

*Dedicated to Academician Aureliu Sandulescu's 80<sup>th</sup> Anniversary*

## NEW FORMULA OF HALF-LIVES FOR PROTON EMISSION FROM SPHERICAL AND DEFORMED NUCLEI

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A unified formula of half-lives for  $\alpha$  decay and cluster radioactivity has been proposed [Ni, Ren *et al.*, Phys. Rev. C 78, 044310 (2008)]. In this work, a similar formula is proposed for determining half-lives of proton emission. This new formula contains the dependence on the centrifugal barrier and the structure of the daughter nucleus. It not only well reproduces the experimental half-lives of spherical emitters but also shows excellent agreement with the experimental data of deformed emitters. Moreover, we use it to simultaneously describe the data of spherical and deformed emitters. The agreement between the theoretical values and the experimental data is also reasonably good. The aim of this work is not only to reproduce the experimental data better, but also to achieve a unified description of proton emission,  $\alpha$  decay, and cluster radioactivity. The new formula has the form very similar to that for  $\alpha$  decay and cluster radioactivity, and the values of three sets of parameters respectively describing proton emission,  $\alpha$  decay, and cluster radioactivity are close to each other. This indicates that we have reached a unified law for proton emission,  $\alpha$  decay, and cluster radioactivity.

*Key words:* Proton emission, half-lives, unified description.

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### 1. INTRODUCTION

The proton drip line defines one of the fundamental limits of nuclear stability. Nuclei lying beyond this line have the negative proton separation energies with a natural tendency to emit a proton from their ground states [1, 2]. This phenomenon is known as proton emission. The study of this process is very important, since it not only provides nuclear structure information on nuclei very far from the  $\beta$ -stability line, but also offers a unique tool to obtain spectroscopic information due to the unpaired proton. Besides, there is another proton decay mode called two-proton decay, which proceeds when one-proton emission is prohibited either energetically or dynamically and only the simultaneous emission of two protons is possible. Compared with proton emission, it is more complicated as a three-body Coulomb problem in the continuum [3, 4].

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Proton emission,  $\alpha$  decay, and cluster radioactivity, as three important decay modes for unstable nuclei, are physically analogical processes [5].  $\alpha$  decay mostly proceeds in the region of heavy and super-heavy nuclei while proton emission and cluster radioactivity only occur for a small number of nuclei in a certain mass region [6–10]. Proton emission has been studied from odd- $Z$  nuclei in the two regions:  $51 \leq Z \leq 67$  and  $69 \leq Z \leq 83$  [11–16]. In most cases, proton emission is understood in terms of the simple quantum tunneling through a one-dimensional barrier. And the structure property can be evaluated by taking into account the spectroscopic factor, which is similar to the cluster preformation factor in the description of  $\alpha$  decay and cluster radioactivity. In our previous work [17], we started from the quantum tunneling effect and unified the phenomenological laws of  $\alpha$  decay and cluster radioactivity. The unified formula of half-lives for  $\alpha$  decay and cluster radioactivity is written as

$$\log_{10} T_{1/2} = a\sqrt{\mu} Z_c Z_d Q^{-1/2} + b\sqrt{\mu} (Z_c Z_d)^{1/2} + c, \quad (1)$$

where  $a$ ,  $b$ , and  $c$  are the constants to be determined. In this article, considering many resemblances among proton emission,  $\alpha$  decay, and cluster radioactivity, we hope to give a similar formula to describe proton emission. This would lead to a unified law for these three decay modes.

This article is organized in the following way. In Sec. 2, we propose a new formula of half-lives for proton emission, which is very similar to the unified formula for  $\alpha$  decay and cluster radioactivity. The centrifugal barrier and the deformation of daughter nuclei are taken into account in the new formula. In Sec. 3, we use the new formula to perform a systematic study of the available experimental data for spherical and deformed emitters. The experimental data are well reproduced by the formula. Importantly, we bring systematics of proton emission,  $\alpha$  decay, and cluster radioactivity into comparison, and it is concluded that a unified description of them is achieved. A summary is given in Sec. 4.

