SHAPE TRANSITION AND COLLECTIVE EXCITATIONS IN NEUTRON–RICH $^{170-178}$Yb NUCLEI

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Received March 7, 2012

Energy levels, B(E2) values and potential energy surface for even-even $^{170-178}$Yb isotopes have been calculated using the IBM-1. The predicted energy levels, B(E2) values and intrinsic quadrupole moments Q_0 results were reasonably consistent with the experimental data. The contour plot of the potential energy surfaces shows all interest nuclei were deformed and have rotational characters.

Key words: IBM-1, neutron-rich, B(E2) values, energy levels, potential energy surface.

1. INTRODUCTION

The quadrupole collectivity in atomic nucleus exhibits distinct regularities, where the nuclear shape can be spherical, deformed and the situation in between. Like other models and theories [1, 2], the Interacting Boson Model [3] has been successful in reproducing the nuclear collective levels in terms of s and d bosons, which are essentially the collective s and d pairs of valence nucleons [4], respectively. The IBM Hamiltonian has the so-called dynamical symmetry, and the shape of quadrupole deformation can be classified as a spherical vibrator (U(5)), axially symmetric deformation (SU(3)), and γ–unstable deformation (O(6)), if the interaction strengths of the IBM Hamiltonian taken specific values. The medium-to heavy-mass Ytterbium (Yb) isotopes are located in the rear-earth mass region, most of these nuclei were well-deformed and it can be populated to very high spin. Much experimental information on even- odd- mass Yb isotopes has become more abundant [5-10]. For the heavier A=174 to 178 nuclei [11], previous work using deep inelastic reactions and Gammasphere have begun to reveal much information about the high-spin behavior of these neutron-rich Yb isotopes. The yrast states in

the well deformed rare-earth region have been described using the projected shell model [12-18]. In this study, the calculations of energy levels of even-even $^{170-178}$Yb isotopes have been done using the interacting boson model. Positive parity state energies, reduce probabilities of E2 transitions, B(E2) values, intrinsic quadrupole moments Q_0 and potential energy surface were calculated and compared with the experimental data.

2. INTERACTING BOSON MODEL (IBM)

The IBM has become one of the most intensively used nuclear models, due to its ability to describe the changing low-lying collective properties of nuclei across an entire major shell with a simple Hamiltonian. In the IBM the spectroscopies of low-lying collective properties of even-even nuclei were described in terms of a system of interacting s bosons (L=0) and d bosons (L=2) [19]. In addition, the structure of low-lying levels is dominated by excitations among the valence partials outside the major closed shells in this model. The number of proton bosons N_π and neutron bosons N_υ were counted from the nearest closed shell, and the total boson number N= N_π+ N_υ. The underlying structure of the six-dimensional unitary group SU(6) of the model leads to a simple Hamiltonian, capable of describing the three dynamical symmetries. These symmetries are called SU(5) vibrational [20], SU(3) rotational [21] and O(6) γ–unstable [22] and also the transitional nuclei [23] whose structure are intermediate. The IBM-1 Hamiltonian can be expressed as [22]

$$\begin{aligned} H=&\varepsilon_s s^+ s+\varepsilon_d (d^+ d)+\sum_{L=0,2,4} C_L\left[(d^+ d^+)^{(L)}.(dd)^{(L)}\right]+ \\ &+\frac{1}{2}\upsilon_0\left[(d^+ d^+)^{(0)}_0 s^2+(s^+)^2\right. \\ &\left.(dd)^{(0)}_0\right]+\sqrt{\frac{1}{2}}\upsilon_2\left[\left[(d^+ d^+)^{(2)} ds\right]^{(0)}_0\left[s^+ d^+ (dd)^{(2)}\right]^{(0)}_0\right]+ \\ &+\frac{1}{2}u_0 (s^+)^{(2)} s^+ +\frac{1}{\sqrt{5}}u_2 s^+ s\,(d^+ d) \end{aligned} \tag{1}$$

where it can be written in general form as [24]

$$H = \varepsilon n_d + a_0 P^\dagger.P + a_1 L.L + a_2 Q.Q + a_3 T3.T3 + a_4 T4.T4 \tag{2}$$

where $\varepsilon = \varepsilon_d - \varepsilon_s$ is the boson energy. The parameters a_0, a_1, a_2, a_3 and a_4 designated the strength of the pairing, angular momentum, quadrupole, octupole and hexdecupole interaction between the bosons.

