Dedicated to Academician Aureliu Sandulescu's 80th Anniversary

ON THE RELATIVISTIC FIELD-THEORETICAL THREE DIMENSIONAL EQUATIONS FOR THE COUPLED $Nd-3N$ SYSTEMS WITH AND WITHOUT QUARK-GLUON DEGREES OF FREEDOM.

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The new relativistic three-body equations for the amplitude of the coupled nucleon-deuteron $\Longleftrightarrow$ three nucleon $(Nd \Longleftrightarrow 3N)$ reactions are suggested within the standard field-theoretical S-matrix approach. The suggested equations have the form of the three-body time-ordered and three dimensional Lippmann-Schwinger or Schrödinger equations for the connected part of the three-body amplitudes. These relativistic equations allow to construct the complete set of the three-body forces. The form the suggested equations and their potentials is not dependent on the choice of the Lagrangian. Moreover, in the field-theoretical formulation with and without quark-gluon degrees of freedom the corresponding two-body and three-body equations and potentials have the same form.

The considered field-theoretical equations are three dimensional from the beginning and the potential of these equations can be constructed from the one-variable vertex functions. Moreover, these equations satisfy automatically the unitarity and causality conditions even after truncation of multiparticle intermediate states. Therefore, the considered formulation s free of the principal ambiguities which appear in the 4D Bethe-Salpeter equations and their 3D quasipotential reductions.

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1. INTRODUCTION

The relativistic field theory of the particle and nuclear interactions is the unique theory which allows to construct the microscopic generalization of the Schrödinger equation which is based on the relativity, causality, unitarity and other first principles. The potential of the generalized field-theoretical Schrödinger equations consists of the contributions from the particle creation and annihilation effects and contributions from the contact overlapping) interactions. These contributions determine the particle exchange parts and the many particle forces parts of the microscopic potentials. In the nonrelativistic limit the considered microscopic potentials of the field-theoretical equations reproduce the same expression of the non-local and energy depending relativistic potential. Moreover, the different field-theoretical equations reproduce the same S-matrix and other 3D observables within the formulation with the infinite number of the intermediate particles. Therefore, all results obtained in the framework of the one of the field-theoretical equations is not in contradiction with the other field-theoretical approach.

Presently, there exists a huge number of the field-theoretical equations for the two-body systems in the form of the relativistic Schrödinger or the corresponding Lippmann-Schwinger equations. One can classify these equations using on the different origin and construction rules of the corresponding potentials. In particular, one can distinguish three different kind of the field-theoretical equations with sufficient different roots: Bethe-Salpeter equations [1, 2] and their three-dimensional (3D) quasipotential reductions [3, 4], the 3D Tomonaga-Schwinger equations [5, 6]-[11] and the spectral decomposition (or the generalized unitarity condition) for the scattering amplitude [6]-[27]. The last two kind equations are 3D from the beginning and they are reproduced in the framework of the old perturbation theory [5, 6].

The most popular relativistic field-theoretical equations are the four dimensional (4D) Bethe-Salpeter equations [1,29] and their 3D (quasipotential) reductions. The potential of the manifestly Lorentz-covariant Bethe-Salpeter equation and their 3D (quasipotential) reductions [3, 4] consists of the infinite number of the Feynman diagrams. The two-body Bethe-Salpeter equation contain the nonphysical two time variable [3] and need 3D reduction because the cross sections and other observables are given in the 3D Fock space. There exists an infinite 3D reductions of the 4D Bethe-Salpeter equations and any of the 3D quasipotential equations have the different propagators and potentials. Therefore, the practical calculations of the different quasipotential equations with the truncated number of the Feynman diagrams produces the different solutions. An other kind principal ambiguity of the Bethe-Salpeter equations and their quasipotential reductions arise by construction of the potential from the input vertices, because the vertices in the Bethe-Salpeter equations are functions of the three variable and the sought amplitudes of the two-body reactions are the functions of the two variable, *i.e.* one uses more input functions as one can reproduce using the output observables.

An other derivation of the relativistic Lippmann-Schwinger equation based on the field-theoretical generalization of the Schrödinger equation in the form of the functional Tomonaga-Schwinger equations [5, 6]. In the framework of this method a covariant Hamiltonian theory for construction of the relativistic three-dimensional equations was given in Refs. [7,8]. In this approach the covariant equations are three-dimensional from the beginning, and therefore they are free from the ambiguities of the three-dimensional reduction. The potential in these equations is constructed from the vertex functions with all particles on mass shell, *i.e.*, the related "input" vertex functions are also on three variables. Apart from this problem in these equations one has the non-physical "*spurious*" degrees of freedom. Light-front reformulation of these equations was done in Refs. [9, 10]The generalization of the Faddeev equations within this formulation was done in [11].

The modern three-body formulation of the Bethe-Salpeter equations was given in [13,14]. This formulation was performed for the Yukawa (ϕ^3) interactions only *i.e.* the overlapping (contact) four-point (ϕ^4), five-point (ϕ^4) and six-point (ϕ^6) forces was omitted. Besides in this four-dimensional formulation arise an additional the difficulties with the unitary and the current conservations.

The third kind of the three-dimensional Lippmann-Schwinger type integral equations in the quantum field theory is based on the field-theoretical spectral decomposition of the amplitude over the complete set of the asymptotic "*in(out)*" states The resulting equation within this approach have the form of the off shell unitarity conditions. This formulation, suggested first by F. Low [15] in the framework of the old perturbation theory, was developed afterwards in [6, 16–21, 27]. In particular, in [6, 16, 17] these three-dimensional, time-ordered and quadratically nonlinear equations were used for the evaluation of the πN scattering amplitudes. In the [18–20] the analytic linearization procedures of these equations were suggested. The linearized equations have the form of relativistic Lippmann-Schwinger type equations and they were employed for the calculation of the low energy πN and NN scattering reactions. The electromagnetic dipole magnetic moments of the Δ resonances and the cross sections of the πN bremsstrahlung $\pi N \to \gamma'\pi' N'$ was calculated within this approach in [23–25]. The extension of the present approach in the framework of the field theoretical approach with quark degrees of freedom was done in [20, 21, 23, 26]. In particular, it was demonstrated that the generalized unitarity conditions with and without quark-gluon degrees of freedom have the same form and the quark-gluon propagators in the intermediate states does not destroy the unitarity condition for the hadron scattering reactions. The three-body generalization of the considered approach for the $\pi N - \pi\pi N - \gamma\pi N$ coupled channels was given in [27]. The corresponding two-body and three-body equations are also three-dimensional and time ordered from the beginning and their potentials are constructed from completely dressed matrix elements with two on mass shell particles. Using the on-mass shell methods, such as the dispersion relations, sum rules, current algebra etc. one can determine the required "input" one-variable vertices. The total potential of the considered equations consists of the two parts: 1) the on mass shell particle exchange amplitudes and 2) the equal-time commutators of the two external interacting operators which generates the on mass shell particle exchange diagrams. Therefore, in this approach it is necessary to use some model Lagrangian and the equal-time commutation rule in order to determine this part of the potential or to construct this term using the quark-gluon degrees of freedom. The resulting operators calculated by the equal-time commutators are sandwiched between the real asymptotic on-mass shell states, *i.e.* these equal-time commutators are also determined by the one-variable vertex functions which can be considered as "input" vertices.