## 2. NEW FORMULA FOR PROTON-EMISSION HALF-LIVES

Proton emission can be described as a barrier penetration phenomenon. The decay width(or decay constant) can be expressed as a product of three quantities [18–20]

$$\Gamma \equiv \hbar \ln 2 / T_{1/2} = SFP, \quad (2)$$

where  $S$  is the spectroscopic factor, which does not change very much for spherical emitters but becomes complex with a clear dependence upon the deformation parameter for deformed emitters [30].  $F$  is the frequency of the proton inside the barrier, which remains practically constant, and  $P$  is the probability of transmission through the barrier, which is given in the spherical Wentzel-Kramers-Brillouin (WKB) ap-

proximation by

$$P = \exp(-G) = \exp\left(-\frac{2}{\hbar} \int_{R_t}^{R_C} \sqrt{2\mu[V(r) - Q]} dr\right). \quad (3)$$

Here  $R_t$  is the touching radius,  $R_t = 1.2249(A_1^{1/3} + A_2^{1/3})$  fm, where  $A_1$  and  $A_2$  are the mass numbers of the proton and the daughter nuclei, respectively. The classical turning point  $R_C$  is given by  $R_C \approx Z_1 Z_2 e^2 / Q$ , considering that the long-range Coulomb potential ( $\sim Z_1 Z_2 e^2 / r$ ) dominates in the range  $r \geq R_t$ .  $\mu$  is the reduced mass of the proton-daughter system measured in unit of the nucleon mass,  $\mu = A_1 A_2 / (A_1 + A_2)$ .

Proton emission is characterized by the lowest Coulomb potential and the smallest reduced mass among all charged particle emissions. And in most cases the emitted proton carries a nonvanishing angular momentum. Based on these facts, the effect of the centrifugal barrier in proton emission is much more important than in  $\alpha$  decay and cluster radioactivity. Therefore, it is necessary to take into account the effect of the centrifugal barrier in the systematics of proton emission. The potential is hence given by  $V(r) = Z_1 Z_2 e^2 / r + \ell(\ell + 1)\hbar^2 / 2\mu r^2$ , where  $\ell$  is the orbital angular momentum of the proton outside the nucleus. The Gamow factor is approximately

$$G \simeq \frac{2R_C \sqrt{2\mu Q}}{\hbar} \left[ \arccos x - x\sqrt{1-x^2} \right] + c(\ell) \sqrt{1-x^2}/x, \quad (4)$$

where  $x = \sqrt{R_t/R_C}$ ,  $c(\ell) = 2\hbar\ell(\ell + 1) / \sqrt{2\mu QR_C^2}$ . Defining a reduced half-life as

$$T_{red} \equiv T_{1/2} / \exp\left(c(\ell) \sqrt{1-x^2}/x\right), \quad (5)$$

one obtains

$$\log_{10} T_{red} = \log_{10}(\hbar \ln 2 / FS) + \frac{2}{\ln 10} \frac{\sqrt{2\mu} e^2}{\hbar} Z_1 Z_2 Q^{-1/2} \times \left[ \arccos(x) - x\sqrt{1-x^2} \right]. \quad (6)$$

It is known that  $T_{1/2}$  is strongly  $\ell$ -dependent, but  $T_{red}$  shows no dependence upon the angular momentum  $\ell$ , as shown in equation (6). This is because the influence of the centrifugal barrier is completely contained in the definition of  $T_{red}$ . Moreover, looking at the right part of equation (6), one can notice that the property of  $\log_{10} T_{red}$  is the approximately same as that of  $\log_{10} T_{1/2}$  for  $\alpha$  decay and cluster radioactivity. In a similar manner, we derive a new formula for proton emission of spherical emitters as follows

$$\log_{10} T_{red} = a\sqrt{\mu} Z_1 Z_2 Q^{-1/2} + b\sqrt{\mu} (Z_1 Z_2)^{1/2} + c, \quad (7)$$

where  $a$ ,  $b$ , and  $c$  are the constants to be determined. It should be particularly noted

that this new formula (7) has the same form as the unified formula (1) for  $\alpha$  decay and cluster radioactivity.

