Dedicated to Academician Aureliu Sandulescu's 80th Anniversary

GROUND-STATE CORRELATIONS AND STRUCTURE OF THE LOW-LYING STATES IN ODD-EVEN SPHERICAL AND TRANSITIONAL NUCLEI

S. MISHEV, V.V. VORONOV

Bogoliubov Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research,
6 Joliot-Curie str., Dubna 141980, Russia

Received October 3, 2011

The influence of the Pauli principle and the nucleon correlations in the ground states of spherical and transitional even-even nuclei on the structure of the low-lying states in odd-even nuclei is examined. We study correlations caused by the quasiparticle-phonon interaction in the ground state beyond the pairing correlations. The effects owing to the ground-state correlations (GSC) are becoming essential as the number of nucleons in the unclosed shells increase. We reason about this subject using the language of the quasiparticle phonon model which we extend to account for the existence of quasiparticle$\times$phonon configurations in the ground states of the even-even cores.

Key words: Ground state correlations, odd-even nuclei, quasiparticle phonon nuclear model.

PACS: 21.60.Jz, 21.60.Ev.

1. INTRODUCTION

A major challenge in front of the theories beyond the mean-field approximation is the diagonalization of the residual interaction in a tractable way. Thanks to the tremendous increase in computer power, exact diagonalization of the residual interaction with given effective forces in a huge model space is becoming available for up to medium-mass nuclei including those far from the stability line. However, in order to extract physical picture of the nucleons dynamics from huge numerical information, it is necessary to have recourse to the concept of collectivity on which we will elaborate in the sequel.

In the present research we consider two residual interaction modes - short-range pairing and long-range interactions. The diagonalization of the corresponding Hamiltonian is performed using the approximations contained in the quasiparticle phonon model (QPM) stack [1]. In particular the pairing Hamiltonian is diagonalized using the Bardeen-Cooper-Schrieffer(BCS) ansatz [2]. Although it falls short of accurately reproducing the number of nucleons it possesses the appealing feature that much of the simplicity of the independent particle model is regained after the introduction of virtual particles called quasiparticles (qp). After establishing the independent quasiparticle picture the next step in the standard version of QPM is to diagonalize the long-range part of the Hamiltonian. In line with this goal, one defines the phonon (ph) operators as a linear superposition of pure two-quasiparticle states and performs diagonalization in the multi-phonon space. In the present work, we limit our research in the one-phonon space in which the variational calculations yield the quasiparticle random phase approximation (QRPA). Once the properties of both types of objects are fixed, one can calculate the interaction between them which, in turn, determines the dynamics of odd-even nuclei. A notable merit of this model is that the quasiparticle-phonon interaction depends solely on the internal structure of both the quasiparticles and the phonons thus introducing no extra free parameters.

The action of the residual interaction to the ground states is to bring diffuseness around the Fermi level. It does so by mixing multiple-particle-multiple-hole pure states inside the nuclear ground state. These several particle-hole admixtures into the independent particle wave function are called ground-state correlations (GSC). The impact of pairing interaction on the ground state properties has been largely discussed since the first applications of the BCS theory to nuclear systems [3,4]. The long-range force, on the other hand, adds further correlations to the ground states. A widely used method to account for presence of these correlations is QRPA.

In standard QPM, the sequential approach towards diagonalizing the Hamiltonian, outlined above, has the advantage of being manageable in the sense that analysis of intermediate results can be performed at each step of the process. One disadvantage in this setting, however, is that the influence of the long-range force to the quasiparticles' properties is neglected. Numerous improvements to the quasiboson approximation, underlying the RPA [5, 6], have been attempted, as for example in [7–9]. In the present work we utilize the extended random phase approximation (ERPA), suggested by Hara [10] and Ikeda *et al.* [11] a long time ago. The approach they have proposed broadens the area of applicability of the conventional theory which relies on the suggestion that the true ground state must not be very different from the quasiparticle vacuum state. Further developed [12] and applied to concrete nuclei [13] this approach proved successful in improving the theoretical results for most measurable quantities near the nuclear ground states as, for example, the transition charge densities in the nuclear interior.

The other improvement to the conventional QPM model we perform, which is closely allied to odd-even nuclei, is to extend the configurational space so as to allow for quasiparticle and quasiparticle$\times$phonon states to reside in the even-even nuclear ground states. As a result, the last unpaired nucleon couples not only with the excited vibrational states but also with states in the ground state, referred to as zero-point motion. In close analogy to the RPA for even-even nuclei this suggests non-vanishing backward amplitudes in the odd-A nucleus wave function. Such an approach, in which 3qp backward amplitudes were included in the wave function, was initially developed in Ref. [14] in studying the low-lying states in tin isotopes.

The effects from this extension, however, turned out to be small due to the relatively small collectivity of the first phonon state in these nuclei. Progress in this direction was made by Van der Sluys *et al.* [15], who applied this method to ^{142}Nd and showed that the ground state correlations influence the single-particle fragmentation shifting the strength to higher excitation energies. In their study, however, the quasiparticle and phonon operators were taken as commuting ones, thus neglecting the Pauli principle, which can be unsatisfactory in a number of nuclei, since in them serious deviations from the independent harmonic motion occur. In such cases the disregard of the innate fermion structure of the phonons is unjustified. In Ref. [16] we performed analytical calculations following the exact commutation relations between the quasiparticle and phonon operators and evaluated the effects of the resulting corrections on the spectra and single-particle spectroscopic strength in a number of nuclei in the barium region. In this treatment the Pauli exclusion principle manifests itself in the emergence of factors $(1-\mathcal{L}(Jj\lambda i))$ which turn to zero whenever a particular three-quasiparticle state is disallowed as shown in [17].

The present paper is organized in the following way. In section 2, a brief review of the ERPA to spherical even-even nuclei is given. The application of this method to odd-A nuclei in an extended configurational space is presented in section 3. Calculations on various nuclear properties in some odd-even Te, Xe, and Ba nuclei, where experimental data are available, are presented in section 4. Section 5 summarizes our findings.

2. EVEN-EVEN NUCLEI

In this section we discuss the limitations of the quasiboson approximation and introduce the extended boson approximation, which removes some of the inconsistencies of the former. This approach aims to provide a more consistent way to treat the fermion properties of the phonon operators as compared to RPA. The RPA is widely considered as a good first approximation to study small fluctuations in atomic nuclei due to its simplicity and numerous successful applications. However this simple model enjoys only a limited success when one needs to describe properties of states from the lowest part of the spectrum in nuclei remote from the magic configurations. In ERPA the quasiparticle occupation numbers enter the basic equations of the theory explicitly which leads to a co-dependence between the different layers of the framework otherwise separated in QRPA. More specifically, in ERPA one defines the quantities ρ_j, which are proportional to the quasiparticle occupation numbers in the ground state on the level j:

$$\rho_j = \frac{1}{\sqrt{2j+1}} \sum_m \langle|\alpha^\dagger_{jm}\alpha_{jm}|\rangle, \tag{1}$$

where α denotes quasiparticle defined as

$$\alpha_{jm} = u_j a_{jm} - (-)^{j-m} v_j a^{\dagger}_{j-m}. \tag{2}$$

The other key constituent of the theory are the phonon operators defined as:

$$Q^{\dagger}_{\lambda\mu i} = \frac{1}{2}\sum_{jj'}\left[\psi^{\lambda i}_{jj'} A^{\dagger}(jj';\lambda\mu) - (-1)^{\lambda-\mu}\varphi^{\lambda i}_{jj'} A(jj';\lambda-\mu)\right]. \tag{3}$$

The ground state $|\rangle$ in equation (1) is the vacuum state for the phonon operators (3), *i.e.* $Q_{\lambda\mu i}|\rangle = 0$. The explicit account of the quasiparticle densities in the ground state modifies all basic QPM equations.

