left(6 - 4\omega\alpha - \omega\alpha^2\right)}{(3-\omega\alpha)}\frac{1}{R}u = \frac{2\Lambda_0}{(3-\omega\alpha)} R^{1-n} \tag{30}$$

On solving this Leibnitz linear differential equation one can have expression for scale factor as

$$R = R_0 t^{2/n} \tag{31}$$

where $R_0 = \left(\dfrac{2\Lambda_0}{12 - 6\omega\alpha - \omega\alpha^2 - 3n + n\omega\alpha}\right)^{1/n}$

Using this value of scale factor, the scalar field and cosmological “constant” assumes the form

$$\phi = \phi_0 R_0^{\alpha} t^{\frac{2\alpha}{n}} \tag{32}$$

$$\Lambda = \frac{\Lambda_0}{R_0^{\,n} t^2} \tag{33}$$

This value of Λ is similar to the result obtained by authors [39 and references there in]. In this case energy density and pressure are expressed as

$$\rho = \left[\frac{\phi_0 R_0^{\alpha}\left(12 + 12\alpha - 2\omega\alpha^2 - n^2\Lambda_0 R_0^{-n}\right)}{8\pi n^2}\right] t^{\frac{2\alpha}{n}-2} \tag{34}$$

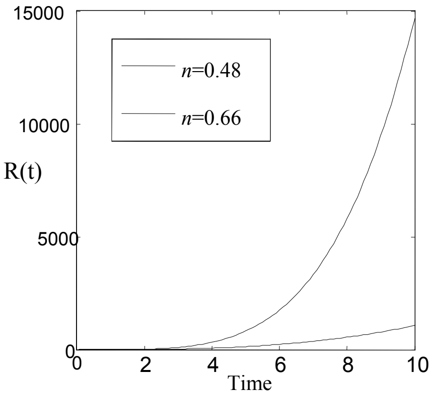
$$p=\left[\frac{-\phi_0 R_0{}^{\alpha}\left(12-4n+8\alpha-2\alpha n+4\alpha^2+2\omega\alpha^2-n^2\Lambda_0 R_0{}^{-n}\right)}{8\pi n^2}\right]t^{\frac{2\alpha}{n}-2} \tag{35}$$

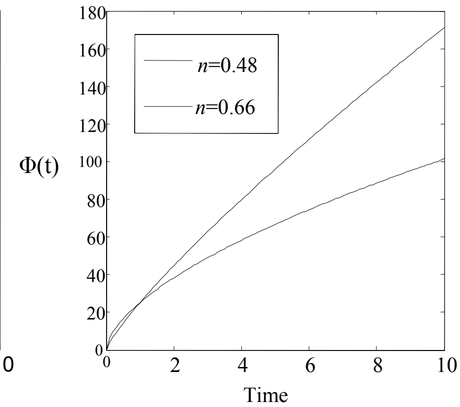
Here the geometrical quantities of observational interest are

$$H=\frac{2}{nt}, \tag{36}$$

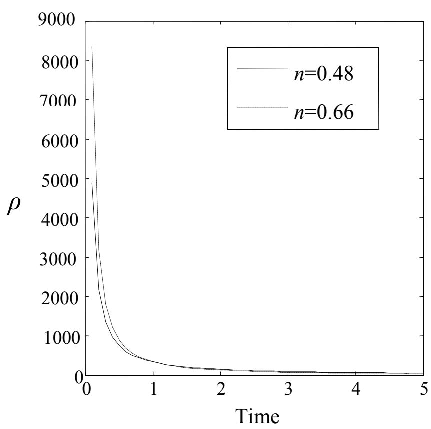
$$q=-1+\frac{n}{2}. \tag{37}$$

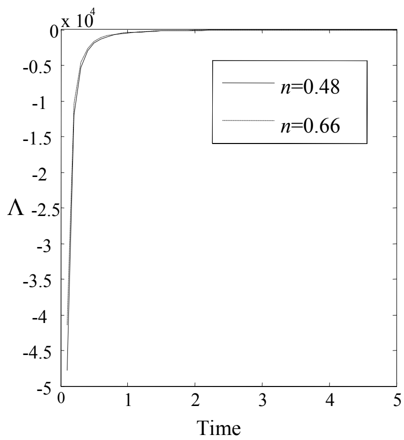
The range for deceleration parameter suggested by the *SNIa* observation [38] as –0.715 to ± 0.045 gives $0.48 \le n \le 0.66$.





