Dedicated to Academician Aureliu Sandulescu's 80th Anniversary

COLLECTIVE DIPOLE MODES IN NUCLEAR SYSTEMS

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In this work we explore some new features of dipole dynamics in nuclear systems. Within a harmonic oscillator shell model, we generalize an approach introduced first by Brink and identify the dipole normal modes in neutron rich nuclei. Consequently, we obtain an upper limit for the energy weighted sum rule exhausted by the pygmy dipole resonance. By solving numerically the Landau-Vlasov kinetic equations for neutrons and protons with specific initial conditions, we study the structure of the dipole vibrations in neutron rich nucleus ^{132}Sn identified in the previous model. Our calculations point out the existence of a distinctive collective dipole mode with an energy well below giant dipole resonance. In the last part, based on the same microscopic transport model, we focus on the charge equilibration dynamics in fusion reactions and evidence its relation to the excitation of a large amplitude dipole motion in the entrance channel.

Key words: Pygmy dipole resonance, giant dipole resonance, Landau theory of Fermi liquids, symmetry energy.

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1. INTRODUCTION

One of the important tasks in nuclear many-body physics is to describe the emergence of collective features, as well as their structure in terms of the individual motion of constituents. Giant Dipole Resonance (GDR) is one of the most prominent collective motions, present in all nuclei, whose centroid position varies as $80A^{-1/3}$MeV, while the width takes values around $4-5$ MeV at $T=0$ MeV. It was intensively studied and considered as a precious tool to probe properties of nuclei in extreme conditions of deformation or temperature. Until recent times, the $E1$ strength for heavy nuclei was measured only in stable nuclei with moderate neutron excess. The steady progress of experimental methods of investigation opens now the possibility to study very neutron rich nuclei, at the limits of stability.

New exotic collective excitations show up when one moves away from the valley of stability [1]. Their experimental characterization and theoretical description is a challenge for modern nuclear physics. Recent experiments provided several evidences about their existence, but the available information is still incomplete and their nature is a matter of debate.

An interesting exotic mode is the pygmy dipole resonance (PDR) which was observed as an unusual concentration of dipole response at energies clearly below the values associated to giant dipole resonance (GDR). Adrich *et al.* [2] reported the observation of a resonant-like shape distribution with a pronounced peak around 10 MeV in ^{130}Sn and ^{132}Sn isotopes exhausting few percentages from the energy-weighted sum rule (EWSR). In general, the corresponding sum rule increases with the proton-to-neutron asymmetry. This behavior was related to the symmetry energy properties and therefore, connected to the size of neutron skin [5], [6]. However, other theoretical analysis suggests a weak connection between the PDR and skin thickness [7].

In spite of theoretical progress in the interpretation of this mode within phenomenological studies based on hydrodynamical equations [8–10] within non-relativistic microscopic approaches using random phase approximation (RPA) with various effective interactions [11–13] or relativistic quasi-particle RPA [14, 15] and new experimental informations [17–21], one still has to answer a number of critical questions concerning the nature of Pygmy Dipole Resonance like the macroscopic picture of neutrons and protons vibrations, the exact location of PDR excitation energy and the degree of collectivity of low-energy dipole states, the role of symmetry energy [22]. Some microscopic studies predict large fragmentation of GDR strength and the absence of collective states in the low-lying region in ^{132}Sn [23].

In the first part, within the harmonic oscillator shell model for neutron rich nuclei, we identify the possible oscillation modes of neutrons core, protons core and excess neutrons. We show that the coordinates associated to neutron excess vibration against the core and to the dipole core mode are separable and obtain an upper limit for the TRK (Thomas-Reiche-Kuhn) sum-rule exhausted by each of them. Then, we adopt a description based on Fermi liquid theory and Landau-Vlasov kinetic equation to investigate the dynamics and the interplay between these dipole modes. The self-consistent treatment allows us to inquire on the role of symmetry energy upon the dipole response of neutron rich nuclei and predict the centroid energy of each collective motion.