3. CALCULATED RESULTS

3.1. ENERGY LEVELS

The rotational limit of the IBM-1 has been applied for the even-even $^{170\text{-}178}$Yb isotopes due to the values of $E4_1^+/E2_1^+$ ratio, therefore, these isotopes have a dynamical symmetry SU(3) respecting to IBM-1. The calculations have been performed with no distinction made between neutron and proton bosons. For the analysis of excitation energies in Yb isotopes it was tried to keep to minimum the number of free parameters in Hamiltonian. The explicit expression of Hamiltonian adopted in calculations is [24].

$$H = a_1\, L.L + a_2\, Q.Q \tag{3}$$

In the framework of the IBM-1, the isotopic chains of Yb with Z=70 nuclei, having a number of proton bosons holes 6, a number of neutron bosons particles varies from 9 to 11 for $^{170\text{-}174,178}$Yb, and number of neutron boson hole for ^{176}Yb is 10. The coefficient values which have a good agreement with the experimental results are shown in Table 1.

The calculated ground band and experimental data of low lying states were plotted in Figure 1 for the even-even $^{170\text{-}178}$Yb isotopes. Good agreements from the comparison of the IBM-1 calculations (energies, spin and parity) with the experimental data. But it is deviated in the high spin (energies) of the experimental data. The IBM-1 model is successful to predict the β– and γ– bands for all nuclei interest as shown in Tables 2 and 3, respectively. Levels with '*' correspond to cases for which the spin and/or parity of the corresponding states are not well established experimentally. The IBM-1 calculations were in good agreements with the experimental results in these tables. The root mean square deviation (RMSD) [25] was used to calculate deviation between IBM energy levels and the experimental results;

$$\mathrm{RMSD} = \left[\frac{1}{m} \left(\sum E_{\mathrm{Cal.}} - E_{\mathrm{Exp.}} \right)^2 \right]^{1/2} \tag{4}$$

where m is the number of levels. Table 4 shows the best agreement of the ground state and γ– bands was found in the ^{178}Yb isotope, while the smallest value of RMSD, for β– bands in the ^{174}Yb.

3.2. THE REDUCED PROBABILITY TRANSITIONS (B(E2) VALUE)

The reduced matrix elements of the E2 operator (T^{E2}) has the form [20]

$$T^{E2} = \alpha 2\, [d^\dagger s + s^\dagger d]^{(2)} + \beta 2 [\, d^\dagger d]^{(2)} \tag{5}$$

where ($s^\dagger$, $d^\dagger$) and (s, d) are creation and annihilation operators for s and d bosons, respectively, while $\alpha 2$ and $\beta 2$ are two parameters. The B(E2) values are then given by

$$B(E2, J_i \rightarrow J_f) = \frac{1}{2J_i+1} \left| \langle J_f \| T^{E2} \| J_i \rangle \right|^2 \quad (6)$$

For the calculations of the absolute B(E2) values two parameters $\alpha 2$ and $\beta 2$ of equation (5) were adjusted according to the experimental B(E2; $2_1^+ \rightarrow 0_1^+$). Table 5 shows the values of the parameters $\alpha 2$ and $\beta 2$, which were obtained in the present calculations. The calculated results of the reduced probability transitions, B(E2) values, and the experimental data [31] are given in Table 6 for all nuclei of interest, with the exception the ^{178}Yb isotope because it has no experimental B(E2) value.

3.3. QUADRUPOLE MOMENTS

The value of B(E2) in units of e^2b^2, is related to B(E2) in units of Weisskopf single particle transition (W.u) [32].

$$B(E2)W.u = 5.94 \text{ X } 10^{-6} \text{ x } A^{4/3} \text{ x } B(E2)\ e^2b^2 \quad (7)$$

The intrinsic quadrupole moments of nuclei can be derived from the transition rate B(E2, J→J-2) values according to Eq. (8) [33].

$$B(E2) = \frac{15}{32\pi} \frac{(J-1)}{(2J-1)} \frac{J}{(2J+1)} e^2 Q_0^2 (J \rightarrow J-2) \quad (8)$$

Table 7 presents the calculation of the intrinsic quadrupole moments Q_0, within frame work of IBM-1 for the even-even $^{170\text{-}178}$Yb isotopes. The presented results for Q_0 is consistent with expectations and from phenomenological systematic and are compared with previous experimental results [34].