Figs. 5 and 6 show plot of Scale factor and scalar field against cosmic time *t*.





Figs. 7 and 8 show plot of energy density and cosmological constant against cosmic time *t*.

In this case $R_0 = 1$, ω = 600, $\alpha = 0.2$, $\frac{\phi_0}{8\pi} = 1$ and $n = 0.48$ and 0.66 is considered.
These assumed values, gives $\Lambda_0 = -477.8189$ and -414.4102 which shows similar behaviour of Λ as in previous case.
It can be easily seen that energy density, pressure are decreasing with evolution of the universe. The deceleration parameter suggests that for all n < 2 the model presents accelerating expansion of the universe.

4. COSMOLOGICAL MODELS FOR NON-FLAT FRW SPACE-TIME

4.1. CASE I: MODEL WITH $\Lambda \propto H^2$

In this case a cosmological model is obtained by considering the expression for cosmological constant as in equation (16).
Equation (14) along with equation (16) gives

$$2(3-\omega\alpha)\frac{\ddot{R}}{R} + \left(6 - 4\omega\alpha - \omega\alpha^2 - 2\beta\right)\frac{\dot{R}^2}{R^2} - = \frac{-6k}{R^2} \tag{38}$$

With the change of variable $u = \dot{R}^2$, equation (38) takes the form

$$\frac{du}{dR} + \frac{\left(6 - 4\omega\alpha - \omega\alpha^2 - 2\beta\right)}{(3-\omega\alpha)}\frac{1}{R}u = \frac{-6k}{(3-\omega\alpha)}\frac{1}{R} \tag{39}$$

Further on integration equation (39), yield

$$R = R_1 t, \tag{40}$$

where $R_1 = \frac{6k}{\left(\omega\alpha^2 - 6 + 4\omega\alpha + 2\beta\right)}$.

This value of scale factor suggests $\phi = \phi_0 R_1{}^{\alpha} t^{\alpha}$, (41)

$$\Lambda = \frac{\beta}{t^2}, \tag{42}$$

Here the expressions for energy density and pressure are

$$\rho = \frac{\phi_0 R_1{}^{\alpha}}{8\pi}\left[\frac{R_1{}^{\alpha}(6 + 6\alpha - \omega\alpha^2) + 6k - 2R_1{}^2\beta)}{2R_1{}^2}\right]t^{\alpha-2} , \tag{43}$$

$$p = -\frac{\phi_0 R_1{}^{\alpha}}{8\pi}\left[\frac{R_1{}^{\alpha}(2 + 2\alpha^2 + 2\alpha + \omega\alpha^2) + 2k - 2R_1{}^2\beta)}{2R_1{}^2}\right]t^{\alpha-2}. \tag{44}$$

The geometrical quantities of observational interest are

$$H = \frac{1}{t}, \tag{45}$$

$$q = 0. \tag{46}$$

It can easily seen that energy density, pressure are decreasing with evolution of the universe. The deceleration parameter suggests that the model presents uniform expansion of the universe.

4.2. CASE II: MODEL WITH $\Lambda \propto R^{-n}$

Assuming expression for cosmological constant as in equation (28), equation (14) becomes

$$2(3-\omega\alpha)\frac{\ddot{R}}{R} + \left(6 - 4\omega\alpha - \omega\alpha^2\right)\frac{\dot{R}^2}{R^2} + \frac{6k}{R^2} = \frac{2\Lambda_0}{R^n}. \tag{47}$$

Using $u = \dot{R}^2$, equation (47) takes the form

$$\frac{du}{dR} + \frac{\left(6-4\omega\alpha-\omega\alpha^2\right)}{(3-\omega\alpha)}\frac{1}{R}u = \frac{-6k}{R(3-\omega\alpha)} + \frac{2\Lambda_0}{(3-\omega\alpha)}R^{1-n}. \tag{48}$$

This Liebnitz linear differential equation has solution

$$u = \frac{-6k}{(6-4\omega\alpha-\omega\alpha^2)} + \frac{2\Lambda_0}{6-4\omega\alpha-\omega\alpha^2-3n+6+\omega n\alpha-2\omega\alpha}R^{2-n}. \tag{49}$$

Here in order to obtained solutions two particular cases are considered for n that are $n = 1$ and $n = 2$.