In the second part of this work, based on the same transport approach, we shall enquire on the possibility to observe the excitation of the pre-equilibrium giant dipole resonance along the fusion path at low beam energies. Exotic nuclear beams are a good opportunity for studying these phenomena.

2. PYGMY DIPOLE RESONANCE IN NEUTRON RICH NUCLEI

We investigate the motion of protons core, neutrons core and excess neutrons, respectively, for a nucleus with charge Z and N neutrons. The position vectors of the center of mass of Z_c core protons, of N_c core neutrons and of the center of mass of the N_e excess neutrons, $\vec{R}_{p_c}$, $\vec{R}_{n_c}$ and $\vec{R}_{n_e}$, are:

$$\vec{R}_{p_c} = \frac{1}{Z_c}\sum_{i=1}^{Z_c}\vec{r}_i; \quad \vec{R}_{n_c} = \frac{1}{N_c}\sum_{i=1}^{N_c}\vec{r}_i; \quad \vec{R}_{n_e} = \frac{1}{N_e}\sum_{i=1}^{N_e}\vec{r}_i, \tag{1}$$

where r_i are nucleon coordinates, $Z = Z_c$ and $N = N_c + N_e$. Then, the center of mass (CM) position vector is:

$$\vec{R}_{CM} = \frac{1}{A}(Z_c\vec{R}_{p_c} + N_c\vec{R}_{n_c} + N_e\vec{R}_{n_e}). \tag{2}$$

The dynamics of various dipole modes is described in terms of the following collective coordinates, related to the quantities defined above: the distance between the center of mass of protons core and the center of mass of neutrons core $\vec{X}_c$:

$$\vec{X}_c = \vec{R}_{p_c} - \vec{R}_{n_c}, \tag{3}$$

the distance between the center of mass of the core and the center of mass of excess neutrons $\vec{Y}$:

$$\vec{Y} = \frac{N_c}{N_c+Z_c}\vec{R}_{n_c} + \frac{Z_c}{N_c+Z_c}\vec{R}_{p_c} - \vec{R}_{n_e}, \tag{4}$$

as well as the distance between the protons center of mass and all neutrons center of mass:

$$\vec{X} = \vec{R}_{p_c} - \vec{R}_n = \vec{R}_{p_c} - \frac{N_e}{N}\vec{R}_{n_e} - \frac{N_c}{N}\vec{R}_{n_c}. \tag{5}$$

We remark that these three degrees of freedom are not independent:

$$\vec{X} = \frac{N_e}{N}\vec{Y} + \frac{N_c A}{N A_c}\vec{X}_c, \quad \text{where } A_c = N_c + Z_c. \tag{6}$$

In a seminar paper [24], Brink has shown that for a system of $A = N + Z$ nucleons moving in a harmonic oscillator well with the Hamiltonian:

$$H_{sm} = \sum_{i=1}^{A}\frac{\vec{p}_i^{\,2}}{2m} + \frac{K}{2}\sum_{i=1}^{A}\vec{r}_i^{\,2}, \tag{7}$$

it is possible to perform a separation in four independent parts:

$$H_{sm} = H_{n\ int} + H_{p\ int} + H_{CM} + H_D, \tag{8}$$

where H_D:

$$H_D = \frac{A}{2mNZ}\vec{P}^2 + \frac{KNZ}{2A}\vec{X}^2, \tag{9}$$

describes the protons against the neutrons vibration of Goldhaber-Teller type [26]. The first two terms in Eq. (8) determine the internal motion of protons and neutrons, respectively, depending only on proton-proton and neutron-neutron relative coordinates. The Hamiltonian H_{CM}:

$$H_{CM} = \frac{1}{2Am}\vec{P}_{CM}^2 + \frac{KA}{2}\vec{R}_{CM}^2, \tag{10}$$

characterizes the nucleus center of mass motion.