The present paper is devoted to the extension of the field-theoretical spectral decomposition of the three-body amplitude for the coupled $Nd-NNN$ reactions and for the classification of the corresponding three-body forces. This paper consists of the six sections. In the next section are derived the generalized field-theoretical unitarity conditions for the three body $Nd-NNN$ systems. The linearization of these quadratically nonlinear equations and the corresponding Lippmann-Schwinger equation for the connected part of the three-body amplitude are given in section 3. The section 4 deals to the equal-time anti-commutators and the corresponding contact (overlapping) part of the three-body relativistic potential of the $Nd-NNN$ systems. The extension of the present three-body equation within the formulation with the quark-gluon degrees of freedom is given in the section 5. Finally in the section 6 is presented the short resume and conclusion.

2. THE FIELD-THEORETICAL GENERALIZED UNITARITY CONDITIONS FOR THE COUPLED $Nd \Longleftrightarrow NNN$ AMPLITUDES.

In the standard formulation of the quantum field theory [1, 2, 5, 6] the S-matrix element between the asymptotic "out" m' and "in" m states define the corresponding S-matrix element $S_{m',m}$ and the scattering amplitude $f_{m',m}$.

$$S_{m',m} \equiv \langle_{out}; m'|m;_{in}\rangle =$$
$$= \langle_{in}; m'|\beta;_{in}\rangle - (2\pi)^4 i\delta^{(4)}(P_{m'} - P_m) f_{m',m} \quad (2.1a)$$

$$f_{m',m} = -\langle_{out}; \widetilde{m}'|J_{\mathbf{p}'_{\mathbf{N}}}(0)|m;_{in}\rangle \quad (2.1b)$$

where $P_{m'} \equiv (P^o_{m'}, \mathbf{P}_{m'})$ is the full four-momentum of the asymptotic state m'. The one-nucleon states N' and N are extracted from the asymptotic m' and m states. Therefore,

$$m' = N' + \widetilde{m}'; \qquad m = N + \widetilde{m} \quad (2.2)$$

and the four-momentum of the free one-nucleon state N is $p_N = \Big(\sqrt{m_N^2 + \mathbf{p}_N^2}, \mathbf{p}_N\Big) \equiv \Big(E_{\mathbf{p_N}}, \mathbf{p}_N\Big)$. $J_{\mathbf{p_N}}(x)$ is the current operator of the nucleon N which is determined by the Dirac equation $J_{\mathbf{p}'_{\mathbf{N}}}(x) = Z_{N'}^{-1/2}\overline{u}(\mathbf{p}'_{\mathbf{N}})(i\gamma_\mu \partial^\mu_x - m_N)\Psi(x)$ with the renormalization constant $Z_{N'}$ and the Dirac bispinor $u(\mathbf{p_N})$. Here and afterwards we use the definitions and normalization conditions from the Itzykson and Zuber's book [1].

It is convenient to introduce the operator of the interacted nucleon $b^+_{\mathbf{p}_N}(x_0)$ which transforms into the asymptotic nucleon creation or annihilation operator $b^+_{\mathbf{p}_N}(in)$

or $b_{\mathbf{p}_N}(out)$ in the asymptotic region $x_0=\pm\infty$ and

$$b^+_{\mathbf{p}_N}(x_0)=Z_N^{-1/2}\int d^3x e^{-ip_Nx}\overline{u}(\mathbf{p_N})\gamma_o\psi_N(x);$$
$$i\frac{\partial}{\partial x_0}b^+_{\mathbf{p}_N}(x_0)=\int d^3x e^{-ip_Nx}\overline{J}_{\mathbf{p_N}}(x). \tag{2.3}$$

These operators satisfy the identities

$$b^+_{\mathbf{p}_N}(in)=b^+_{\mathbf{p}_N}(0)-\int d^4x\theta(-x_0)e^{-ip_Nx}\overline{J}_{\mathbf{p_N}}(x); \tag{2.4}$$

or in the matrix form

$$\mathrm{T}_{m',m}\equiv\langle_{in};\widetilde{m}'|J_{\mathbf{p'_N}}(0)b^+_{\mathbf{p_N}}(in)|\widetilde{m};_{in}\rangle=-\langle_{in};\widetilde{m}'|b^+_{\mathbf{p_N}}(in)J_{\mathbf{p'_N}}(0)|\widetilde{m};_{in}\rangle+$$
$$\langle_{in};\widetilde{m}'|\Big\{J_{\mathbf{p'_N}}(0),b^+_{\mathbf{p_N}}(0)\Big\}|\widetilde{m};_{in}\rangle-$$
$$\langle_{in};\widetilde{m}'|\int d^4x\theta(-x_0)e^{-ip_Nx}\Big\{J_{\mathbf{p'_N}}(0),\overline{J}_{\mathbf{p_N}}(x)\Big\}|\widetilde{m};_{in}\rangle, \tag{2.5}$$

where the curly brackets denotes the anti-commutator of the corresponding operators and we have introduced the auxiliary amplitude $\mathrm{T}_{m',m}$.