The theoretical calculations pointed out that the proton emitters in the region  $68 < Z < 84$  are considered to be spherical [21, 22] while the proton emitters in the region  $50 < Z < 68$  are expected to be quite deformed in their ground states [23–26]. For near-spherical emitters, proton decay rates have been well reproduced by simple WKB calculations. However, for deformed emitters, the same interpretation as spherical emitters is not good enough. Indeed, a good description of these nuclei requires the presence of the deformation parameter  $\beta_2$  [27, 28]. This suggests that a complete explanation of the proton emission process should have the ability to describe the dependence of the half-life upon the internal structure of the decaying nucleus. That is, the formula of half-lives for proton emission should contain the term with a direct dependence upon the deformation parameter. With this in mind, we start from the spherical decay half-life calculations and try to find the term associated with the deformation parameter.

First, the  $Q$  value is a much more crucial quantity compared with the deformation parameter. The presence of the deformation parameter is supposed to make no difference to the  $Q$  dependence of the half-life. Therefore, when we introduce the deformation parameter into the formula, the first term in the right part of equation (7), describing the  $Q$  dependence, remains the same. Second, for deformed systems the spectroscopic factor can be calculated in the framework of the Nilsson model within the adiabatic approach [28, 29], and the results show a clear dependence of the spectroscopic factor upon the deformation parameter  $\beta_2$ . For spherical emitters the spectroscopic factor has been investigated both experimentally and theoretically by Åberg *et al.* [30], where both the experimental  $S^{exp}$  values and the  $S^{th}$  values obtained in the Bardeen-Cooper-Schrieffer (BCS) theory generally vary in the narrow range of 0.25–0.75. Stimulated by these facts, we assume that the last term in the right part of equation (7) should contain dependence on the deformation parameter. Here, the dependence is approximated as a sum of the  $|\beta_2|^3$  term and a constant. In this way, we extend the formula (7) to a deformed case and get a new formula for proton emission, which is written as

$$\log_{10} T_{red} = a\sqrt{\mu} Z_1 Z_2 Q^{-1/2} + b\sqrt{\mu} (Z_1 Z_2)^{1/2} + c + d |\beta_2^3|. \quad (8)$$

In this equation, considering that the last term plays a tiny role in the spherical case, it does not matter that one fixes  $d = 0$  for spherical emitters, getting back to the preceding spherical WKB calculations. The new formula (8) with four parameters relates the logarithm of the half-life corrected by the centrifugal barrier with the  $Q$  value and the deformation parameter. It can not only reproduce the experimental data of spherical emitters but also apply to deformed emitters. The meaning of each term in the formula is also very clear. Especially it is very similar to that for  $\alpha$  decay and

cluster radioactivity.

We also notice that a simple formula for proton emission has been proposed by Delion *et al.* [31]:

$$\log_{10} T_{red}^{(k)} = a_k(\chi - 20) + b_k, k = 1 \text{ for } Z < 68 \text{ and } k = 2 \text{ for } Z > 68, \quad (9)$$

where  $\chi = \sqrt{2}e^2(Z-1)\sqrt{\mu}Q^{-1/2}/\hbar$  is the Coulomb parameter,  $Z$  is the charge number of the mother nucleus, and  $\mu$  is the reduced mass of the proton-daughter system. It is worthwhile to point out that the parameters  $a_k$  and  $b_k$  have different values for nuclei with  $Z < 68$  and  $Z > 68$ . With this formula, the experimental data approximately lie on two straight lines. However, if using one single straight line to fit all the available data, the results are not satisfactory. Furthermore, it is not clear whether the formula (9) is valid for  $\alpha$  decay and cluster radioactivity. In this work, we deduce the formula from the WKB barrier penetration probability in the same way as the description of  $\alpha$  decay and cluster radioactivity. Considering the particularity of proton emission, we introduce the centrifugal barrier and the deformation of the daughter nucleus into the formula. The new formula (8) can be considered as a natural generalization of the unified formula (1) from  $\alpha$  decay and cluster radioactivity to proton emission in terms of the similarity among them. On the other hand, the new formula (8) provides a unified description of deformed and spherical emitters, and the agreement between experiment and theory is also reasonably good. This is the development of the formula (9). The aim of this work is not only to reproduce the experimental data better, but also to separate the same ones from the different ones among proton emission,  $\alpha$  decay, and cluster radioactivity in order to achieve a unified description of them. The detailed discussion about the systematics of proton emission,  $\alpha$  decay, and cluster radioactivity is presented at the end of the next section. It is found that not only they have the similar formulas of half-lives but also the parameter values for their systematics are close to each other.