In these expressions were introduced the conjugate momentum to the neutron-proton relative coordinate $\vec{X}$:

$$\vec{P} = \frac{NZ}{A}\left(\frac{1}{Z}\vec{P}_Z - \frac{1}{N}\vec{P}_N\right), \tag{11}$$

as well as the conjugate momentum of the center of mass coordinate:

$$\vec{P}_{CM} = \vec{P}_Z + \vec{P}_N. \tag{12}$$

Here the protons and neutrons total momenta:

$$\vec{P}_Z = \sum_{i=1}^{Z}\vec{p}_i; \quad \vec{P}_N = \sum_{i=1}^{N}\vec{p}_i, \tag{13}$$

are canonically conjugate to the collective coordinates $\vec{R}_p = \frac{1}{Z}\sum_{i=1}^{Z}\vec{r}_i$ and $\vec{R}_n = \frac{1}{N}\sum_{i=1}^{N}\vec{r}_i$, respectively.

Consequently, for this shell model Hamiltonian the states of the nucleus are represented as products of four wave functions $\Psi = \psi_{n\ int}\chi_{p\ int}\alpha(\vec{R}_{CM})\beta(\vec{X})$, simultaneous eigenvectors of the four Hamiltonians constructed above. For an electric dipole absorption, a Goldhaber-Teller collective motion with a specific linear combinations of single particle excitations is produced. Since the dipole operator is a function depending only on $\vec{X}$, the wave function $\beta(\vec{X})$ is changing from ground state to one GDR phonon state. With the help of sum-rule, the total absorption cross section is given by:

$$\sigma_D = \int_0^\infty \sigma(E)dE = \frac{4\pi^2 e^2}{\hbar c}\sum_i E_i|\langle i|D|0\rangle|^2 = \frac{4\pi^2 e^2}{\hbar c}\frac{1}{2}\langle 0|[D,[H_{sm},D]]|0\rangle$$
$$= \frac{2\pi^2 e^2}{\hbar c}\langle 0[D,[H_D,D]]0\rangle = \frac{2\pi^2 e^2\hbar}{mc}\frac{NZ}{A} = 60\frac{NZ}{A}\text{ mb MeV}.$$

If the physical situation allows to treat the nuclear system as composed of three subsystems, neutrons core, protons core and excess neutrons, as in the case of very neutron rich nuclei, an exact separation of the shell model Hamiltonian in a sum of six independent (commuting) quantities can be again performed:

$$H_{sm} = H_{n_c\ int} + H_{p_c\ int} + H_{e\ int} + \frac{1}{2Am}\vec{P}^2_{CM} + \frac{KA}{2}\vec{R}^2_{CM} + \frac{A_c}{2Z_cN_cm}\vec{P}^2_c + \frac{KN_cZ_c}{2A_c}\vec{X}^2_c + \frac{A}{2A_cN_em}\vec{P}^2_y + \frac{KN_eA_c}{2A}\vec{Y}^2 = H_{n_c\ int} + H_{p_c\ int} + H_{e\ int} + H_{CM} + H_c + H_y \ . \tag{14}$$

Here, the terms $H_{n_c\ int}$, $H_{p_c\ int}$ and $H_{e\ int}$ contain only relative coordinates and momenta among the nucleons of each subsystem and, as before, describe their internal motion. H_c characterizes the core isovector vibration while the relative motion of excess neutrons against the core, the so-called pygmy mode, is determined by H_y. The canonically conjugate momenta corresponding to these degrees of freedom are:

$$\vec{P}_c = \frac{N_cZ_c}{A_c}\left(\frac{1}{Z_c}\vec{P}_{Z_c} - \frac{1}{N_c}\vec{P}_{N_c}\right) \ , \tag{15}$$

$$\vec{P}_y = \frac{N_eA_c}{A}\left(\frac{1}{A_c}(\vec{P}_{Z_c} + \vec{P}_{N_c}) - \frac{1}{N_e}\vec{P}_{N_e}\right) \ , \tag{16}$$