Similarly for the scattering amplitude $f_{m'm}$ one obtains according to the reduction formulas [1, 2]

$$f_{m'm}=\langle_{out};\widetilde{m}'|J_{\mathbf{p'_N}}(0)b^+_{\mathbf{p_N}}(in)|\widetilde{m};_{in}\rangle=-\langle_{out};\widetilde{m}'|b^+_{\mathbf{p'_N}}(out)J_{\mathbf{p_a}}(0)|\widetilde{m};_{in}\rangle$$
$$-\langle_{out};\widetilde{m}'|\Big\{J_{\mathbf{p_a}}(0),b^+_{\mathbf{p_b}}(0)\Big\}|\widetilde{m};_{in}\rangle$$
$$+i\int d^4x e^{-ip_bx}\langle_{out};\widetilde{m}'|T\Big(J_{\mathbf{p_a}}(0)\overline{J}_{\mathbf{p_b}}(x)\Big)|\widetilde{m};_{in}\rangle, \tag{2.6}$$

where $T\Big(\ldots\Big)$ denotes the T-product of the corresponding operators.

After substitution of the complete set of the asymptotic states $\sum_n|n;_{in}\rangle\langle_{in};n|=\widehat{1}$ between the current operators in expressions (2.5) and (2.6) and after integration over x we get

$$f_{m',m}=W_{m',m}+(2\pi)^3\sum_{\substack{n=H^3(He^3)\\N''+d'',3N'}}f_{m',n}\frac{\delta^{(3)}(\mathbf{p}_N+\mathbf{P}_{\widetilde{\mathbf{m}}}-\mathbf{P_n})}{E_{\mathbf{p_N}}+P^o_{\widetilde{m}}-P^o_n+i\epsilon}\mathrm{T^*}_{m,n} \tag{2.7}$$

$$\mathrm{T}_{m',m}=\mathcal{W}_{m',m}+(2\pi)^3\sum_{\substack{n=H^3(He^3)\\N''+d'',3N''}}\mathrm{T}_{m',n}\frac{\delta^{(3)}(\mathbf{p}_N+\mathbf{P}_{\widetilde{\mathbf{m}}}-\mathbf{P_n})}{E_{\mathbf{p_N}}+P^o_{\widetilde{m}}-P^o_n+i\epsilon}\mathrm{T^*}_{m,n}, \tag{2.8}$$

where $W_{m',m}$ and $\mathcal{W}_{m',m}$ contain disconnected parts like $\langle_{out};\widetilde{m}'|b^+_{\mathbf{p'_N}}(out)J_{\mathbf{p_a}}(0)$

$|\widetilde{m};_{in}\rangle$ and all contributions of the intermediate states that can appear in the reaction $m\to m'$ except the s-channel $n=H^3(He^3),N''+d'',3N''$-particle exchange states.

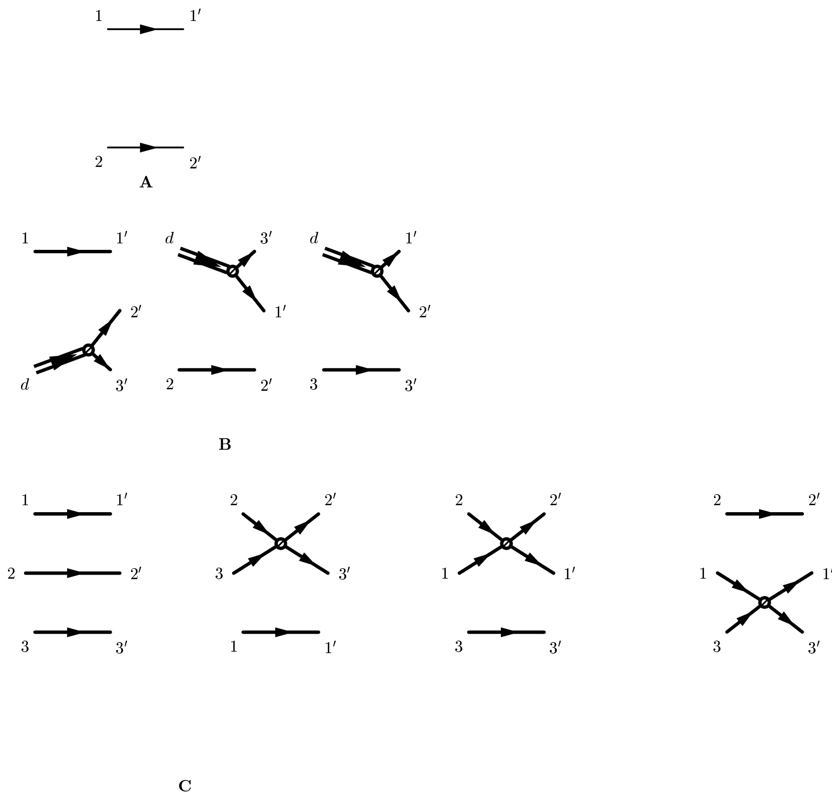

Fig. 1 – The disconnected parts of the two-nucleon scattering amplitude, $N+d$ disintegration amplitude $N+d\to 3N'$ and $3N\to 3N'$ scattering amplitude correspondingly in A, B and C. The shaded circle corresponds to the connected part of the nucleon-nucleon scattering amplitude.

Afterwards we shall consider the three nucleon $(3N)$ and nucleon+deuteron $(N+d)$ asymptotic states

$$m'=3N';N'+d'; \qquad m'=3N;N+d. \tag{2.9}$$

In order to obtain the closed system of the equations we keep the s-channel particle exchange states $n=H^3(He^3),N''+d'',3N''$ with the $3N$ bound states $H^3(He^3)$ in the equation. The other s-channel states with $n=\pi 3N",\pi N''d'',\pi H^3(He^3),\dots$ are included into potential $W_{m',m}$ and $\mathcal{W}_{m',m}$.