Table 1

Adopted values for the parameters used for IBM-1 calculations. All parameters are given in MeV

A	ε	a_0	a_1	a_2	a_3	a_4	CHI
170	0.0	0.0	0.0094	–0.0120	0.0	0.0	–1.30
172	0.0	0.0	0.0089	–0.0112	0.0	0.0	–1.30
174	0.0	0.0	0.0071	–0.0150	0.0	0.0	–1.30
176	0.0	0.0	0.0089	–0.0126	0.0	0.0	–1.30
178	0.0	0.0	0.0090	–0.0130	0.0	0.0	–1.30

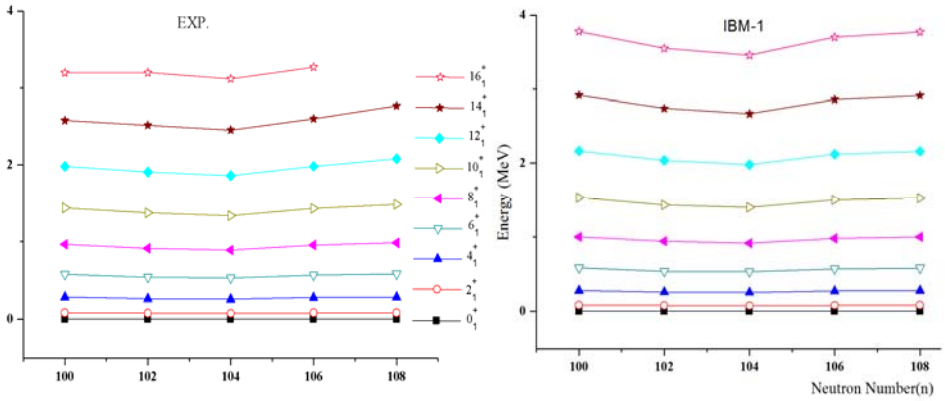

Fig. 1 – (Color online) Comparison the IBM-1 calculations with the available experimental data of the ground band for $^{170\text{-}178}$Yb nuclei. The experimental data are taken from [26-30].

Table 2

β– bands for Yb isotopes (in MeV). The experimental data are taken from [26-30]

	IBM-1	EXP.	IBM-1	EXP.	IBM-1	EXP.
	^{170}Yb		**^{172}Yb**		**^{174}Yb**	
0^+	1.036	1.003	1.033	1.042	1.473	1.467
2^+	1.111	1.138	1.104	1.117	1.538	1.561*
4^+	1.305	1.292*	1.287	1.286	1.716	1.715
6^+	1.611	1.521	1.575	1.537	1.996	
8^+	2.028	1.803*	1.968	1.853	2.377	
10^+	2.556	2.132*	2.465	2.212*	2.860	
12^+	3.195	2.524*	3.068	2.607*	3.445	
14^+	3.945	2.987*	3.77	3.043*	4.132	
16^+	4.806	3.547*	4.586		4.920	
18^+	5.778	4.207*	5.502		5.810	
	^{176}Yb		**^{178}Yb**			
0^+	1.162	1.138*	1.122	1.315		
2^+	1.235	1.199*	1.196	1.404		
4^+	1.426		1.391	1.559*		
6^+	1.725		1.696			
8^+	2.134		2.112			
10^+	2.651		2.639			
12^+	3.278		3.276			
14^+	4.013		4.025			
16^+	4.857		4.885			
18^+	5.810		5.855			

Table 3

γ–bands for Ytterbium nuclei (in MeV). The experimental data are taken from [26-30]