In the following we study the dynamics of nuclear systems governed by the Hamiltonian in the form:

$$H = \sum_{\tau}^{(n,p)}\left\{\sum_{jm}(E_j - \lambda_\tau)a^{\dagger}_{jm}a_{jm} - \frac{1}{4}G^{(0)}_{\tau} : (P^{\dagger}_0 P_0)^{\tau} : -\frac{1}{2}\sum_{\lambda\mu}\kappa^{(\lambda)} : (M^{\dagger}_{\lambda\mu}M_{\lambda\mu}) : \right\}.$$

The constants G_n and G_p measure the strengths of the pairing interaction between the neutrons and the protons respectively. The isoscalar interaction suggests that the long-range proton-proton, neutron-neutron and proton-neutron forces are equally intense

$$\kappa_{\lambda}(n,n) = \kappa^{(\lambda)}(p,p) = \kappa^{(\lambda)}(n,p) \equiv \kappa^{(\lambda)}. \tag{4}$$

E_j are the energies of the nucleons, moving in a mean field, approximated by the spherical Woods-Saxon potential

$$V(r) = -\frac{V_0}{1+e^{\alpha(r-R_0)}}\left(1 + \frac{\kappa\alpha e^{\alpha(r-R_0)}(\vec{\sigma}\vec{l})}{r\left(1+e^{\alpha(r-R_0)}\right)}\right). \tag{5}$$

The multipole operator is defined in the following way

$$M^{+}_{\lambda\mu} = \frac{(-1)^{\lambda-\mu}}{\pi_\lambda}\sum_{\substack{j_1m_1\\ j_2m_2}}(-1)^{j_2-m_2} f^{\lambda}_{j_1j_2}\langle j_1m_1j_2-m_2 \mid \lambda\mu\rangle a^{+}_{j_1m_1}a_{j_2m_2}, \tag{6}$$

with a^{+}_{jm}, a_{jm} being the creation and annihilation nucleon operators respectively and $f^{\lambda}_{j_1j_2}$ standing for the single-particle reduced matrix elements

$$f^{\lambda}_{jj'} = \langle j||R_\lambda(r)i^{\lambda}Y_{\lambda\mu}||j'\rangle. \tag{7}$$

The pair creation operator is defined as

$$P^{\dagger}_0 = \sum_{jm}(-1)^{j-m}a^{\dagger}_{jm}a^{\dagger}_{j-m}. \tag{8}$$

If the pairing vibrations are not taken into consideration then one can obtain (conf. [13]) the following modified QPM equations describing the states in even-even nuclei:

$$\frac{1}{2}\sum_j(2j+1)\left\{1-\frac{(1-2\rho_j)(E_j-\lambda)}{\sqrt{(E_j-\lambda)^2+\Delta^2}}\right\}=n, \tag{9}$$

$$\frac{G}{4}\sum_j\frac{2j+1}{\sqrt{(E_j-\lambda)^2+\Delta^2}}(1-2\rho_j)=1, \tag{10}$$

$$\frac{\kappa_\lambda}{2\lambda+1}\sum_{jj'}(1-\rho_{jj'})\frac{(f^\lambda_{jj'}u^+_{jj'})^2(\varepsilon_j+\varepsilon_{j'})}{(\varepsilon_j+\varepsilon_{j'})^2-\omega^2_{\lambda i}}=1, \tag{11}$$

$$\sum_{jj'}(1-\rho_{jj'})[(\psi^{\lambda i}_{jj'})^2-(\varphi^{\lambda i}_{jj'})^2]=2, \tag{12}$$

$$\rho_j=\frac{1}{2}\sum_{\lambda ij'}\frac{2\lambda+1}{2j+1}(1-\rho_{jj'})(\varphi^{\lambda i}_{jj'})^2.. \tag{13}$$

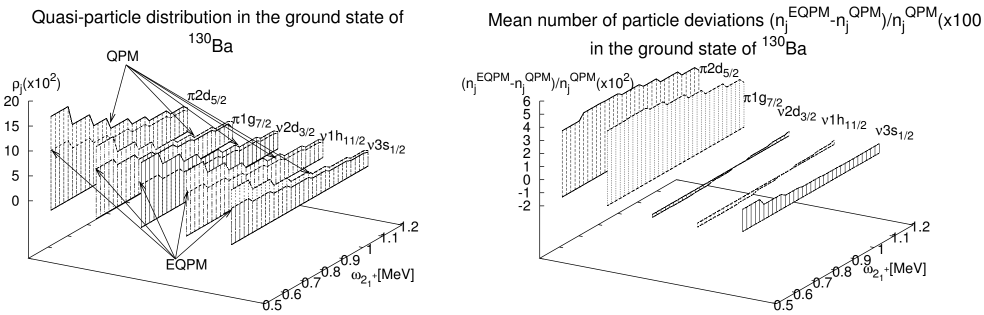

Fig. 1 – Left panel - quasiparticle densities $\rho_j\times 100$ in the ground state of ^{130}Ba within one-phonon QRPA and ERPA theories for the sub-shells in the valence shell. Right panel - same as in the left panel but for the quantities $\left(n_j^{ERPA}-n_j^{QRPA}/n_j^{QRPA}\right)\times 100$, where n_j is the number of particles on the level j.

The pairing gap Δ and the quasiparticle energies can be expressed as

$$\Delta=\frac{G}{2}\sum_j(1-2\rho_j)\pi_j^2u_jv_j,\quad \varepsilon_j=\sqrt{\Delta^2+(E_j-\lambda)^2}. \tag{14}$$

The emergence of the factors $(1-\rho_{jj'})$ takes into account the blocking effect due to the Pauli principle and requires one to solve these equations as a system of coupled equations.

The multipole-multipole interaction strengths $\kappa^{(\lambda)}$ are treated as free parameters in our study. In the numerical calculations we kept the quadrupole-quadrupole term only because from one side it gives the dominant part of the long-range interaction for the determination of the low-lying states' properties and from the other - it is manageable in terms of adjusting a single constant $\kappa^{(2)}$. One way to fix the latter is to have it reproduce the energy of the first 2^+ state ($\omega_{2_1^+}$) . Since a one-to-one correspondence between $\omega_{2_1^+}$ and $\kappa^{(2)}$ exists, we show most of the calculated quantities as a function of $\omega_{2_1^+}$ because its values are more intuitive and closer to the experiment than the corresponding interaction strength values. One major advantage of the ERPA approach over QRPA is that a non-trivial solution of the system (9)-(13) exists at any values of the multipole-multipole interaction strengths $\kappa^{(\lambda)}$ which is in contrast with the QRPA equation which predicts physically interesting solutions only at $\kappa < \kappa_c^{(\lambda)}$, where $\kappa_c^{(\lambda)}$ are critical values of these strength.