The graphical representation of the disconnected parts of the three-body amplitudes (2.5) and (2.6) are given in Fig.1. In the field-theoretical S-matrix approach the disconnected parts of the scattering amplitudes are exactly separated. This means that there exists so called cluster decomposition procedure [12, 17] which allows to obtain the connected and disconnected parts of the amplitudes and potentials in (2.7) and (2.8)

$$f_{m',m} = f^C_{m',m} + f^D_{m',m}; \qquad \mathrm{T}_{m',m} = \mathrm{T}^C_{m',m} + \mathrm{T}^D_{m',m}; \tag{2.10a}$$

$$W_{m',m} = W^C_{m',m} + W^D_{m',m}; \qquad \mathcal{W}_{m',m} = \mathcal{W}^C_{m',m} + \mathcal{W}^D_{m',m}. \tag{2.10b}$$

Therefore, one can rewrite (2.7) and (2.8) as

$$f^C_{m',m} = W^C_{m',m} + (2\pi)^3 \sum_{\substack{n=H^3(He^3)\\ N''d,3N''}} f^C_{m',n} \frac{\delta^{(3)}(\mathbf{p}_N + \mathbf{P}_{\widetilde{\mathbf{m}}} - \mathbf{P}_{\mathbf{n}})}{E_{\mathbf{p_N}} + P^o_{\widetilde{m}} - P^o_n + i\epsilon} \mathrm{T}^{*C}_{\ \ m,n} \tag{2.11}$$

$$\mathrm{T}^C_{m',m} = \mathcal{W}^C_{m',m} + (2\pi)^3 \sum_{\substack{n=H^3(He^3)\\ N''d,3N''}} \mathrm{T}^C_{m',n} \frac{\delta^{(3)}(\mathbf{p}_N + \mathbf{P}_{\widetilde{\mathbf{m}}} - \mathbf{P}_{\mathbf{n}})}{E_{\mathbf{p_N}} + P^o_{\widetilde{m}} - P^o_n + i\epsilon} \mathrm{T}^{*C}_{\ \ m,n}, \tag{2.12}$$

where the upper indexes ^{C} and ^{D} indicate the connected and disconnected parts of the corresponding expressions correspondingly.

$$W^C_{m',m} = -\langle_{out};\widetilde{m}'|\Big\{J_{\mathbf{p}'_\mathbf{N}}(0), b^+_{\mathbf{p_N}}(0)\Big\}|\widetilde{m};_{in}\rangle^C$$

$$+ (2\pi)^3 \sum_{\substack{n=\pi 3N'',\\ \pi N''d'',\ldots}} \langle_{out};\widetilde{m}'|J_{\mathbf{p}'_\mathbf{N}}(0)|n;_{in}\rangle^C \frac{\delta^{(3)}(\mathbf{p}_N + \mathbf{P}_{\widetilde{m}} - \mathbf{P}_n)}{E_{\mathbf{p_N}} + P^o_{\widetilde{m}} - P^o_n + i\epsilon} \langle_{in};n|\overline{J}_{\mathbf{p_N}}(0)|\widetilde{m};_{in}\rangle^C$$

$$- (2\pi)^3 \sum_{\substack{l=N'',\\ \pi N",\ldots}} \langle_{out};\widetilde{m}'|\overline{J}_{\mathbf{p_N}}(0)|l;_{in}\rangle^C \frac{\delta^{(3)}(-\mathbf{p}_\pi + \mathbf{P}_{\widetilde{\mathbf{m}}'} - \mathbf{P}_{\mathbf{l}})}{-E_{\mathbf{p}_\pi} + P^o_{\widetilde{m}'} - P^o_l} \langle_{in};l|J_{\mathbf{p}'_\mathbf{N}}(0)|\widetilde{m};_{in}\rangle^C$$

$$+ \text{all possible transpositions 2',3' into initial vertex and 2,3 into final vertex.} \tag{2.13}$$

After replacing $\langle_{out};\widetilde{m}'|$ with $\langle_{in};\widetilde{m}'|$ the connected potential $W^C_{m',m}$ (2.13) transforms into $\mathcal{W}^C_{m',m}$ in (2.12).

The inhomogeneous term $W^C_{m',m}$ (2.13) of the spectral decomposition of the three-body connected amplitude $f^C_{m',m}$ (2.11) consists from the product of the two connected vertices and the linear propagator between them. This propagator arise after Fourier conjugation of the relative time step function $\theta(x_o)$ in (2.6). Therefore, (2.13) and (2.11) are the 3D time-ordered relativistic expressions. Besides, the intermediate particle states $n = 4N'', 2d'', \ldots$, $l = N'', \pi N", \ldots$ in (2.13) are on mass shell, because these particle are extracted from the "in" states of the corresponding vertices.

All possible transpositions (crossings) of the particles $1'^*$, $3'$, 1^* and 3 of the s-channel diagram in Fig. 2A generate the diagrams in Fig. 2. In particular, the second term in (2.13) describes the on mass shell multi-particle $\pi+3N'', \pi+N''d, \dots$ s-channel exchange diagrams which is depicted in Fig. 2A and the third part of eq.(2.13) in Fig. 2E corresponds to the u-channel interaction which are obtained after crossing of the N' and N nucleons from the final "out" and the initial "in" states.