	IBM-1	EXP.	IBM-1	EXP.	IBM-1	EXP.
	^{170}Yb		^{172}Yb		^{174}Yb	
2^+	1.119	1.145	1.112	-----	1.550	-----
3^+	1.194	1.225	1.182	1.172	1.614	1.606*
4^+	1.313	1.329	1.295	1.263	1.728	1.701
5^+	1.444	1.460	1.418	1.375	1.843	1.819*
6^+	1.619	1.601	1.583	1.510	2.007	1.959*
7^+	1.806	1.780	1.758	1.666*	2.174	
8^+	2.036	1.954	1.976	1.841*	2.389	
9^+	2.278	2.215	2.204	2.039*	2.606	
10^+	2.564	2.372	2.473	2.256*	2.872	
11^+	2.862	2.603	2.754	2.492*	3.140	
12^+	3.203	2.826	3.075	2.746*	3.456	
13^+	3.556	3.067	3.408	3.020*	3.776	
14^+	3.953	3.307	3.782	3.309*	4.143	
	^{176}Yb		^{178}Yb			
2^+	1.244	1.260	1.205	1.221*		
3^+	1.317	1.336*	1.280			
4^+	1.435	1.435*	1.399			
5^+	1.562	1.558*	1.529			
6^+	1.734		1.704			
7^+	1.916		1.890			
8^+	2.143		2.120			
9^+	2.379		2.361			
10^+	2.660		2.647			
11^+	2.951		2.944			
12^+	3.286		3.285			
13^+	3.632		3.637			
14^+	4.022		4.034			

Table 4

The root mean square deviation RMSD between experimental and calculated energy levels

A	RMSD (MeV)		
	g-band	β-band	γ-band
170	0.234	0.050	0.265
172	0.147	0.020	0.234
174	0.141	0.013	0.030
176	0.175	0.030	0.012
178	0.059	0.190	0.016

Table 5

Parameters (in eb) obtained from T^{E2} operator to calculate the absolute $B(E2)$ values

A	$\alpha 2$	$\beta 2$
170	0.1060	-0.140
172	0.1037	-0.137
174	0.0960	-0.126
176	0.0980	-0.129

Table 6

$B(E2)$ values for Yb isotopes (in W.u.)

	IBM-1	EXP.	IBM-1	EXP.
	^{170}Yb		**^{172}Yb**	
$2_1^+ \to 0_1^+$	198.543	201(6)	211.689	212 (2)
$4_1^+ \to 2_1^+$	280.768		299.697	301 (20)
$6_1^+ \to 4_1^+$	303.549		324.746	320 (30)
$8_1^+ \to 6_1^+$	309.178	360 (30)	331.835	400 (40)
$10_1^+ \to 8_1^+$	306.15	356 (25)	329.971	375 (23)
$12_1^+ \to 10_1^+$	296.956	268 (21)	322.160	430 (60)
$14_1^+ \to 12_1^+$	283.181		309.724	394 (+60-45)
$16_1^+ \to 14_1^+$	265.349		293.311	
$18_1^+ \to 16_1^+$	243.819		273.310	
$20_1^+ \to 18_1^+$	218.751		249.967	
	^{174}Yb		**^{176}Yb**	
$2_1^+ \to 0_1^+$	199.908	201 (7)	182.916	183 (7)
$4_1^+ \to 2_1^+$	283.321	280 (9)	258.969	270 (25)
$6_1^+ \to 4_1^+$	307.532	370 (50)	280.618	298 (22)
$8_1^+ \to 6_1^+$	315.122	388(21)	286.743	300 (5)
$10_1^+ \to 8_1^+$	314.533	335 (22)	285.139	
$12_1^+ \to 10_1^+$	308.624	369 (23)	278.384	
$14_1^+ \to 12_1^+$	298.641	320 (8)	267.636	
$16_1^+ \to 14_1^+$	285.192		253.459	
$18_1^+ \to 16_1^+$	268.659		236.177	
$20_1^+ \to 18_1^+$	249.249		215.995	

Table 7

Intrinsic quadrupole moments Q_0 (in b) for ground state band

A	Q_0 (b) IBM	Q_0 (b) EXP.[34]
170	7.475	7.630 *9*
172	7.779	7.792 *45*
174	7.616	7.727 *39*
176	7.342	7.300 *13*

3.4. POTENTIAL ENERGY SURFACE (PES)

In recent years, the potential energy surface by Skyrme mean field method was mapped onto the PES of the IBM Hamiltonian [35–38]. The expectation value of the IBM-1 Hamiltonian with the coherent state ($|N,\beta,\gamma\rangle$) is used to create the IBM energy surface [24]. The state is a product of boson creation operators ($b_c^\dagger$), with

$$|N,\beta,\gamma\rangle=\frac{1}{\sqrt{N!}}(b_c^\dagger)^N|0\rangle \tag{9}$$

$$b_c^\dagger=(1+\beta^2)^{-1/2}\{s^\dagger+\beta[\cos\gamma(d_0^\dagger)+\sqrt{1/2}\sin\gamma(d_2^\dagger+d_{-2}^\dagger)]\} \tag{10}$$