Model For n = 1: In this case the expression for scale factor is obtained as

$$R = a_1 t^2 + b_1 t + c_1, \tag{50}$$

which gives the expression for the scalar function and cosmological constant as

$$\phi = \phi_0 (a_1 t^2 + b_1 t + c_1)^\alpha, \tag{51}$$

$$\Lambda = \frac{\Lambda_0}{(a_1 t^2 + b_1 t + c_1)}. \tag{52}$$

Here the energy density and pressure take the following form

$$\rho = \frac{\phi_0 (a_1 t^2 + b_1 t + c_1)^\alpha}{8\pi}\left[\frac{(6+6\alpha-\omega\alpha^2)(2a_1 t + b_1)^2 + 6k}{2(a_1 t^2 + b_1 t + c_1)^2} - \frac{\Lambda_0}{(a_1 t^2 + b_1 t + c_1)}\right], \tag{53}$$

$$p = \frac{-\phi_0 (a_1 t^2 + b_1 t + c_1)^{\alpha}}{8\pi}\left[\frac{(2+\alpha)2a_1 - \Lambda_0}{(a_1 t^2 + b_1 t + c_1)} + \frac{(2 + 2\alpha^2 + \omega\alpha^2 + 2\alpha)(2a_1 t + b_1)^2 + 2k}{2(a_1 t^2 + b_1 t + c_1)^2}\right]. \tag{54}$$

The geometrical quantities of observational interest are

$$H = \frac{2(a_1 t + b_1)}{a_1 t^2 + b_1 t + c_1}, \tag{55}$$

$$q = -\frac{2a_1 (a_1 t^2 + b_1 t + c_1)}{(2a_1 t + b_1)^2}. \tag{56}$$

Equation (56) suggests that deceleration parameter is dynamical. Variable deceleration parameter has already been presented by Singh and Kale [41]. Considering a variable deceleration parameter Singh *et al.* [42] and Pradhan *et al.* [43] have presented cosmological models in Lyra's manifold.

Model For n = 2: Here the scalar factor assumes the form

$$R = R_2 \mathrm{e}^{a_2 t}. \tag{57}$$

Again using this, one can easily obtained the expressions for the scale factor and cosmological constant as

$$\phi = \phi_0 R_2^{\ \alpha} \mathrm{e}^{a_2 \alpha t}, \tag{58}$$

$$\Lambda = \Lambda_0 R_2^{-2} \mathrm{e}^{-2a_2 t}. \tag{59}$$

In this case the energy density and pressure have the following form

$$\rho = \frac{\phi_0 R_2^{\ \alpha} e^{a_2 \alpha t}}{8\pi}\left[\frac{(6 + 6\alpha - \omega\alpha^2)a_2^{\ 2}}{2} + \frac{3k - \Lambda_0}{R_2^{\ 2} e^{2 a_2 t}}\right], \tag{60}$$

$$p = -\frac{\phi_0 R_2^{\ \alpha} e^{a_2 \alpha t}}{8\pi}\left[\frac{(2 + 2\alpha + 2\alpha^2 + \omega\alpha^2)a_2^{\ 2}}{2} + \frac{k - \Lambda_0}{R_2^{\ 2} e^{2 a_2 t}}\right]. \tag{61}$$

The Hubble parameter and deceleration parameter are obtained as

$$H = a_2, \tag{62}$$

$$q = -1. \tag{63}$$

5. DISCUSSION

In this paper cosmological models have been obtained in the context of Brans-Dicke theory by considering two expressions for cosmological "constant". The cosmological solutions are obtained for flat and non-flat FRW space time. In both cases, $k = 0$ and $k \neq 0$ energy density, pressure, cosmological “constant” are decreasing with evolution of the universe. These results are in fair agreement with the observations. In case of non-flat models a dynamic deceleration parameter has been obtained.

In section 3.1.1. subcase-I model with power law solution had been discussed. Considering observational value of deceleration parameter Fig.1 shows rapid growth in scale factor $R(t)$ with respect to cosmic time, while Fig. 2 indicates that scalar field $\varphi(t)$ is increasing with evolution of the universe. It can be easily seen from Fig. (3) and (4) respectively that energy density is decreasing and dynamic cosmological term is always negative for all values of “a” which satisfy observational limits of deceleration parameter.

Further, section 3.1.2. subcase-II is not interesting due to the fact that energy density ρ and pressure p takes uniform values for all time which are not consistent with observational results. Hence in this case exact solutions are not presented.

The case-II deals with cosmological models where $\Lambda \propto R^{-n}$. In this case behaviour of scale factor R(t), scalar field $\varphi(t)$, energy density ρ and cosmological term Λ behave similar to the previous case with different rate during evolution of the universe.

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