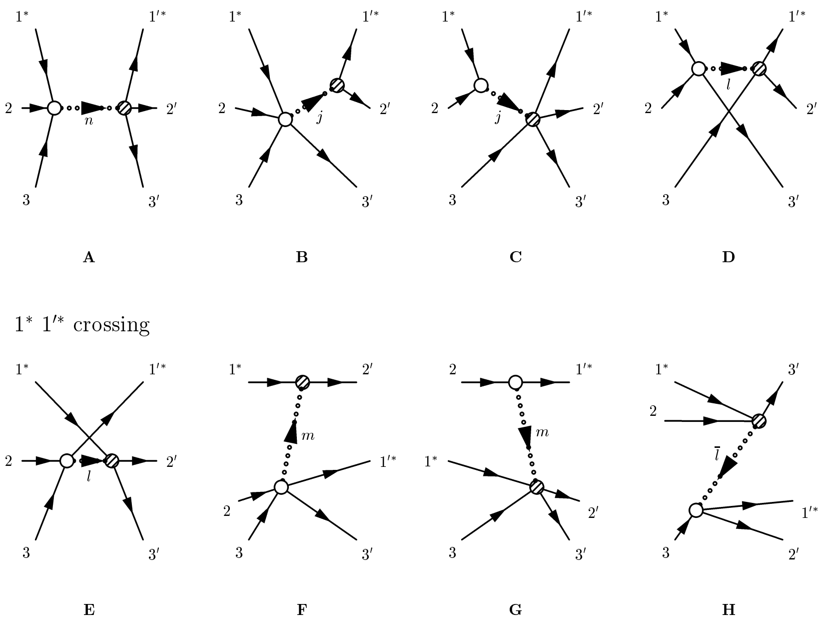

Fig. 2 – The on mass shell particle exchange diagrams which are included in the effective potential (2.13) for the three-nucleon reaction $1+2+3 \to 1'+2'+3'$. The empty circle stands for the primary transition amplitude and the dashed circle corresponds to the following transition amplitude. The nucleons with the index * generate the source operators $J_{\mathbf{p}'_{\mathbf{N}}}(0)$ and $\overline{J}_{\mathbf{p_N}}(0)$ in (2.13). The corresponding nucleons $1'^*$ and 1^* are considered off mass shell in the corresponding amplitude or vertex. All of diagrams have the three-dimensional time-ordered form with the physical or "*dressed*" vertices. Due to time ordering in all of the diagrams the initial empty circle is depicted in left-hand side and the following circle takes a place in the right-hand side. The intermediate on mass shell nucleon $l = N'', \pi+N'', \dots$ and anti-nucleon $\bar{l} = \overline{N}'', \pi+\overline{N}'', \dots$ exchange diagrams D and H appear after $3' \Longleftrightarrow 3$-crossing and $1'^* + 3' \Longleftrightarrow 1+3$-crossing transformations. The two-nucleon exchange diagrams B and C with $j = 2N'', d, \pi d, \dots$ appear after the crossing of the nucleon $3'$ and 3 correspondingly. The crossing of the nucleons $1'^* + 3' \Longleftrightarrow 1$ and $1'^* \Longleftrightarrow 1+3$ generate the diagrams F and G with the meson intermediate states $m = \pi, 2\pi, \dots$.

Other kind of diagrams, which appear after all possible c transposition of the pairs nucleons $(2'+3')$ and $(2+3)$ from and from the diagrams in Fig. 2A and Fig.2E are depicted in Fig. 3.

The nucleons $1'$ and 1 are extracted from the asymptotic states and they determine the source operators in the amplitude, *i.e.* they are considered in (2.13) off shell. These off mass shell particle in Fig. 2 and 3 are marked by $*$. The diagrams in Fig.2A and in Fig. 2E have different chronological sequences of a absorption and a emission of the nucleons 1 and $1'$. In particular, the diagram 2A corresponds to the chain of reactions, where firstly the initial three body state 1^*+2+3 transforms into intermediate on mass shell n-particle states which afterwards produces the final $1^{*\prime}+2'+3'$ state. In the diagram 2E at first the final fermion $1^{*\prime}$ is generated with the intermediate states l from the initial $2+3$ states and afterwards we obtain final $2'+3'$ state from the intermediate $l+1^*$ states.

The cluster decomposition procedure [12, 17] determines the chronological sequence of absorption of the initial and final nucleons. In diagrams 2B,2C and 2D are presented all transposition (crossing) of the particles 3 and $3'$ from the initial s-channel diagram in Fig. 2A. In particular, Fig.2B is obtained after transposition of the nucleon $3'$ *i.e.* after substitution of the disconnected part of amplitude $\langle_{out};\mathbf{p_2'}|J_{\mathbf{p_1'}}(0)b_{\mathbf{p_3'}}(in)|n;_{in}\rangle$. Fig.2C is generated by transposition of the nucleon 3 and Fig. 2D is result of the permutation of a both particles 3 and $3'$. The same procedure of transposition of the nucleons 3 and $3'$ from the u-channel diagram in Fig.2E generates the diagrams 2F. 2G and 2H. An other kind of permutations of the both particles $2+3$ and $2'+3'$ from s-channel diagram in Fig. 2A produces the diagrams 3A, 3B and 3C. The complete set of the diagrams which can be obtained after transpositions of the nucleons $1^*, 2, 3$ and $1'^*, 2'3'$ consists from the different disposition of these particles at the first and second vertex functions. One has the following combinations of the dispositions of particles $(2,3);(2'3')$ at the vertices: $1^*{\Longrightarrow}1'^*+\text{four particles}$, $1^*+\text{one particle}{\Longrightarrow}1'^*+\text{three particles}$, $1^*+\text{two particle}{\Longrightarrow}1'^*+\text{two particles}$, $1^*+\text{three particles}{\Longrightarrow}1'^*+\text{one particle}$ and $1^*+\text{four particles}{\Longrightarrow}1'^*$. For instance, we have four diagrams $1^*+2{\Longrightarrow}1'^*+3,2'3'$ (Fig. 2C), $1^*+3{\Longrightarrow}1'^*+2,2'3'$, $1^*+2'{\Longrightarrow}1'^*+23,3'$ and $1^*+3'{\Longrightarrow}1'^*+23,2'$ (Fig. 3G) for the disposition $1^*+\text{one particle}{\Longrightarrow}1'^*+\text{three particles}$. The particle distribution $1^*+\text{three particles}{\Longrightarrow}1'^*+\text{one particle}$ have also the four diagrams $1^*+3,2'3'{\Longrightarrow}1'^*+2$, $1^*+2,2'3'{\Longrightarrow}1'^*+3$ (Fig. 3H), $1^*+23,3'{\Longrightarrow}1'^*+2'$ (fig. 2B), $1^*+23,2'{\Longrightarrow}1'^*+3'$. The particle distribution $1^*+\text{two particle}{\Longrightarrow}1'^*+\text{two particles}$ can be observed in the six diagrams $1^*+23{\Longrightarrow}1'^*+2'3'$ (Fig.2A), $1^*+2,3'{\Longrightarrow}1'^*+3,2'$ (Fig. 2D), $1^*+2,2'{\Longrightarrow}1'^*+3,3'$, $1^*+3,2'{\Longrightarrow}1'^*+2,3'$, $1^*+3,3'{\Longrightarrow}1'^*+2,2'$ and $1^*+2'3'{\Longrightarrow}1'^*+23$ (Fig. 3C). And one diagram $1^*{\Longrightarrow}1'^*+23,2'3'$ (Fig. 3A) and one diagram $1^*+23,2'3'{\Longrightarrow}1'^*$ (Fig.3B) relates to the distributions $1^*{\Longrightarrow}1'^*+\text{four particles}$ and $1^*+\text{four particles}{\Longrightarrow}1'^*$ correspondingly. Finally, the s-channel term in (2.13) generates the $2\times 4+6+2=16$ connected terms after cluster decomposition. The other 16 connected terms produces the u-channel term in (2.13). Thus the cluster decomposition procedure performed in the second and in the third terms of eq.(2.13) generate 32 independent diagrams. Diagrams 3C, 3D, 3E, 3F, 3I and 3J contain the antiparticle intermediate states, because the time-ordered field-theoretical formulation includes the complete set of the intermediate particle propagators with a different time sequences. This means that for any diagrams with $n,l,j.m..$-particle intermediate states appear the corresponding diagrams with the antiparticle $\overline{n},\overline{l},\ldots$ intermediate states.