$$\vec{P}_{CM} = \vec{P}_Z + \vec{P}_{N_c} + \vec{P}_{N_e} \ . \tag{17}$$

The total wave function is now a product of the eigenfunctions associated to each of these motions. Three of them characterize the internal state of each subsystem and one is related to the nucleus center of mass motion. The remaining two are eigenstates of H_c and H_y, respectively, describing two independent collective dipole excitations. Both of them will contribute to the dipole response since, with (6), the total dipole momentum can be expressed as:

$$\vec{D} = \frac{NZ}{A}\vec{X} = \frac{Z_cN_c}{A_c}\vec{X}_c + \frac{Z_cN_e}{A}\vec{Y} \equiv \vec{D}_c + \vec{D}_y \ . \tag{18}$$

In this picture, the excitation of the pygmy mode results in a collective motion of Goldhaber-Teller type with the excess neutrons oscillating against the core. The dipole absorption leads also to the change of the wave function depending on the relative coordinate $\vec{Y}$. The total cross section for the pygmy dipole excitation, in term of the corresponding TRK sum-rule, is:

$$\sigma_y = \int_0^\infty \sigma_y(E)dE = \frac{4\pi^2e^2}{\hbar c}\sum_i E_i|\langle i|D_y|0\rangle|^2 \frac{4\pi^2e^2}{\hbar c}\frac{1}{2}\langle 0|[D_y,[H_{sm},D_y]]|0\rangle = = \frac{2\pi^2e^2}{\hbar c}\langle 0|[D_y,[H_y,D_y]]|0\rangle = \frac{N_eZ_c}{NA_c}\frac{2\pi^2e^2\hbar}{mc}\frac{NZ}{A} = \frac{N_eZ_c}{NA_c}\sigma_D \ . \tag{19}$$

This calculation shows that a fraction $f_y = \dfrac{N_e Z_c}{N A_c}$ of the TRK energy-weighted sum rule is exhausted by the pygmy mode. This result is consistent with the molecular sum rule obtained by Alhassid *et al.* [25]. In the case of tin isotope ^{132}Sn, with $N_e = 32$, $A_c = Z_c + N_c = 100$, $f_y = 19.5\%$ is obtained. This value is greater than the value estimated experimentally which is around 5%. A possible explanation for the difference between our estimation and the experimental observations is that actually only part of excess nucleons, $N_{eff} < N_e$, will contribute to pygmy resonance. It may be assumed that a core with 50 protons and 50 neutrons is quite unstable and an additional number of neutrons are still bound to the core. To carry only 5% of TRK sum-rule we should assume that a number of $N_{eff} = 10$ neutrons is entering in the expression for f_y, while the rest of 22 excess neutrons belong to the core, see also [2].

A more accurate picture of the Giant Dipole Resonance in nuclei corresponds to an admixture of Goldhaber-Teller and Stenweidel-Jensen vibrations. The latter is a volume type oscillation of the isovector density $\rho_i = \rho_n - \rho_p$ keeping the total density $\rho = \rho_n + \rho_p$ constant [27]. A microscopic, self-consistent study of various dipole collective features of nuclear systems can be performed within Landau theory of Fermi liquids. This is based on two coupled Landau-Vlasov kinetic equations for neutrons and protons one-body distribution functions $f_q(\vec{r},\vec{p},t)$ with $q = n,p$:

$$\frac{\partial f_q}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial f_q}{\partial \mathbf{r}} - \frac{\partial U_q}{\partial \mathbf{r}}\frac{\partial f_q}{\partial \mathbf{p}} = I_{coll}[f]. \tag{20}$$

Such a model was applied quite successfully in describing various features of GDR, including their properties in hot nuclei [28] and pre-equilibrium excitation in fusion entrance channel [29], or in the description of transition from isovector zero sound to first sound mode in nuclear matter [30]. Recently, within a linear response approach, it was also considered to investigate some properties of pygmy mode [31,32].