In the following we discuss the results obtained within ERPA for the particle and quasiparticle occupation numbers as well as for the transition probabilities in even-even nuclei.

The differences between the quasiparticle and particle occupation numbers in ^{130}Ba as a function of the first quadrupole phonon's energy within the QRPA and ERPA are presented in figure 1. This figure is noteworthy because it shows that the smearing of the Fermi surface increases together with the strength of the field force (and, correspondingly, with the decrease of $\omega_{2_1^+}$). In the right panel of this figure we point out that the relative difference of the particle occupation numbers calculated within the two model variants can reach up to 5 %, as is the case for the proton subshell $2d_{5/2}$. In figure 2 a similar graph is presented for a fixed value of the parameter $\kappa^{(2)}$ which sets the energy of the first quadrupole phonon in ^{130}Ba to be 0.5 MeV. This graph complements figure 1 with additional bars representing the occupation numbers obtained using the BCS approximation and shows the large impact of the long-range force on these quantities.

In view of calculating the transition probabilities in odd-even nuclei using a core described as a solution of the system (9)-(13), we give the expression for $B(E2|0_1^+ \rightarrow \lambda_i)$ within ERPA:

$$B(E\lambda|0_1^+ \rightarrow \lambda_i) = \left[\frac{1}{2} \sum_{jj'} (1 - \rho_{jj'}) f_{jj'}^{\lambda} u_{jj'}^{+} g_{jj'}^{\lambda i} \right]^2 . \tag{15}$$

Setting $\rho_j = 0$ will make the equations from this section coincide with the more agreeable ones from the standard QPM [1].

3. ODD-EVEN NUCLEI

This section embraces the main subject of the present work - GSC in odd-even nuclei. In particular, in subsection 3.1 we elaborate on the role of the backward

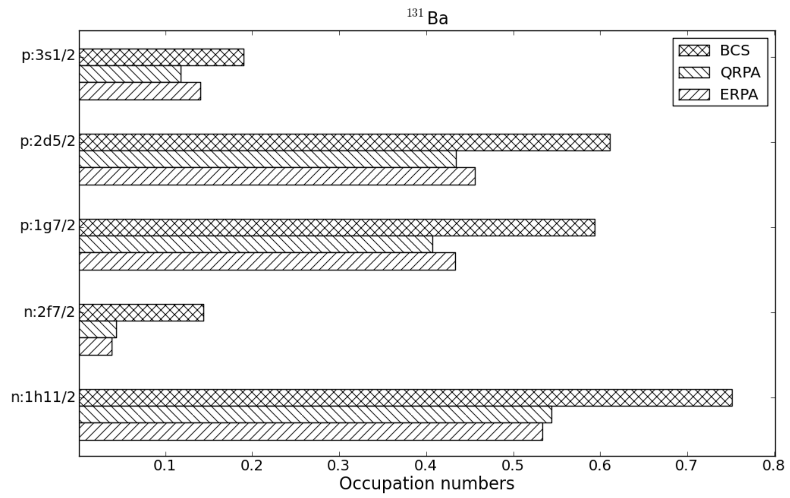

Fig. 2 – Nucleon occupation numbers for several valent subshells in ^{130}Ba. The parameter $\kappa^{(2)}$ in QRPA and ERPA is fixed so as to set the energy of the first quadrupole phonon to be 0.5 MeV.

amplitudes on the structural changes of the wave functions, leading to significant modifications in the lowest part of the spectra as well as in the transition probabilities in these nuclei. The ERPA is applied in calculating qp-ph interaction vertices in the forward and backward direction as well as other matrix elements in subsection 3.2. The physical consequences are discussed.

3.1. GROUND STATE CORRELATIONS IN ODD-A NUCLEI WITHIN QRPA

The odd-even nuclei description presented in standard QPM [1] neglects the nucleon correlations in the ground state to a large extend. In this section we shall discuss long-range correlations taken into account by suggesting non-vanishing components of the wave function arising from the existence of quasiparticle and qp$\times$ph states in the nuclear ground states. It will be shown in section 4 that the effects, owing to the ground-state correlations, are becoming essential as the number of nucleons in the unclosed shells increase in both even-even and odd-even nuclei. The wave function employed in this and the following subsection incorporates terms which allow quasiholes and qp$\times$ph states living in the ground states of even-even nuclei $|\rangle$:

$$\Psi_\nu(JM) = C_{J\nu}\alpha^\dagger_{JM} + \sum_{j\lambda i} D_{j\lambda i}(J\nu) P^\dagger_{j\lambda i}(JM) - E_{J\nu}\tilde{\alpha}_{JM} - \sum_{j\lambda i} F_{j\lambda i}(J\nu)\tilde{P}_{j\lambda i}(JM)|\rangle, \quad (16)$$

where the qp×ph creation operator $P^{\dagger}_{j\lambda i}(JM)$ is defined in as

$$P^{\dagger}_{j\lambda i}(JM) = [\alpha^{\dagger}_{j} Q^{\dagger}_{\lambda i}]_{JM} \tag{17}$$

and ˜ stands for time conjugation according to the convention $\tilde{a}_{jm} = (-1)^{j-m} a_{j-m}$.

The matrix elements evaluation in this section is performed conforming to the QRPA. The exclusion of certain 3qp states inhibited by the Pauli principle, as well as other effects, related to this principle are taken into account by using the exact commutation relations between the quasiparticle and phonon operators:

$$\left[\alpha_{jm}, Q^{\dagger}_{\lambda\mu i}\right] = \sum_{j'm'} \langle jmj'm' \mid \lambda\mu\rangle \, \psi^{\lambda i}_{jj'} \alpha^{\dagger}_{j'm'}. \tag{18}$$

The amplitudes in (16) and the states' energies are determined using the equation-of-motion method (EOMM)

$$\langle|\{\delta O_{JM\nu}, H, O^{\dagger}_{JM\nu}\}|\rangle = \eta_{J\nu} \langle|\{\delta O_{JM}, O^{\dagger}_{JM}\}|\rangle. \tag{19}$$

Following the linearization procedure described in [18], at the final state of calculation of the matrix elements, we consider the even-even nucleus ground state to be a vacuum state for both operators α_{jm} and $Q_{\lambda\mu i}$, *i.e.* $|\rangle \equiv |qp\rangle \equiv |ph\rangle$. In that the double commutator in the left-hand side of (19), representing the energy of the systems in state $\Psi_{\nu}(JM)$, evaluates to

$$\begin{aligned}
\langle|\{O_{JM\nu}, H, O^{\dagger}_{JM\nu}\}|\rangle &= (C^2_{J\nu} - E^2_{J\nu}) \langle\{[\alpha_{JM}, H], \alpha^{+}_{JM}\}\rangle + \\
&+ 2(C_{J\nu} \sum_{j\lambda i} D_{j\lambda i} - E_{J\nu} \sum_{j\lambda i} F_{j\lambda i}) V(Jj\lambda i) + \\
&+ \sum_{j\lambda i} \sum_{j'\lambda' i'} (D_{j\lambda i} D_{j'\lambda' i'} - F_{j\lambda i} F_{j'\lambda' i'}) I_J(j\lambda i|j'\lambda' i') + \\
&+ 2(C_{J\nu} \sum_{j\lambda i} F_{j\lambda i} - E_{J\nu} \sum_{j\lambda i} D_{j\lambda i}) W(Jj\lambda i).
\end{aligned} \tag{20}$$