The energy surface, as a function of β and γ, has been given by [3]

$$E(N,\beta,\gamma)=N\varepsilon_d\beta^2/(1+\beta^2)+N(N-1)/(1+\beta^2)^2(\alpha_1\beta^4+\alpha_2\beta^3\cos3\gamma+\alpha_3\beta^2+\alpha_4) \tag{11}$$

Where the α_i 's are related to the coefficients C_L, ν_2, ν_0, u_2 and u_0 of equation (1).
The calculated potential energy surfaces for Yb isotopes were presented in Figure 2. It shows that all nuclei are deformed and have rotational-like characters. The prolate deformation is deeper than oblate in all nuclei.

4. SUMMARY

Interacting boson model (IBM-1) was used to calculate the energy levels (positive parity), the reduced probability of E2 transitions, intrinsic quadrupole moments Q_0 and potential energy surface for $^{170\text{-}178}$Yb isotopes. The predicted low-lying levels (energies, spins and parities) and the reduced probability of *E*2 transitions results were reasonably consistent with the experimental results. The potential energy surfaces for Yb isotopes shows that all nuclei are deformed and have rotational-like characters.

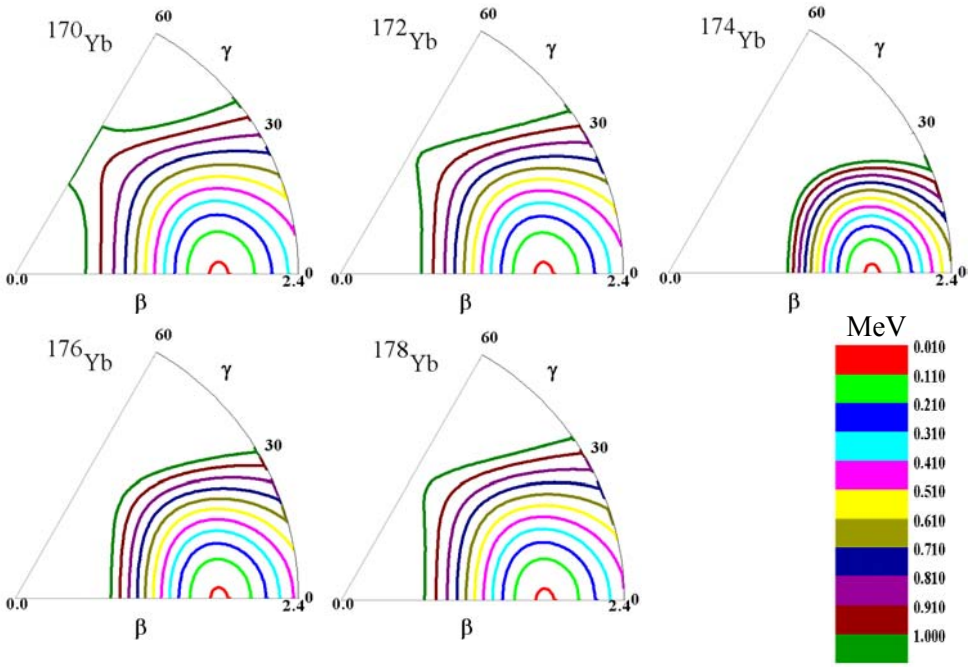

Fig. 2 – (Color online) The potential energy surfaces for even-even $^{170\text{-}178}$Yb isotopes.

Acknowledgments. We thank the Islamic Development Bank (IDB) for supporting this work under grant no. 36/11201905/35/IRQ/D31. Furthermore, we thank University of Malaya - Faculty of Science as well as Department of Physics and University of Kerbala - College of Science - Department of Physics for supporting this work.

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