The collision integral $I_{coll}[f]$ for fermionic systems, responsible for the collisional damping at finite temperature, is set here equal to zero. The dynamics is dictated by the mean-field for which we consider a Skyrme-like parametrization:

$$U_q = A\frac{\rho}{\rho_0} + B\left(\frac{\rho}{\rho_0}\right)^{\alpha+1} + C(\rho)\frac{\rho_n - \rho_p}{\rho_0}\tau_q + \frac{1}{2}\frac{\partial C}{\partial \rho}\frac{(\rho_n - \rho_p)^2}{\rho_0}, \tag{21}$$

where $\tau_q = +1$ for $q = n$ and $\tau_q = -1$ for $q = p$. The values of the coefficients appearing in Eq. (21) are $A = -356$ MeV, $B = 303$ MeV, $\alpha = 1/6$. The effective nucleon mass is equal to the bare mass, 940 MeV. Then, the saturation density of symmetric nuclear matter is 0.16 fm^{-3}, the binding energy is $E_B = -16$ MeV/n, while the compressibility modulus takes the value $K = 200$ MeV. For the isovector sector we employed various parametrizations with density, see [33]. For the calculations presented here we select the asysoft case with $\frac{C(\rho)}{\rho_0} = 482 - 1638\rho$.

The numerical procedure to integrate the transport equations is based on pseudo-particle (or test-particle, t.p.) method. For a good spanning of phase-space we work with 1500 t.p. *per* nucleon. We consider the neutron rich nucleus ^{132}Sn. To inquire on the collective properties of pygmy dipole, after the ground state preparation, at $t = t_0$ (we considered $t_0 = 30$ fm/c), we excite the nuclear system by boosting along z direction all excess neutrons and in opposite direction all core nucleons, while keeping the center of mass of the nucleus at rest. The excess neutrons were identified just before the perturbation as the most distant $N_e = 32$ neutrons from the nucleus center of mass. Further, the system is left to evolve and the time evolution of the quantities D_y, D_c and D, associated to different dipole modes is followed for 600 fm/c by solving numerically the equations (20). The results are reported in Figure 1. We observe quite undamped oscillations of the dipole D_y associated to the pygmy mode coordinate Y. Moreover, as a departure from a simplified picture provided by various phenomenological models, we also remark the appearance of an oscillatory motion of the core coordinate X_c and during the coupled motions, the three coordinates Y, X_c, X verify the relation (6).

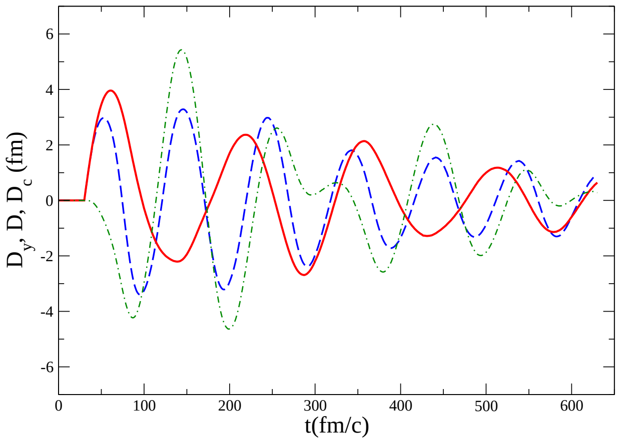

Fig. 1 – The time evolution of the total dipole momentum D (blue, dashed line), of the pygmy dipole momentum D_y (red, solid line) and of the core dipole momentum D_c (green, dot-dashed) for asysoft EOS.