The normalization condition for the wave function takes the form

$$\begin{aligned}
\langle|\{O_{JM\nu}, O^{+}_{JM\nu}\}|\rangle &= C^2_{J\nu} + E^2_{J\nu} + \sum_{j\lambda i} [D_{j\lambda i}(J\nu)]^2 + \sum_{j\lambda i} [F_{j\lambda i}(J\nu)]^2 + \\
&+ \sum_{j\lambda i j'\lambda' i'} [D_{j\lambda i}(J\nu) D_{j'\lambda' i'}(J\nu) + F_{j\lambda i}(J\nu) F_{j'\lambda' i'}(J\nu)] \mathcal{L}_J(j\lambda i|j'\lambda' i') = 1.
\end{aligned} \tag{21}$$

In what follows we use a diagonal approximation for the quantities $\mathcal{L}_J(j\lambda i|j'\lambda' i')$ [17]:

$$\mathcal{L}_J(j\lambda i|j'\lambda' i') = \mathcal{L}(Jj\lambda i) \delta_{jj'} \delta_{\lambda\lambda'} \delta_{ii'}, \tag{22}$$

where

$$\mathcal{L}(Jj\lambda i) = \pi_\lambda^2 \sum_{j'} \left(\psi_{j'j}^{\lambda i}\right)^2 \left\{ \begin{array}{ccc} j & j' & \lambda \\ j & J & \lambda \end{array} \right\}. \tag{23}$$

Applying the EOMM (19) one can obtain the following generalized eigenvalue problem for each state Ψ_ν with fixed quantum numbers J and M:

$$\begin{pmatrix} \varepsilon_J & V(Jj'\lambda'i') & 0 & -W(Jj'\lambda'i') \\ V(Jj\lambda i) & K_J(j\lambda i|j'\lambda i') & W(Jj\lambda i) & 0 \\ 0 & W(Jj'\lambda'i') & -\varepsilon_J & -V(Jj'\lambda'i') \\ -W(Jj\lambda i) & 0 & -V(Jj\lambda i) & -K_J(j\lambda i|j'\lambda i') \end{pmatrix} \begin{pmatrix} C_{J\nu} \\ D_{j'\lambda'i'}(J\nu) \\ -E_{J\nu} \\ -F_{j'\lambda'i'}(J\nu) \end{pmatrix}$$
$$= \eta_{J\nu} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-\mathcal{L}(Jj\lambda i) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1-\mathcal{L}(Jj\lambda i) \end{pmatrix} \begin{pmatrix} C_{J\nu} \\ D_{j'\lambda'i'}(J\nu) \\ -E_{J\nu} \\ -F_{j'\lambda'i'}(J\nu) \end{pmatrix}. \tag{24}$$

The explicit expressions for the quantities entering the formulas above will be examined one at a time. The matrix elements $V(Jj\lambda i)$, expressed as

$$V(Jj\lambda i) = \left\langle \left| \left\{ [\alpha_{JM}, H], P^+_{j\lambda i}(JM) \right\} \right| \right\rangle = -\frac{1}{\sqrt{2}}[1+\mathcal{L}(Jj\lambda i)]\Gamma(Jj\lambda i), \tag{25}$$

represent the forward qp-ph interaction.

The matrix elements $W(Jj\lambda i)$ appear after the introduction of the backward-going terms in the wave function (16) and play a central role in the present work. The explicit form of these quantities is

$$W(Jj\lambda i) = \left\langle \left| \left\{ \left[\alpha^+_{JM}, H\right], \tilde{P}^+_{j\lambda i}(JM) \right\} \right| \right\rangle =$$
$$= -\frac{1}{4}\frac{\pi_\lambda}{\pi_J}[1+\mathcal{L}(Jj\lambda i)-\mathcal{L}(jJ\lambda i)] \sum_{i'\tau_0} \mathcal{A}_{\tau_0}\left(\lambda i i'\right) \varphi_{Jj}^{\lambda i'}. \tag{26}$$

The calculation of the diagonal matrix elements yields:

$$K_J\left(j\lambda i j'\lambda' i'\right) = \frac{1}{2}\left[I_J\left(j\lambda i|j'\lambda' i'\right) + I_J\left(j'\lambda' i'|j\lambda i\right)\right] =$$
$$= \delta_{jj'}\delta_{\lambda\lambda'}\delta_{ii'}\left(1+\mathcal{L}(Jj\lambda i)\right)\left(\omega_{\lambda i}+\varepsilon_j - \mathcal{R}(Jj\lambda i)\right), \tag{27}$$

where

$$I_J\left(j\lambda i|j'\lambda' i'\right) = \left\langle \left| \left\{ P_{j\lambda i}(JM), \left[H, P^+_{j'\lambda' i'}(JM)\right] \right\} \right| \right\rangle \tag{28}$$

with

$$\mathcal{R}(Jj\lambda i) = \frac{\mathcal{R}_J(j\lambda i|j\lambda i)}{1+\mathcal{L}(Jj\lambda i)} = \frac{1}{4}\sum_{\tau i'} \mathcal{A}_\tau(\lambda i i')\mathcal{L}(j\lambda i|j\lambda i'). \tag{29}$$

Below are given the definitions for notations used in Eqs. (26) and (29):

$$\mathcal{A}_\tau(\lambda i i') = \frac{X_{\lambda i}(\tau) + X_{\lambda i'}(\tau)}{\sqrt{\mathcal{Y}_{\lambda i}(\tau)\mathcal{Y}_{\lambda i'}(\tau)}}, \quad X_{\lambda i}(\tau) = \sum_{jj'}^{\tau} \frac{(f^{\lambda}_{jj'} u^{+}_{jj'})^2 \varepsilon_{jj'}}{\varepsilon^2_{jj'} - \omega^2_{\lambda i}},$$
$$\mathcal{Y}_{\lambda i}(p) = \mathcal{Y}_{\lambda i}(n) = \omega_{\lambda i} \sum_{jj'} \frac{(f^{\lambda}_{jj'} u^{+}_{jj'})^2 \varepsilon_{jj'}}{(\varepsilon^2_{jj'} - \omega^2_{\lambda i})^2}, \quad \Gamma(jj'\lambda i) = \frac{\pi_\lambda}{\pi_j} \frac{v^{(-)}_{jj'} f^{(\lambda)}_{jj'}}{\sqrt{Y^{\lambda i}}}, \tag{30}$$

where

$$v^{(-)}_{jj'} = u_j u_{j'} - v_j v_{j'}, \quad u^{(+)}_{jj'} = u_{j'} v_j + u_j v_{j'}, \quad \varepsilon_{jj'} = \varepsilon_j + \varepsilon_{j'}$$

and $\pi_j = \sqrt{2j+1}$. $f^{\lambda}_{jj'}$ are the single-particle reduced matrix elements of the residual forces with radial dependence $\sim \partial V / \partial r$ [19].