### 3. NUMERICAL RESULTS AND DISCUSSIONS

Before presenting numerical results, let us compare the formula for proton emission with the famous Viola-Seaborg formula for  $\alpha$  decay [32]. The Viola-Seaborg formula is

$$\log_{10} T_{1/2} = (aZ + b)Q^{-1/2} + cZ + d + h_i, \quad (10)$$

where  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $h_i$  are adjustable parameters.  $h_i$  are the average hindrance factors for even-odd, odd-even, and odd-odd nuclei, and for even-even nuclei  $h_i = 0$ . In our formula the last term  $d|\beta_2|^3$  describes the influence of nuclear deformation and for spherical emitters  $d = 0$ . This is similar to the hindrance factor  $h_i$  for  $\alpha$  decay. The evident difference between these two formulas is that an additional parameter

$b$  is introduced in the Viola-Seaborg formula. The reason for this is that  $\alpha$ -decay half-lives span over many orders of magnitude, from  $10^{-8}$  to  $10^{24}$  s. By contrast, half-lives of proton emission vary in the narrow range of  $10^{-6} \sim 10$  s. Hence, the parameter  $b$  is not necessary for proton emission at present. Although the additional parameter  $b$  is used for the description of  $\alpha$  decay, it is worthwhile to point out that, its value is quite small with respect to the values of  $aZ$  in the Viola-Seaborg formula.

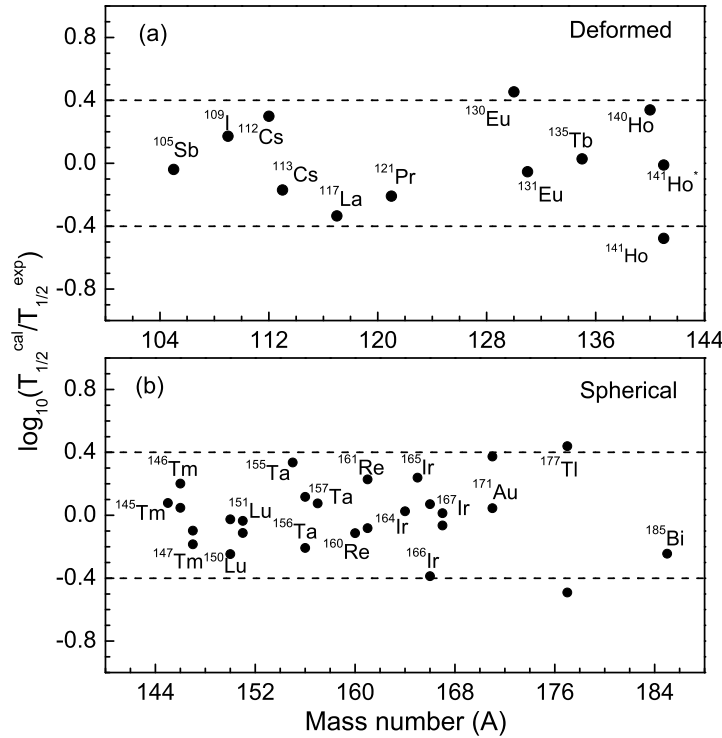


Fig. 1 – The deviations of proton-emission half-lives calculated with the new formula (8) from the experimental data for deformed and spherical emitters, respectively. The upper part (a) shows the results for the deformed case, and the lower part (b) shows the ones for the spherical case.