In Fig. 1 the pygmy dipole moment D_y and of the total dipole moment D should stay equal if the core remains inert, in agreement with Eq. (18). Our self-consistent simulations provide, however, a different scenario. While the pygmy dipole approaches its maximum value, an dipolar motion of the core initiate. This core vibration is amplified while the oscillation of the excess neutron against the charge-symmetric bulk is pursuing with its own frequency. To estimate the energy associated to each collective dipole motion, we calculated the power spectrum of D_y:

$$|D_y(\omega)|^2 = \left| \int_{t_0}^{t_{max}} D_y(t) e^{-i\omega t} dt \right|^2 , \tag{22}$$

and similarly for D. The results are shown in Fig. 2. The position of the centroids corresponding to the giant dipole resonance shifts toward larger values (around 14 MeV for this isovector interaction), while the energy centroid associated to pygmy dipole is situated well below the GDR peak, at around 8.5 MeV. A conclusion consistent with this observations was also reported within an relativistic mean-field approach, see [35]. Therefore, the skin thickness seems to play a minor influence on the position of the pygmy resonance, but can affect the amount of exhausted TRK sum-rule.

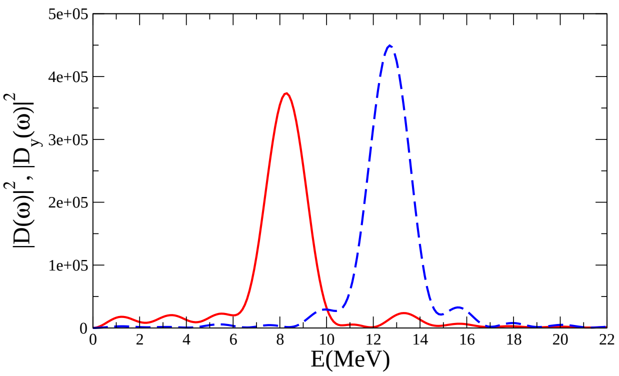

Fig. 2 – The power spectrum of total dipole momentum D (blue, dashed line) and of the pygmy dipole D_y (red, solid line). The spectra correspond to asysoft EOS choice.

3. DIPOLE MODE IN FUSION DYNAMICS

In the early stages of heavy ion collisions at Fermi energies, large amplitude collective motions can be excited, including monopole or quadrupole modes. If the colliding nuclei have different N/Z ratio, the charge equilibration initiates and several works suggested that the this process has the features of the Giant Dipole Resonance [37–42]. Here, we would also like to mention the study of the quantum and statistical aspects of charge equilibration in deep-inelastic collisions performed by prof. A. Sandulescu and his collaborators, see [43]. With the advent of microscopic transport models and their numerical implementation was possible to explore in a self-consistent manner the interplay between fusion dynamics and charge equilibration process and to study the dependence of this process on mass combination, beam energy or impact parameter.

We shall focus on the dynamical dipole mode in entrance channel in a semi-classical approach based on Landau-Vlasov kinetic equation introduced above. We consider the reaction ^{36}Ar + ^{96}Zr at 6 AMeV, 9 AMeV and 16 AMeV. It is observed that after the approaching phase of the two nuclei, when they still keep their own response, follows a dinuclear, non-equilibrium stage, when the conversion of the relative motion energy in thermal motion initiate, see Figure 3.