The quantities $\mathcal{L}(Jj\lambda i)$ and $\mathcal{R}(Jj\lambda i)$ vanish if the Pauli principle is not respected.

If one is interested in the single-quasiparticle components only, the problem in Eq. (24) can be reduced to the following 2×2 eigenvalue problem [15] :

$$\left[\begin{pmatrix} \varepsilon_J & 0 \\ 0 & -\varepsilon_J \end{pmatrix} + \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}\right] \begin{pmatrix} C_{J\nu} \\ -E_{J\nu} \end{pmatrix} = \eta_{J\nu} \begin{pmatrix} C_{J\nu} \\ -E_{J\nu} \end{pmatrix}, \tag{31}$$

where

$$M_{11} = \sum_{j\lambda i} \frac{1}{(1+\mathcal{L}(Jj\lambda i))} \times \left(\frac{V^2(Jj\lambda i)}{\eta_{J\nu} - (\omega_{\lambda i} + \varepsilon_j - \mathcal{R}(Jj\lambda i))} + \frac{W^2(Jj\lambda i)}{\eta_{J\nu} + \omega_{\lambda i} + \varepsilon_j - \mathcal{R}(Jj\lambda i)} \right), \tag{32}$$

$$M_{22} = \sum_{j\lambda i} \frac{1}{(1+\mathcal{L}(Jj\lambda i))} \times \left(\frac{W^2(Jj\lambda i)}{\eta_{J\nu} - (\omega_{\lambda i} + \varepsilon_j - \mathcal{R}(Jj\lambda i))} + \frac{V^2(Jj\lambda i)}{\eta_{J\nu} + \omega_{\lambda i} + \varepsilon_j - \mathcal{R}(Jj\lambda i)} \right), \tag{33}$$

$$M_{12} = \sum_{j\lambda i} \frac{V(Jj\lambda i) W(Jj\lambda i)}{(1+\mathcal{L}(Jj\lambda i))} \times \left(\frac{1}{\eta_{J\nu} + \omega_{\lambda i} + \varepsilon_j - \mathcal{R}(Jj\lambda i)} - \frac{1}{\eta_{J\nu} - (\omega_{\lambda i} + \varepsilon_j - \mathcal{R}(Jj\lambda i))} \right) = M_{21}. \tag{34}$$

From (31) one can further derive a secular equation for the energies $\eta_{J\nu}$:

$$M_{12} M_{21} = (\varepsilon_J + M_{11} - \eta_{J\nu})(M_{22} - \varepsilon_J - \eta_{J\nu}). \tag{35}$$

3.2. GROUND STATE CORRELATIONS IN ODD-A NUCLEI WITHIN ERPA

This section is intended to provide an improved treatment of the way the Pauli principle is taken into account in odd-mass nuclei utilizing the ERPA formalism, introduced in section 2. In that a refined version to the quasiparticle-phonon model for odd-even nuclei is derived. The interaction strengths between the quasiparticles and phonons in the developed model depend on the number of the quasiparticles in the ground state. In this way the core-particle equations (24) couple with the generalized equations describing the pairing correlations and the excited vibrational states of the even-even core (9)-(13) thus forming a large non-linear system. This development is a step forward to broadening the area of applicability of the model because it allows for larger backward amplitudes $\varphi^{\lambda i}_{jj'}$ in the wave functions in even-even nuclei which are characteristic to most open-shell spherical and transitional odd-A nuclei.

The wave functions, considered in this section, have the same prototypes (16) as the one used in section 3.1. Making use of the EOMM (19) as we did in the previous section and conforming to the relation (1) when calculating matrix elements, we obtain a similar eigenvalue problem as in Eq. (24) for the states' energies and structure coefficients:

$$\begin{pmatrix} \varepsilon_J & V(Jj'\lambda'i') & 0 & -W(Jj'\lambda'i') \\ V(Jj\lambda i) & K_J(j\lambda i|j'\lambda'i') & W(Jj\lambda i) & 0 \\ 0 & W(Jj'\lambda'i') & -\varepsilon_J & -V(Jj'\lambda'i') \\ -W(Jj\lambda i) & 0 & -V(Jj\lambda i) & -K_J(j\lambda i|j'\lambda i') \end{pmatrix} \begin{pmatrix} C_{J\nu} \\ D_{j'\lambda'i'}(J\nu) \\ -E_{J\nu} \\ -F_{j'\lambda'i'}(J\nu) \end{pmatrix} =$$
$$= \eta_{J\nu} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-\mathcal{L}^*(Jj\lambda i) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1-\mathcal{L}^*(Jj\lambda i) \end{pmatrix} \begin{pmatrix} C_{J\nu} \\ D_{j'\lambda'i'}(J\nu) \\ -E_{J\nu} \\ -F_{j'\lambda'i'}(J\nu) \end{pmatrix}. \quad (36)$$

Adhering to the diagonal approximation for $\mathcal{L}^*_J(j\lambda i|j'\lambda'i')$

$$\mathcal{L}^*(Jj\lambda i) = \mathcal{L}^*_{J|j'}(j\lambda i|j'\lambda'i')\delta_{jj'}\delta_{\lambda\lambda'}\delta_{ii'} =$$
$$= \pi_{\lambda\lambda}\sum_{j_1}(1-\rho_{j_1j'})(\psi^{\lambda i}_{j_1j})^2 \begin{Bmatrix} j & j_1 & \lambda \\ j & J & \lambda \end{Bmatrix}, \quad (37)$$

the elements from the matrix in the left-hand side of (36) acquire the form

$$V(Jj\lambda i) = \langle|\{[\alpha_{JM},H],P^{\dagger}_{j\lambda i}\}|\rangle = -\frac{1}{\sqrt{2}}[1-\rho_j+\mathcal{L}^*(Jj\lambda i)]\Gamma(Jj\lambda i), \quad (38)$$

$$W(Jj\lambda i) = \langle|\{[\alpha^\dagger_{JM}, H], \tilde{P}^\dagger_{j\lambda i}\}|\rangle =$$
$$= \frac{\pi_\lambda}{\pi_J}\varepsilon_J \rho_j \varphi^{\lambda i}_{Jj} - \frac{1}{4}[1-\rho_j + \mathcal{L}^*(Jj\lambda i)]\frac{\pi_\lambda}{\pi_J}\sum_{i_1}\mathcal{A}(\lambda i_1 i)\varphi^{\lambda i_1}_{Jj}, \quad (39)$$

$$K_J(j\lambda i|j'\lambda' i') = \frac{1}{2}[I_J(j\lambda i|j'\lambda' i') + I_J(j'\lambda' i'|j\lambda i)] =$$
$$= \delta_{jj'}\delta_{\lambda\lambda'}\delta_{ii'}[1-\rho_j + \mathcal{L}^*(Jj\lambda i)](\varepsilon_j + w_{\lambda i})$$
$$-\delta_{jj'}\delta_{\lambda\lambda'}\delta_{ii'}(1+\mathcal{L}(Jj\lambda i))\frac{1}{4}\sum_{i_1}\mathcal{A}(\lambda i i_1)\mathcal{L}^*_{J|j}(j\lambda i|j\lambda i_1), \quad (40)$$

It is worthwhile to notice that in the limiting case, where the number of the quasiparticles in the ground state is set to zero, *i.e.* $\rho_j = 0$, the system of equations (36) decouple to reduce to the equations of the model described in section 3. In the following we discuss the effect of the correlations in the nuclear ground state on the behavior of the matrix elements in (36).