Next, we determine the three parameters  $a$ ,  $b$ , and  $c$  for the spherical case ( $d = 0$ ) and the four parameters  $a$ ,  $b$ ,  $c$ , and  $d$  for the deformed case, respectively. Through a least-square fit to the experimental data of 27 spherical emitters and 12 deformed ones separately, we obtain two sets of parameters:  $a = 0.39930$ ,  $b = -0.35097$ ,  $c = -26.92758$  for spherical emitters and  $a = 0.39745$ ,  $b = -1.81241$ ,  $c = -14.51383$ ,  $d = 28.72385$  for deformed ones. To evaluate the proposed relation, the calculated half-lives are compared with the experimental data as shown in fig. 1. It can be seen that the values of  $\log_{10}(T_{1/2}^{exp}/T_{1/2}^{cal})$  are generally within the range of about

$\pm 0.4$ , corresponding to the values of the ratio  $T_{1/2}^{exp}/T_{1/2}^{cal}$  within the range of about 0.4–2.5. This means that the calculated half-lives are in good agreement with the experimental data for both spherical and deformed emitters. Table 1 displays the values of  $\log_{10} T_{1/2}$  calculated with the new formula and these values are compared with the experimental data. In table 1 the logarithms of the experimental half-lives and of the calculated ones are listed in columns 6 and 7, respectively. One can see that the calculated half-lives agree well with the experimental ones.

*Table 1.: The comparison of the logarithm of the calculated half-lives with the logarithm of the experimental data for proton emission. The angular momentum and parity of the emitted proton ( $j^\pi$ ), the  $Q$  value ( $Q_p$ ), and the experimental half-life ( $T_{1/2}^{exp}$ ) are taken from [33–35]. The quadrupole deformation  $\beta_2$  parameters are taken from the theoretical values of Möller [36]. The calculated results ( $\log_{10} T_{1/2}^{cal}$ ) are obtained with the formula (8). All half-lives are in seconds. The stars in the emitters represent the excited states.*

Emitter	$\ell$	$j^\pi$	$Q_p$ (keV)	$\beta_2$	$\log_{10} T_{1/2}^{exp}$	$\log_{10} T_{1/2}^{cal}$
$^{105}_{51}\text{Sb}$	2	$5/2^+$	491	0.081	2.049	2.010
$^{109}_{53}\text{I}$	2	$3/2^+$	829	0.160	-3.987	-3.815
$^{112}_{55}\text{Cs}$	2	$3/2^+$	824	0.208	-3.301	-3.001
$^{113}_{55}\text{Cs}$	2	$3/2^+$	978	0.207	-4.777	-4.946
$^{117}_{57}\text{La}$	2	$3/2^+$	823	0.290	-1.602	-1.937
$^{121}_{59}\text{Pr}$	2	$3/2^+$	900	0.318	-2.000	-2.209
$^{130}_{63}\text{Eu}$	2	$3/2^+$	1028	0.331	-3.046	-2.592
$^{131}_{63}\text{Eu}$	2	$3/2^+$	951	0.331	-1.575	-1.629
$^{135}_{65}\text{Tb}$	3	$7/2^-$	1188	0.325	-3.027	-2.999
$^{140}_{67}\text{Ho}$	3	$7/2^-$	1106	0.297	-2.222	-1.883
$^{141}_{67}\text{Ho}$	3	$7/2^-$	1190	0.286	-2.387	-2.864
$^{141}_{67}\text{Ho}^*$	0	$1/2^+$	1256	0.286	-5.180	-5.191
$^{145}_{69}\text{Tm}$	5	$11/2^-$	1753	0.249	-5.409	-5.331
$^{146}_{69}\text{Tm}$	5	$11/2^-$	1148	-0.199	-0.456	-0.408
$^{146}_{69}\text{Tm}^*$	5	$11/2^-$	1210	-0.199	-1.276	-1.075
$^{147}_{69}\text{Tm}$	5	$11/2^-$	1071	-0.190	0.591	0.494
$^{147}_{69}\text{Tm}^*$	2	$3/2^+$	1139	-0.190	-3.444	-3.628
$^{150}_{71}\text{Lu}$	5	$11/2^-$	1283	-0.164	-1.180	-1.206
$^{150}_{71}\text{Lu}^*$	2	$3/2^+$	1317	-0.164	-4.523	-4.770
$^{151}_{71}\text{Lu}$	5	$11/2^-$	1255	-0.156	-0.896	-0.932
$^{151}_{71}\text{Lu}^*$	2	$3/2^+$	1332	-0.156	-4.796	-4.908
$^{155}_{73}\text{Ta}$	5	$11/2^-$	1791	0.008	-4.921	-4.585
$^{156}_{73}\text{Ta}$	2	$3/2^+$	1028	-0.053	-0.620	-0.828
$^{156}_{73}\text{Ta}^*$	5	$11/2^-$	1130	-0.053	0.949	1.066
$^{157}_{73}\text{Ta}$	0	$1/2^+$	947	0.045	-0.523	-0.448
$^{160}_{75}\text{Re}$	2	$3/2^+$	1284	0.080	-3.046	-3.160
$^{161}_{75}\text{Re}$	0	$1/2^+$	1214	0.080	-3.432	-3.203
$^{161}_{75}\text{Re}^*$	5	$11/2^-$	1338	0.080	-0.488	-0.570