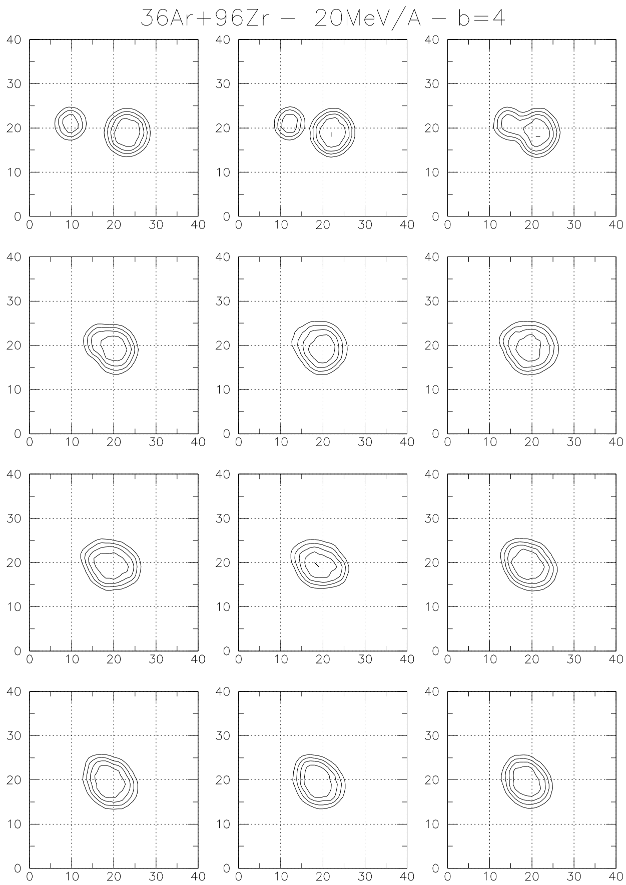

Fig. 3 – The reaction Ar+Zr at 20 AMeV and impact parameter b=4fm. The density contour plots are constructed at time intervals of 20 fm/c.

Along the fusion path, in agreement with Brink-Axel hypothesis, the excitation of a collective state of GDR type on top of a non-equilibrium state may take place. We obtained the time evolution of the total dipole moment $D(t) = \frac{NZ}{A}X(t)$ and of the quantity $DK(t) = \frac{NZ}{A\hbar}\left(\frac{P_p}{Z} - \frac{P_n}{N}\right)$, see Fig. 4. Here $X = R_p - R_n$ is the distance between the centers of mass of protons and neutrons, while $P_p(P_n)$ are the center of mass in momentum space for protons (neutrons). We remark a sudden rising of the latter quantity when the collective dynamics is triggered. The evolution corresponds clearly to a damped oscillator, suggesting the collective character of this mode.

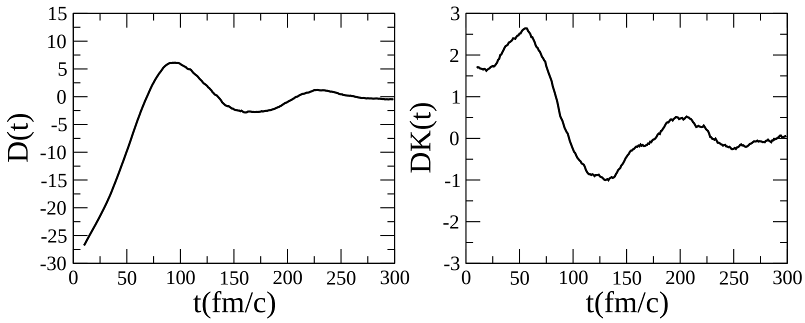

Fig. 4 – The time evolution of the dipole moment $D(t)$ (left panel, in fm units) and of $DK(t)$ (right panel, in fm^{-1} units) for impact parameter $b = 2$fm. See the text.

The total emission probability can be obtained from a bremsstrahlung formula as:

$$\frac{dP}{dE_\gamma} = \frac{2e^2}{3\pi\hbar c^3 E_\gamma}\left(\frac{NZ}{A}\right)^2 |X''(\omega)|^2 \quad , \tag{23}$$

where $X''(\omega)$ is the Fourier transform of the acceleration $X''(t)$. In Fig. 5 we report the photon spectra at different energies and impact parameter. We conclude that the fusion dynamics may favor the dipole emission, of importance being the competition between the time scales for the passage from the dinuclear system to compound nucleus and time scales for collective dipole oscillations, respectively. A slow evolution of dinuclear system may reduce the collective dipole excitation. At the same time, due to collisional damping, the same effect is expected to high beam energies.