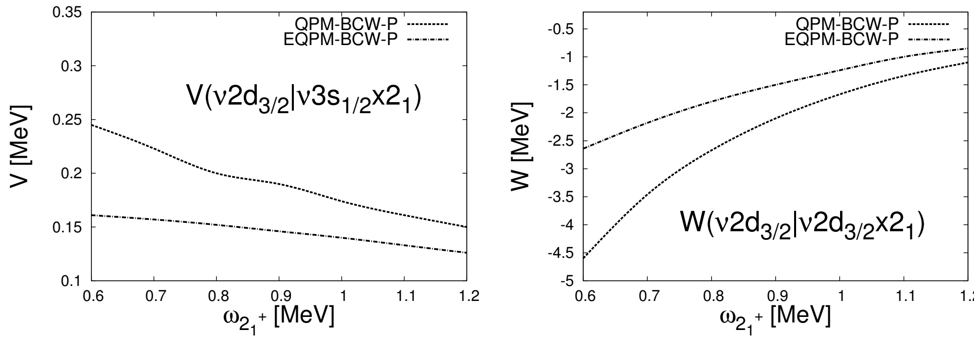

Fig. 3 – The matrix elements $V(\nu 2d_{3/2}|\nu 3s_{1/2}\otimes 2^+_1)$ and $W(\nu 2d_{3/2}|\nu 2d_{3/2}\otimes 2^+_1)$ in ^{131}Ba plotted against the energy $\omega_{2^+_1}$ of the first quadrupole phonon in ^{130}Ba.

The interaction between the quasiparticles and the phonons will naturally become stronger when the smearing around the Fermi level increases. In figure 3, the dependence of sample qp-ph interaction strengths on $\omega_{2^+_1}$ is plotted. The weakening of this interaction within the extended model, as compared to the interaction derived within the QRPA, is getting more salient as the ground state correlations increase. It is also worth noting that the strengths in the backward direction depend not only on the structure of the phonon state $|\lambda i\rangle$ building the matrix element $W(Jj\lambda i)$ but also on all other phonons entering into the sum in the second summand of the r.h.s. of equation (39) . This implies that the higher-lying phonon states influence the properties of the states near the ground state. We estimated that the contribution of the higher-lying phonons to the quantities $W(Jj\lambda i)$ can be up to 25%. The diagonal matrix elements $K_J(j\lambda i|j'\lambda' i')$ exhibit a similar effect due to the second summand of the r.h.s. of (40). This sum generates an energy shift, which can contribute to the appearance of intruder states in the lower part of the energy spectrum as shown in [16, 17].

In [16] we found out that, within the model based on QRPA, the decrease in energy of the first 2^+ state leads to a considerable growth of the quantities $W(Jj\lambda i)$, thus pushing the first solution very close to the first qp×ph pole. This did not allow us to correctly reproduce the properties of both the odd-even nucleus and its even-even core using the same values for the multipole constants $\kappa^{(\lambda)}$ and correspondingly $\omega_{\lambda_1^\pi}$. We noticed that the values of $\omega_{2_1^+}$ in the even-even core, which let us reproduce the energies of the lowest part of the spectrum in the odd-even nucleus with reasonable accuracy, were much higher than their experimental counterparts. The reason for this we attribute to the large values of the $W(Jj\lambda i)$ matrix elements. In this regard the weakened interaction between the pure qp and qp×ph configurations (38) and (39), caused by the quasiparticle blocking yields, as will be shown in the sequel, better agreement between the theory and experiment.

One important application of this extended model is in reproducing the reduced transition probabilities given by the formula

$$B(E\lambda; J_1\nu_1 \to J_2\nu_2) = \frac{1}{\pi_{J_1}^2}\Bigg(C_{J_1\nu_1}C_{J_2\nu_2}e_{np}f^{\lambda}_{J_1J_2}v^{-}_{J_1J_2} + \sum_i U(J_1\nu_1J_2\nu_2\lambda i)\sqrt{B(E\lambda; g.s. \to \lambda_i)}\Bigg)^2 . \quad (41)$$

The coefficients $U(J_1\nu_1J_2\nu_2\lambda i)$ here represent not only transitions between pure states, excited with respect to an uncorrelated ground state, but also additional transitions between pure states existing in the ground states of even-even nuclei. The explicit form of these coefficients is

$$U(J_1\nu_1J_2\nu_2\lambda i) = \frac{\pi_{J_1}}{\pi_\lambda}[C_{J_2\nu_2}D_{J_2\lambda i}(J_1\nu_1) - E_{J_2\nu_2}F_{J_2\lambda i}(J_1\nu_1)][1+\mathcal{L}(J_1J_2\lambda i)] + (-1)^{J_1-J_2+\lambda}\frac{\pi_{J_2}}{\pi_\lambda}[C_{J_1\nu_1}D_{J_1\lambda i}(J_2\nu_2) - E_{J_1\nu_1}F_{J_1\lambda i}(J_2\nu_2)][1+\mathcal{L}(J_2J_1\lambda i)]. \quad (42)$$

In systems where the last particle is a neutron we make the approximation:

$$B_{odd}(E\lambda; J_1\nu_1 \to J_2\nu_2) = \frac{1}{\pi_{J_1}^2}\left[\sum_i U(J_1\nu_1J_2\nu_2\lambda i)\sqrt{B(E\lambda; g.s. \to \lambda_i)}\right]^2 \approx \approx \frac{1}{\pi_{J_1}^2}U^2(J_1\nu_1J_2\nu_2\lambda 1)B(E\lambda; g.s. \to \lambda_1), \quad (43)$$

which stems from the fact that the coefficients $U(J_1\nu_1 J_2\nu_2\lambda i)$ are non-negligible for the lowest-lying states only and out of these states the transition to the first excited state is the strongest.

4. NUMERICAL RESULTS

In this chapter we quantify the ground-state correlations effects in both even-even and odd-even nuclei, based on the formalism presented in the previous sections. The nuclei in consideration are those from the barium region with $A \sim 130$. In the following we give technical details on the calculations carried out for even-even nuclei.

The single-particle basis and energies are obtained using the Woods-Saxon potential (5) for simplicity and the all bound and quasibound states with energies from the bottom of the well up to 10 MeV are included in the calculations. The parameters, entering this potential, are fitted to reproduce the nuclear binding energies. In a similar way, the pairing strengths G_τ are obtained to match the odd-even mass differences.