According to the field-theoretical axiom about the equivalence of the one-particle "in" and "out" states $\langle_{in};\mathbf{p}| \equiv \langle_{out};\mathbf{p}|$ (see footnote on the page 55 in [27]) we have

$$\mathrm{T}_{N'd',Nd} = f_{N'd',Nd} \text{ since}$$
$$\langle_{out};\mathbf{p_d}'|J_{\mathbf{p}'}(0)|\mathbf{p_N}\mathbf{p_d};_{in}\rangle = \langle_{in};\mathbf{p_d}'|J_{\mathbf{p}'}(0)|\mathbf{p_N}\mathbf{p_d};_{in}\rangle, \quad (2.14a)$$

$$\mathrm{T}_{N'd',3N} = f_{N'd',3N} \text{ since}$$
$$\langle_{out};\mathbf{p_d}'|J_{\mathbf{p}'}(0)|\mathbf{p_{N1}}\mathbf{p_{N2}}\mathbf{p_{N3}};_{in}\rangle = \langle_{in};\mathbf{p_d}'|J_{\mathbf{p}'}(0)|\mathbf{p_{N1}}\mathbf{p_{N2}}\mathbf{p_{N3}};_{in}\rangle, \quad (2.14b)$$

But, for the three-nucleon transition amplitudes

$$\mathrm{T}_{3N',3N} \neq f_{3N',3N}. \quad (2.14c)$$

In the considered approach the asymptotic deuteron is constructed similarly with the one-particle asymptotic state according to the Haag-Nishijima-Zimmermann formulation for the composed particle [32–35]. This formulation will be considered in the section 5. The relations (2.14a,b) allow to obtain the convenient system of the equations for the amplitudes $f_{Nd,N'd'}$ and $f_{Nd,3N'}$

$$f^C_{N'd',Nd} = W^C_{N'd',Nd} + (2\pi)^3 \sum_{\substack{n=H^3(He^3),\\ N''d'',3N''}} f^C_{N'd',n} \frac{\delta^{(3)}(\mathbf{p_1}+\mathbf{P_d}-\mathbf{P_n})}{E_{\mathbf{p_1}}+P^o_d-P^o_n+i\epsilon} {f^*_{1d,n}}^C \quad (2.15a)$$

$$f^C_{3N',Nd} = W^C_{3N',Nd} + (2\pi)^3 \sum_{\substack{n=H^3(He^3),\\ N''d'',3N''}} f^C_{3N'',n} \frac{\delta^{(3)}(\mathbf{p_1}+\mathbf{P_d}-\mathbf{P_n})}{E_{\mathbf{p_1}}+P^o_d-P^o_n+i\epsilon} {f^*_{1d,n}}^C \quad (2.15b)$$

In the quadratically nonlinear system of the equations as well as for the corresponding potentials $W^C_{N'd',Nd}$ and $W^C_{3N',Nd}$ (2.13) the auxiliary amplitude $\mathrm{T}^C_{m',m}$ is excluded due to the relations (2.14a,b). The inhomogeneous term of (2.15a) $W^C_{N'd',Nd}$ has the same structure as the corresponding term of the two-body cluster decomposition. This term is depicted in Fig. 4 and it contains the eight skeleton diagrams diagrams only. In particular, $W^C_{N'd',Nd}$ contains of the s, u, $\overline{u}$ and $\overline{s}$ diagrams in Fig. 4A,4E,4D and 4H correspondingly. The diagrams in Fig. 4B,4C,4F and 4G are produced by corresponding transpositions of the on mass shell deuteron's d and d'.

3. THE 3D TIME-ORDERED RELATIVISTIC SCHRÖDINGER AND LIPPMANN-SCHWINGER EQUATIONS FOR THE COUPLED $Nd \Longleftrightarrow NNN$ SYSTEMS.

Equations (2.15a,b) represent the spectral decomposition of the three-body amplitudes over the complete set of the asymptotic $"in"$ states. Simultaneously, these equations present the generalized unitarity condition for the half off shell amplitudes $f^C_{N'd',Nd}$ and $f^C_{3N',Nd}$. The similar 3D time-ordered relations ware considered in the textbooks in the quantum field theory [1, 2, 6, 12] and in the nonrelativistic collision theory [30, 31] for the two-body amplitudes. Therefore, one can treat (2.15a,b) as the three-body generalization of the field-theoretical spectral decomposition formulas (or off shell unitarity conditions). The field-theoretical formulation allows to get analytically the three-body potential $W^C_{m'm}$ which determine the three-body forces in the relativistic and nonrelativistic formulations.

In the nonrelativistic collision theory [30, 31] the spectral decompositions of the multichannel amplitude is

$$T_{\alpha,\beta}(P^o_\beta) = V_{\alpha,\beta} + (2\pi)^3 \sum_\gamma T_{\alpha,\gamma}(P^o_\gamma) \frac{\delta^{(3)}(\mathbf{P}_\alpha - \mathbf{P}_\gamma)}{P^o_\beta - P^o_n + i\epsilon} T^*_{\gamma,\beta}(P^o_\gamma) \tag{3.1}$$

where $P^o_\beta, \mathbf{P}_\beta$, $P^o_\alpha, \mathbf{P}_\alpha$ and $P^o_\gamma \mathbf{P}_\gamma$ denotes the full energy and the full momentum of the states β, α and γ correspondingly.