In the last decade, several experimental results provided clear evidences for the pre-equilibrium dipole emission along fusion path, see [44–46]. The angular distribution of this radiation, obtained in more exclusive experiments, can be a valuable ”clock” for dinuclear system dynamics.

4. CONCLUSIONS

Summarizing, in the first part of this paper we explored several facets of the dipole motions in neutron rich nuclei. Within a harmonic oscillator shell model, we found an upper limit of the fraction of TRK energy sum-rule carried by the pygmy mode, $f_y = \frac{N_e Z_c}{N A_c}$. For ^{64}Ni, this is around $f_y = 11\%$, while for ^{132}Sn rises at $f_y = 19\%$. However, a comparison with experimental data shows that this expression overestimates the observed values.

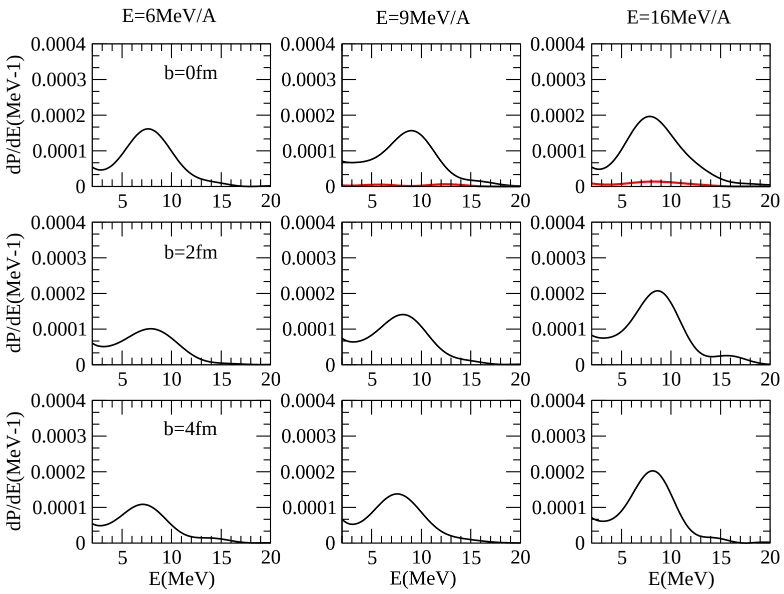

Fig. 5 – The photon spectra for the system Ar+Zr system at three beam energies $E_{beam} = 6, 9$ and 16 AMeV and for three impact parameters $b = 0, 2, 4$ fm. For comparison, the red lines are associated to the total photon probability for a symmetric N/Z combination, forming the same compound nucleus at the same excitation energy.

The difference can be related to a smaller number of neutrons which get involved in the dynamics of this mode. For ^{132}Sn if only 10 neutrons in excess are involved in pygmy mode, the sum rule diminishes until 5%. Within a semi-classical, self-consistent, microscopic approach based on Landau-Vlasov kinetic equations and Skyrme-like mean-fields, was also evidenced the existence of the collective motion of the excess neutrons against the charge-symmetric bulk with a frequency much lower than those associated with giant dipole resonance. A complex pattern, involving coupling of PDR with the core dipole mode was noticed. In the case of the nucleus ^{132}Sn, our simulations predict the position of centroid of pygmy resonance around 8.6 MeV for asy-soft EOS. This observation rises the question of the dependence of the centroid energy of the mode on the parametrization with density of symmetry energy below saturation. Our investigations, using three different parametrizations, show that this mode is rather insensitive to the density dependence of the symmetry energy and to the neutron skin [47].

In the second part we presented an approach which treats self-consistently the fusion dynamics and the isospin degree of freedom equilibration. In this way, a realistic estimate of the pre-equilibrium dipole radiation due to charge asymmetry in entrance channel can be obtained. We mention that the dipole radiation emitted in the entrance channel in fusion reaction can represent a cooling mechanism in "warm" fusion reactions, of interest for the synthesis of heavy elements [48].

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