From the computational perspective solving the algebraic system of equations (9)-(13) is a more challenging task than solving the equations of the one-phonon standard QPM. As an initial approximation to the solution of the coupled problem we take the solutions obtained from the uncoupled equations (*i.e.* $\rho_j = 0$). One important difference between ERPA and QRPA is that the equations (13), determining the quasiparticle densities in the ground state ρ_j, link phonons with all multipolarities. In other words, the properties of a given phonon state $|\lambda\mu i\rangle$ depend on all phonons with multipolarities $\lambda' \neq \lambda$. The results in Ref. [13] show, however, that it is sufficient to include in the calculations the first quadrupole and octupole phonons only.

The upshot of the comparative study for the reduced transition probabilities $B(E2|0_1^+ \to 2_1^+)$ in several even-even nuclei within QRPA and ERPA is presented in figure 4. The nuclides in this figure are chosen so that their shapes are close to spherical, having $E(4_1^+)/E(2_1^+) < 2.5$. From this figure one can see that the blocking effect due to the Pauli principle exerts a large impact on this measurable quantity. The apparent superiority of ERPA in this region serves as a motivation to study odd-even systems with a core described within the framework of this model (see formula (43)). A further argument to adapt the extended approximation to odd-even nuclei is that in reproducing the low-lying energy spectrum in these nuclei, the energies of the first quadrupole phonons in the corresponding even-even cores would come closer to the experimental values than what we obtained for the models, based on QRPA [16]. This assumption is fed by the weakened interaction strengths $W(Jj\lambda i)$ within ERPA (conf. figure 3), which in turn, are largely responsible for the level ordering.

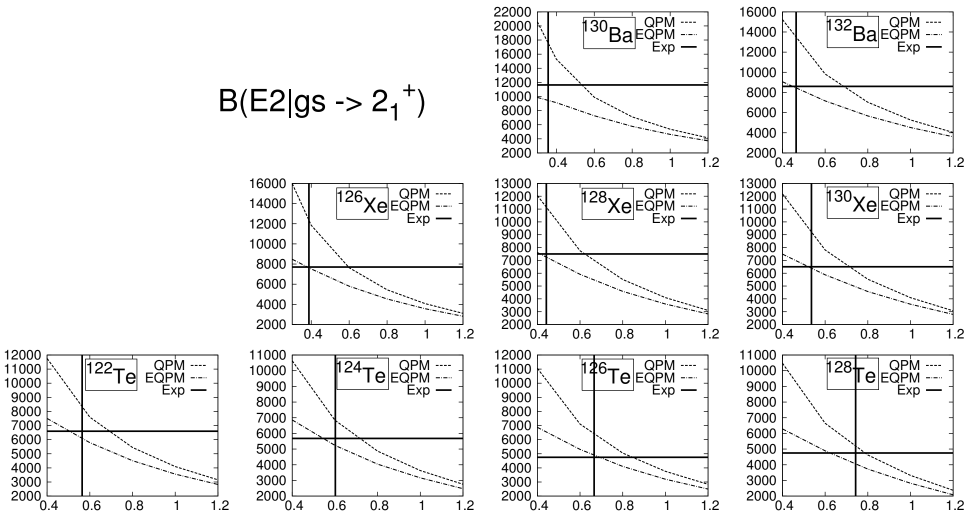

Fig. 4 – The reduced transition probabilities $B(E2|0_1^+ \to 2_1^+)$ (in units e^2fm^4) in several Te, Xe and Ba isotopes plotted against the energy $\omega_{2_1^+}$ of the first quadrupole phonon. The dashed lines represent the experimental energies and transitions.

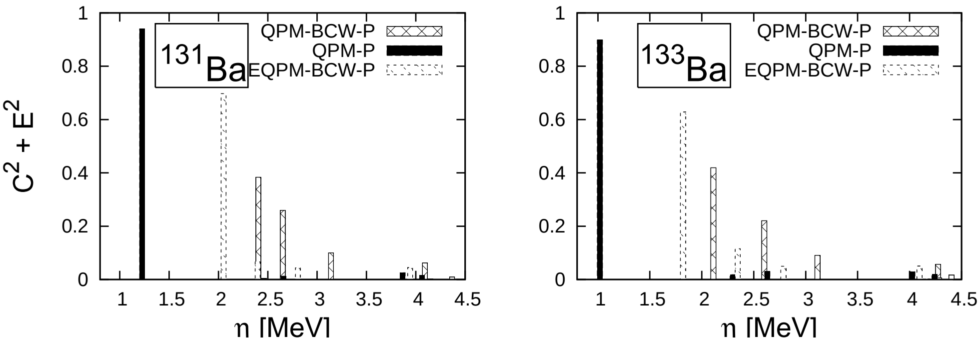

Fig. 5 – Quasiparticle strength distribution $(C^2 + E^2)$ of the state $\nu 2d5/2$ in ^{131}Ba and ^{133}Ba. The quadrupole-quadrupole interaction strength $\kappa^{(2)}$ is kept constant in the calculations within the three model versions.

In the numerical calculations for odd-even nuclei we included quadrupole phonons only, since from one side the quadrupole-quadrupole interaction, along with the pairing one, plays a dominant role for the low-lying collective states in even-even nuclei and from the other - we wanted to have a minimal number of free parameters in the calculations in order to elucidate the effects in a clearer form. In keeping with this, we let the quadrupole strength $\kappa^{(2)}$, correspondingly $\omega_{2_1^+}$, vary and analyze the dependence of the quantities of interest on $\omega_{2_1^+}$. The calculations performed include phonons with energies of up to 15 MeV.

For reasons of conciseness, we introduce the following notations, indicating different variants of the model for odd-A nuclei:

- QPM_P - one-phonon model, including Pauli principle corrections (as in [17])
- QPM_BCW_P - one-phonon model, including backward amplitudes and Pauli principle corrections using QRPA (as in [16])
- EQPM_BCW_P - one-phonon model, including both backward amplitudes and Pauli principle corrections having a core described within ERPA.

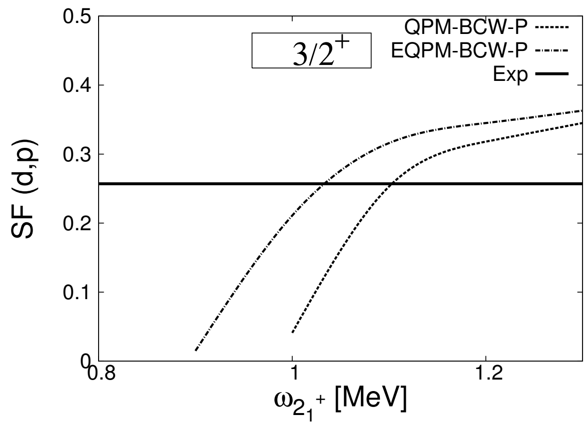

Fig. 6 – Spectroscopic factor for the (d,p) reaction in ^{131}Ba as function of $\omega_{2_1^+}$.