The spectral decomposition (3.1) with the hermitian multichannel potential $V_{\alpha,\beta}$ was used as basis for the consistent derivation of the following Lippmann-Schwinger type equations [31]

$$T_{\alpha\beta}(P^o_\beta) = V_{\alpha\beta} + (2\pi)^3 \sum_\gamma V_{\alpha\gamma} \frac{\delta^{(3)}(\mathbf{P}_\alpha - \mathbf{P}_\gamma)}{P^o_\beta - P^o_n + i\epsilon} T_{\gamma\beta}(P^o_\beta), \tag{3.2}$$

The three-body field-theoretical potential (2.13) is not hermitian due to the particle propagators in the intermediate states. Nevertheless, we have shown in ref.[18–20, 27] that quadratically nonlinear spectral decomposition formulas (2.15a,b) are equivalent to the following relativistic Lippmann-Schwinger-type equations

$$\mathcal{T}_{N'd',Nd}(P^o_{Nd}) = \mathcal{U}_{N'd',Nd}(P^o_{Nd}) + \sum_{\substack{n=N''d'',\\ 3N''}} \mathcal{U}_{N'd',n}(P^o_{Nd}) \frac{\delta^{(3)}(\mathbf{P_{Nd}} - \mathbf{P_n})}{P^o_{Nd} - P^o_n + i\epsilon} \mathcal{T}_{n,Nd}(P^o_{Nd}), \tag{3.3a}$$

$$\mathcal{T}_{3N',Nd}(P^o_{Nd}) = \mathcal{U}_{3N',Nd}(P^o_{Nd}) + \sum_{\substack{n=N''d'',\\ 3N''}} \mathcal{U}_{3N'',n}(P^o_{Nd}) \frac{\delta^{(3)}(\mathbf{P_{Nd}} - \mathbf{P_n})}{P^o_{Nd} - P^o_n + i\epsilon} \mathcal{T}_{n,Nd}(P^o_{Nd}). \tag{3.3b}$$

The explicit form of the linear energy depending potentials

$$U_{n,m}(P^0)=A_{n,m}+P^oB_{n,m}; \qquad n,m=Nd,3N \tag{3.4}$$

with hermitian A and B matrices

$$A_{n,m}=A^*_{m,n}; \qquad B_{n,m}=B^*_{m,n}, \tag{3.5}$$

is single-valued determined by the three-body connected potentials $W^C_{n,m}$ (2.13) according to the relation

$$U_{n,m}(P^o=P^o_m)=W_{n,m}. \tag{3.6}$$

Therefore, for any potential $W^C_{m.n}$ one can unambiguously construct $U_{m.n}(E)$.

Solutions of the equations (2.15a,b) and (3.3a,b) coincide on energy shell

$$\mathcal{T}_{m,n}(P^o_m=P^o_n)=f^c_{n,m}|_{P^o_m=P^o_n} \tag{3.7a}$$

and in the half on energy shell region these amplitudes are simply connected

$$f^C_{n,Nd}=W^C_{n,Nd}+(2\pi)^3\sum_{\substack{m=N''d'',\\3N''}}W^C_{n,m}\frac{\delta^{(3)}(\mathbf{P_{Nd}}-\mathbf{P_{m'}})}{P^o_{Nd}-P^o_{m'}+i\epsilon}\mathcal{T}_{m,Nd}(P^o_{Nd}). \tag{3.7b}$$

One can rewrite the three-body Lippmann-Schwinger type equations (3.3a,b) in the form of the Schrödinger equations if one introduces the wave functions as

$$\mathcal{T}_{n,Nd}(P^o_{Nd})=\sum_{\substack{m=N''d'',\\3N''}}\mathcal{U}_{n,m}(P^o_{Nd})\langle m|\Psi_{Nd}\rangle, \tag{3.8}$$

then one obtains the following three-body Schrödinger equations with the connected potential

$$\sum_{\substack{m=N''d'',\\3N''}}\Big(\delta_{nm}P^o_n+\mathcal{U}_{n,m}(P^o_{Nd})\Big)\langle m|\Psi_{Nd}\rangle=P^o_{Nd}\langle n|\Psi_{Nd}\rangle \tag{3.9a}$$

or

$$(H_o+A)|\Psi_{Nd}\rangle=P^o_{Nd}(1-B)|\Psi_{Nd}\rangle \tag{3.9b}$$

According to the relations (3.7a,b) the solutions of the Lippmann-Schwinger type equations (3.3a,b) or the corresponding Schrödinger equations (3.9a.b) reproduces exactly the sought connected scattering amplitude $f^C_{n,Nd}$. On the other hand, the potential of the three-body Faddeev equation contains the sum of the disconnected parts and iteration of these disconnected parts contributes in the sought full three-body amplitude. The same properties have the total potentials of the three-body Bethe-Salpeter equations [13, 14, 28] which are constructed in the framework of the graphical method [29]. In the considered 3D time-ordered approach the total potential consists of the product of the two amplitudes or vertex functions and the last cut lemma of the graphical method does not work in this case. Therefore, for the derivation of the three-body equations (3.3a,b) and (3.9a,b) with the connected potential $W^C_{m',m}$ was used the cluster decomposition method [12, 17] that separates analytically the connected and disconnected parts of amplitudes. As a result of this procedure the disconnected and connected parts of the amplitude $f_{m',m}$ (2.6) can be calculated independently from each other according to the eq.(2.10a,b). In other words, the contributions of the products of the disconnected amplitudes $f^D_{m,n}$ (Fig.1) and the connected three-body amplitudes are taken into account in $W^C_{m.n}$ (2.13) as it can be observed in Fig.2B and Fig.2C. Therefore, in order to reproduce the Faddeev equation from the three-body Lippmann Schwinger equation (2.8) we consider the three-particle irreducible amplitude

$$\begin{aligned} T^F_{3N',3N} &= \mathrm{T}^C_{3N',3N} + \mathrm{T}^D_{3N',3N}; \\ \mathrm{T}^D_{3N',3N} &= \sum_{i=1}^{3} f_{N_i^{*\prime} N_j',2N"} \langle N_k'|N_k\rangle, \quad ijk = 123, 231, 312, \end{aligned} \tag{3.10}$$

where $T^D_{3N',3N}$ contain the disconnected terms in Fig. 1C and $T^F_{3N',3N}$ contain in the intermediate state more as 3 particle.

The linearization procedure can be formally performed for the full amplitude $T^F_{3N',3N}$. Then one obtains

$$\begin{aligned} \mathcal{T}^F_{3N',3N}(P^o_{3N}) = &\ \mathcal{U}^F_{3N',3N}(P^o_{3N}) + \\ &(2\pi)^3 \sum_{n=3N''} \mathcal{U}^F_{3N',n}(P^o_{3N}) \frac{\delta^{(3)}(\mathbf{P_{3N}} - \mathbf{P_n})}{P^o_{3N} - P^o_n + i\epsilon} \mathcal{T}^F_{n,3N}(P^o_{3N}), \end{aligned} \tag{3.11}$$

where the linear energy depending potential $\mathcal{U}^F_{3N',3N}$ is single-valued determined by $W^C_{3N',3N}$ (2.13) and $\mathcal{T}^F_{3N',3N}$ and $T^F_{3N',3N}$ coincides on the energy shell

$$\mathcal{U}^F_{3N',3N}(P^o_{3N}) = \mathcal{W}^F_{3N',3N}; \quad \mathcal{T}^F_{3N',3N}(P^o_{3N}) = T^F_{3N',3N}. \tag{3.12}$$

and $\mathcal{W}^F_{3N',3N}$ does not contain the terms in Fig. 2B and Fig. 2C with $j = 2N''$.