(Continued on next page)

<b>Table 1 – continued from previous page</b>						
Emitter	$\ell$	$j^\pi$	$Q_p$ (keV)	$\beta_2$	$\log_{10} T_{1/2}^{\text{exp}}$	$\log_{10} T_{1/2}^{\text{cal}}$
$^{164}_{77}\text{Ir}$	5	11/2 <sup>-</sup>	1844	0.089	-3.959	-3.933
$^{165}_{77}\text{Ir}$	5	11/2 <sup>-</sup>	1733	0.099	-3.456	-3.217
$^{166}_{77}\text{Ir}$	2	3/2 <sup>+</sup>	1168	0.107	-0.824	-1.211
$^{166}_{77}\text{Ir}^*$	5	11/2 <sup>-</sup>	1340	0.107	-0.076	-0.005
$^{167}_{77}\text{Ir}$	0	1/2 <sup>+</sup>	1086	0.116	-0.959	-0.945
$^{167}_{77}\text{Ir}^*$	5	11/2 <sup>-</sup>	1261	0.116	0.875	0.810
$^{171}_{79}\text{Au}$	0	1/2 <sup>+</sup>	1469	-0.105	-4.770	-4.397
$^{171}_{79}\text{Au}^*$	5	11/2 <sup>-</sup>	1718	-0.105	-2.654	-2.608
$^{177}_{81}\text{Tl}$	0	1/2 <sup>+</sup>	1180	-0.053	-1.174	-0.735
$^{177}_{81}\text{Tl}^*$	5	11/2 <sup>-</sup>	1986	-0.053	-3.347	-3.838
$^{185}_{83}\text{Bi}$	0	1/2 <sup>+</sup>	1624	-0.052	-4.229	-4.474

To measure the goodness of the agreement, we introduce the standard deviation and the average deviation, which are defined as

$$\sqrt{\langle \sigma^2 \rangle} = \sqrt{\sum_{i=1}^N \left[ \log_{10}(T_{exp}^i / T_{cal}^i) \right]^2 / N} \quad (11a)$$

$$\langle \sigma \rangle = \sum_{i=1}^N \left| \log_{10} T_{exp}^i - \log_{10} T_{cal}^i \right| / N \quad (11b)$$

Their detailed values are listed in table 2. In table 2 the first and second columns are the type and number of the parent nuclei. The third and fourth columns denote the standard deviations and the mean deviations, respectively. It is seen that the experimental data of spherical and deformed emitters are well reproduced by the formula with a mean factor of less than two.

Table 2.

Results for proton emission obtained with the new formula. The deviations 0.2, 0.3, and 0.4 of the logarithms correspond to the deviations between the experimental half-lives and the theoretical ones by factors of 1.6, 2.0, and 2.5, respectively.