We first head off to investigate the single particle components of the wave function 16. In the model versions which take into account the backward amplitudes we found serious depletion of the quasiparticle strengths, as exemplified in figure 5, for the case of the qp state $\nu 2\mathrm{d}_{5/2}$. We found a similar behavior for the rest of the states from the valence shell in all nuclei from the considered region. Direct measurable quantities related to the quasiparticle fragmentation are the spectroscopic factors for the single-particle transfer reactions given by

$$S_{J\nu}^{(d,p)} = (C_{J\nu}u_J - E_{J\nu}v_J)^2, \quad S_{J\nu}^{(d,t)} = (C_{J\nu}v_J + E_{J\nu}u_J)^2. \tag{44}$$

From figure 6 we see that the value of $\omega_{2_1^+}$, at which the experimentally measured spectroscopic factor is reproduced, is lower in the case of EQPM_BCK_P than in QPM_BCK_P by about 50 keV and is therefore closer to the energy of the first 2^+ state in ^{130}Ba.

Another instructive argument supporting the backward vertices and ERPA is given by the reduced electric transition probabilities. While the spectroscopic factors are influenced mainly by the properties of the last, unpaired particle, the electric transition probabilities depend strongly on the bulk properties of the even-even core. The largest contribution to these quantities is due to transitions between pure qp and qp$\times$ph states represented by the sum in the right-hand side of formula (41). We make use of the approximate expression (43) because the last particle in the considered isotopes is a neutron.

The dependence $B_{odd}(E2|3/2_1^+ \to 1/2_1^+) = B_{odd}(E2|3/2_1^+ \to 1/2_1^+)(\omega_{2_1^+})$ is plotted in figure 7 within the three model versions. This function shows an almost linear behavior in the case of QPM_P, while in the calculations which take into account the backward amplitudes a peak emerges. This peak is a result of the increased fragmentation in the latter pair of model versions (cf. figure 5) which contributes to the enhanced values of the coefficients $U(J_1\nu_1 J_2\nu_2\lambda i)$. As a result, the maximum value of the presented transition probabilities in EQPM_BCK_P and QPM_BCK_P is about three times as large as the maximum value obtained within QPM_P bringing us closer to the experimental values. It is also worth noting that the values of $\omega_{2_1^+}$, which correspond to the peak values obtained within EQPM_BCK_P, are about 100keV lower than in QPM_BCK_P. We therefore conclude that the effect of the renormalization yields better results with respect to the experimentally measured energy of the corresponding even-even core though it is still rather higher from it.

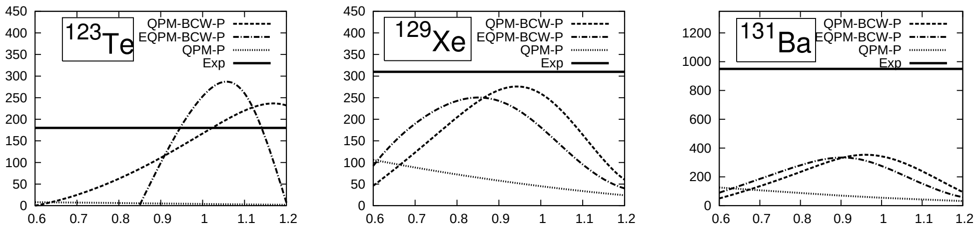

Fig. 7 – Same as figure 4 but for $B(E2|3/2_1^+ \to 1/2_1^+)$ in ^{123}Te, ^{129}Xe and ^{131}Ba.

5. SUMMARY

The primarily goal of the present work is the development of an approach to study the effects due to the ground state correlations on the properties of the low-lying states in odd-even spherical and transitional nuclei. To this end we combine two improvements to the conventional QPM model:

- extended the configurational space so as to allow for quasiparticle and quasiparticle+phonon states to reside in the even-even nuclear ground states
- applied the ERPA to calculate the elements of the energy matrix in the extended configurational space.

In all calculations, the effect of the Pauli principle on certain three-quasiparticle configurations is taken into account by applying the exact commutation relations between the quasiparticle and phonon operators. Utilizing these techniques, we derived quasiparticle-phonon interaction strengths in the forward and backward directions within both QRPA and ERPA using a Hamiltonian with two residual interaction modes - pairing and long-range interactions.

A comparison between models, built on QRPA and ERPA, is established on the basis of the reduced transition probabilities from the ground to the first 2^+ state in even-even nuclei with $A \sim 130$. The results of this study show a superiority of the model, based on ERPA, in describing these quantities, which serves as a motivation to further apply the extended model in investigating the transition probabilities in several odd-even Te, Xe and Ba nuclei for which experimental data are available. The results from the numerical calculations indicate improvements, coming mainly from the increased fragmentation, due to the qp$\times$ph admixtures in the ground state, while the quasiparticle blocking happens to be less important in the nuclei in consideration. An enhancement of this result, due to ERPA, is that the theoretical values, matching their experimental counterparts best, are obtained at strengths of the quadrupole-quadrupole interaction which yield better agreement between the energy of the first quadrupole phonon and the experimental values for the energy of the first 2^+ state in the corresponding even-even cores. It is argued that this effect is a result of the weakened quasiparticle-phonon interaction due to the quasiparticle blocking.

REFERENCES

1. V. Soloviev, *Theory of Complex Nuclei* (Pergamon, Oxford, 1976).
2. J. Bardeen, L. N. Cooper, J. R. Schrieffer, Phys. Rev. **108**(5), 1175-1204 (1957).
3. V. G. Soloviev, Nucl. Phys. **9**(4), 655-664 (1958).
4. S. Belyaev, Mat. Fys. Medd. Dan. Vid. Selsk. **31**, 11 (1959).
5. D. J. Thouless, Nucl. Phys. **22**(1), 78-95 (1961).
6. G. E. Brown, J. A. Evans, D. J. Thouless, Nucl. Phys. **45**, 164-176 (1963).
7. J. D. Providencia, Nucl. Phys. A **108**, 589-608 (1968).
8. H. Lenske, J. Wambach, Phys. Lett. B **249**(3-4), 377-380 (1990).
9. J. Dukelsky, P. Schuck, Phys. Lett. B **387**, 233-238 (1996).
10. K. Hara, Prog. Theor. Phys. **32**, 88 (1964).
11. K. Ikeda *et. al.*, Prog. Theor. Phys. **33**, 22 (1965).
12. R. V. Jolos and W. Rybarska, Z. Phys. A **296**, 73 (1980).
13. D. Karadjov, V. V. Voronov and F. Catara, Phys. Lett. B **306**, 197 (1993).
14. T. T. S. Kuo, E. U. Baranger, M. Baranger, Nucl. Phys. A **79**, 513-549 (1965).
15. V. V. der Sluys, D. V. Neck, M. Waroquier, J. Ryckebusch, Nucl. Phys. A **551**(2), 210-240 (1993).
16. S. Mishev and V. V. Voronov, Phys. Rev. C **78**, 024310 (2008).
17. C. Z. Khuong, V. G. Soloviev, V. V. Voronov, J. Phys. G **7**, 151-163 (1981).

18. D. Rowe, *Nuclear Collective Motion* (Menthuen, London, 1970).
19. A. Bohr, B. Mottelson, *Nuclear structure* (vol. 2, Benjamin, New York, 1975).