Nuclei type	Number	$\sqrt{\langle \sigma^2 \rangle}$	$\langle \sigma \rangle$
Deformed	12	0.267	0.216
Spherical	27	0.217	0.170
All	39	0.384	0.302

As shown above, the new formula (8), including the dependence upon the deformation of daughter nuclei, separately reproduce the experimental data of spherical and deformed emitters, and the values of the parameter  $a$  associated with the crucial quantity  $Q$  in these two cases are very close to each other ( $a = 0.39930$  for spherical



emitters and  $a = 0.39745$  for deformed ones). These suggest that, when one uses it to simultaneously describe spherical and deformed emitters, the agreement between the theoretical values and the experimental data is expected to be reasonably good, though poorer than that of the separate description in which the larger number of parameters is used. Let us use four parameters  $a$ ,  $b$ ,  $c$ , and  $d$  to describe all the available experimental data of proton emission. By analyzing 39 experimental values, consisting of the data for spherical and deformed emitters, we obtain the following parameter set

$$\{a = 0.40740, b = -1.49669, c = -17.66041, d = 27.26907.\}$$

The standard deviation and the mean deviation are, respectively, listed in table 2 for comparison. As expected from our discussion above, the deviations in this case are larger than in the separate description of spherical and deformed emitters. Nevertheless, it is still seen that the calculated half-lives agree well with the experimental data within a mean factor of about two. The deviation of the calculated half-lives from the experimental values is shown in fig. 2 as a function of the proton number of the parent nucleus. In order to gain clear insight into the influence of nuclear deformation, we also illustrate in fig. 2 the results obtained with the spherical formula (7) for comparison. As can be seen in fig. 2(b), there is a pronounced underestimation of the half-lives in neighborhood of  $Z = 82$ , showing the strong shell effect at the proton magic number  $Z = 82$ . Another conspicuous deviation occurs at  $Z = 69$ . This may be caused either by an abrupt change in the  $Q_p$  values or in the structure of the different emitters, or by both [31]. For details, let us get back to table 1, where the values of the decay energy  $Q_p$  and the deformation parameter  $\beta_2$  are shown. At  $Z = 69$  (Tm), there is an abrupt change in the  $Q_p$  values from 1.256 ( $^{141}\text{Ho}^*$ ) to 1.753 ( $^{145}\text{Tm}$ ) to 1.148 ( $^{146}\text{Tm}$ ) MeV, and a pronounced change in the deformation parameter  $\beta_2$  is also seen from 0.249 ( $^{145}\text{Tm}$ ) to -0.199 ( $^{146}\text{Tm}$ ).

Although the formula for proton emission is derived with some approximations, the excellent agreement between the calculated half-lives and the experimental data gives an active response to the validity of the simple formula. This is illustrated in fig. 3, where the  $x$  axis is  $\mu^{1/2}Z_pZ_dQ^{-1/2}$  and the  $y$  axis is the other parts of the formula. Figs. 3(a) and 3(b) show the results for the separate description of deformed and spherical emitters while fig. 3(c) shows the results for the simultaneous description of both deformed and spherical emitters. It is evident that in these three cases the experimental data lie approximately on a straight line as predicted by the formula.

Finally, it is interesting to perform a systematic study of the half-lives of proton emission,  $\alpha$  decay, and cluster radioactivity. Let us compare the values of the parameters  $a$ ,  $b$ , and  $c$  in the three cases. The parameter  $a$ , describing the  $Q$  dependence, is determined to be  $a = 0.41$  for proton emission,  $a = 0.40$  for  $\alpha$  decay,

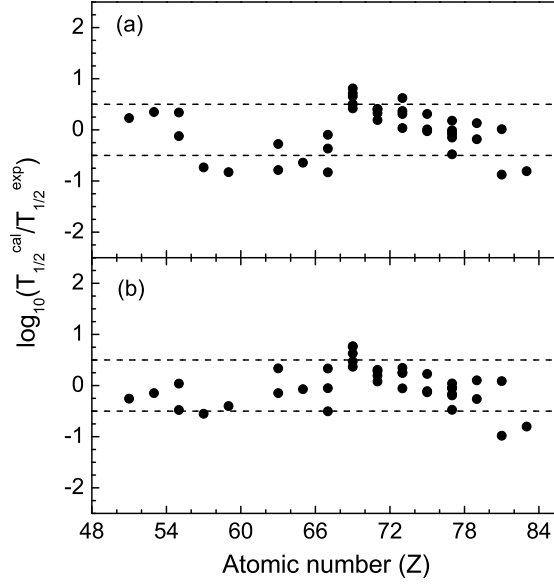


Fig. 2 – The deviations of calculated half-lives from the experimental values for proton emission. The upper part (a) shows the results obtained from the spherical formula (7), and the lower part (b) shows the ones obtained from the new formula (8).

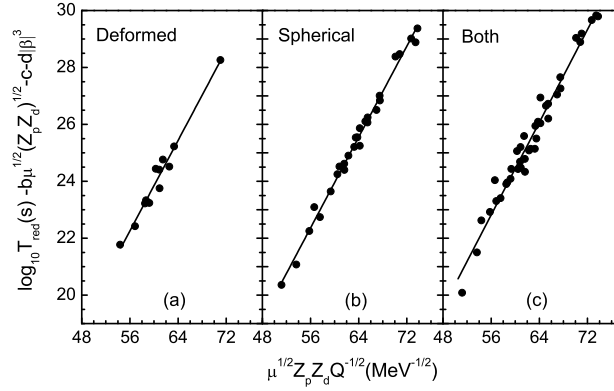


Fig. 3 – The comparison of the logarithm of the calculated half-lives with the logarithm of the experimental data for proton emission. (a) On the left side, the results for deformed emitters. (b) In the middle, the results for spherical emitters. (c) On the right side, the results for both deformed and spherical emitters. Calculations are performed with the new formula (8). The straight line represents the calculated values and the points represent the experimental ones.

and  $a = 0.39$  for cluster radioactivity. The difference of  $a$  in these three cases is very small. The value of the parameter  $b$  is  $-1.50$  for proton emission,  $-1.31$  for  $\alpha$

decay, and -1.09 for cluster radioactivity. For the case of the parameter  $c$ , it has one value ( $c = -17.66$ ) for proton emission, four different values ( $c_{e-e} = -17.01$ ,  $c_{e-o} = -16.26$ ,  $c_{o-e} = -16.40$ , and  $c_{o-o} = -15.85$ ) for  $\alpha$  decay, and two different values ( $c_{e-e} = -21.37$  and  $c_{o-A} = -20.11$ ) for cluster radioactivity. Although there is a clear difference of the preformation probability in the three cases, the differences of  $b$  and  $c$  in the three cases are also small with respect to the values of themselves. With these in mind, it is concluded that there is common behavior for the half-lives of proton emission,  $\alpha$  decay, and cluster radioactivity. This confirms the fact that proton emission,  $\alpha$  decay, and cluster radioactivity share the same underlying mechanism in physics. More importantly, it indicates that we have successfully obtained a unified law for proton emission,  $\alpha$  decay, and cluster radioactivity, because the formulas of half-lives in the three cases are very similar, and the values of the three sets of parameters separately describing them are close to each other.

#### 4. SUMMARY

In summary, a new formula is proposed for the evaluation of proton-emission half-lives. We systematically investigate the experimental data of spherical and deformed emitters with this formula, respectively. The results show excellent agreement between the experimental data and the theoretical values. Moreover, in view of the universality of this new formula, we use it to describe the data of both spherical and deformed emitters at the same time. The results are also satisfactory. More importantly, there are many resemblances between the new formula for proton emission and the unified formula for  $\alpha$  decay and cluster radioactivity. Furthermore, the values of the three sets of parameters, separately describing proton emission,  $\alpha$  decay, and cluster radioactivity, show small differences. This demonstrates that a unified description of proton emission,  $\alpha$  decay, and cluster radioactivity has been achieved.

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