The equation (3.11) can be transformed to the Faddeev equations.

In particular, the diagram in Fig. 2B and Fig. 2C for $j = 2N''$ correspond to the expressions $f^C_{2N',2N''} G_o \mathrm{T}^{*C}_{N'+2N'',3N}$ and $\mathrm{T}^C_{3N',2N''+N} G_o f^{*C}_{2N'',2N}$, where G_o is the propagator of tree nucleons $2N'' + N'$ or $2N'' + N$. After anti-symmetrization of the nucleons in the initial and final states and using the full 3N amplitude, the diagrams in Fig.2B and Fig.2C together with the disconnected diagrams in Fig. 1C reproduces the Faddeev-type equations.

In the contrary with the 4D equations [13, 14] the suggested eq. (3.3a,b) contains the on mass shell particle exchange diagrams in the intermediate states. Therefore, in the practical calculation of the potentials (2.13) one has to truncate the completeness condition of the intermediate the on mass shell “in” states. But the truncated potential of eq. (3.3a,b) remains to be hermitian and the solutions of these equations satisfy the two and three body unitarity.

4. EQUAL-TIME ANTI-COMMUTATORS AS THE SOURCE OF THE OFF MASS SHELL PARTICLE EXCHANGE PART IN THE THREE-BODY POTENTIAL $W^C_{m',m}$ (2.13).

The sufficient part of the effective potential $W^C_{m',m}$ (2.13) is the first term with the equal-time anti-commutator, In particular, for the amplitudes of the reactions $1d \to 1'd'$, $1d \to 1'2'3'$ and $123 \to 1'2'3'$ these terms are

$$Y_{1d,1'd'} = -\langle \mathbf{p'_d}|\Big\{J_{\mathbf{p'_1}}(0), b^+_{\mathbf{p_1}}(0)\Big\}|\mathbf{p_d}; in\rangle. \tag{4.1a}$$

$$Y_{1d,1'2'3'} = -\langle \mathbf{p'_2}, \mathbf{p'_3}|\Big\{J_{\mathbf{p'_1}}(0), b^+_{\mathbf{p_1}}(0)\Big\}|\mathbf{p_d}; in\rangle. \tag{4.1b}$$

$$Y_{123,1'2'3'} = -\langle \mathbf{p'_2}, \mathbf{p'_3}|\Big\{J_{\mathbf{p'_1}}(0), b^+_{\mathbf{p_1}}(0)\Big\}|\mathbf{p_2}, \mathbf{p_3}; in\rangle^C. \tag{4.1c}$$

The expressions (4.1abc) can be determined using the *a priori* given Lagrangian and equal-time anti-commutations rules between the Heisenberg field operators. For the renormalizable Lagrangian models or for the simple phenomenological Lagrangians the equal-time anticommutators are easy to calculate [18–20, 27]. In that case expressions (4.1abc) consists of the off shell one particle exchange potentials (see diagrams 5A, 5C and 5E) and of the contact (overlapping) interactions for the four-point system (Fig.5B), for the five point interaction (Fig.5D) and for the six nucleon scattering (Fig.5F). The overlapping (contact) terms does not contain any particle propagator in the intermediate states. The equal-time commutators are the only part of potentials (2.13) or (3.6) which contains *explicitly* the off mass shell particle exchange contributions, since other terms in the potential (2.13) consists of the on mass shell particle exchange terms. In particular, we shall consider two simplest πN Lagrangians with the pseudo-scalar (ps) and pseudo-vector (pv) coupling

$$\mathcal{L}_{ps} = ig_\pi \overline{\Psi}\gamma_5 \Psi \Phi_\pi; \qquad \mathcal{L}_{pv} = \frac{f_\pi}{m_\pi}\overline{\Psi}\gamma_5\gamma_\mu\Psi\frac{\partial \Phi_\pi}{\partial x_\mu}, \tag{4.2}$$

where g_π is the coupling constant of the πN system and $f_\pi/m_\pi = g_\pi/2m_N$.

The current $J_{\mathbf{p}_N}(x)$ in the both cases are

$$J^{ps}_{\mathbf{p}_N} = ig_\pi\gamma_5\Psi\Phi_\pi; \quad J^{pv}_{\mathbf{p}_N} = \frac{f_\pi}{m_\pi}\gamma_5\gamma_\mu\Psi\frac{\partial\Phi_\pi}{\partial x_\mu} \tag{4.3}$$

The equal time commutator of the nucleon and pion Heisenberg fields Ψ and Φ_π vanish. But the equal time commutator of Ψ and $\partial\Phi_\pi/\partial x_o$ vanish for the pseudo-scalar coupling and it is equal to $-(f_\pi/m_\pi)\overline{\Psi}\gamma_5\gamma_0\Psi$ Therefore, for the equal-time anti-commutators (4.1a,b,c) we get

$$Y^{ps}_{m',m} = \frac{-ig_\pi}{\sqrt{Z_1 Z_{1'}}}\overline{u}(\mathbf{p_1'})\gamma^5 u(\mathbf{p_1})\langle_{in};\widetilde{m}'|\phi_\pi(0)|\widetilde{m};_{in}\rangle = \frac{-ig_\pi}{\sqrt{Z_1 Z_{1'}}}\frac{\overline{u}(\mathbf{p_1'})\gamma^5 u(\mathbf{p_1})}{(P_{\widetilde{m}'} - P_{\widetilde{m}})^2 - m_\pi^2}\langle_{in};\widetilde{m}'|j_\pi(0)|\widetilde{m};_{in}\rangle, \quad (4.4)$$

$$Y^{pv}_{m',m} = Y^{ps}_{m',m} - \frac{f_\pi}{m_\pi\sqrt{Z_1 Z_{1'}}}\overline{u}(\mathbf{p_1'})\langle_{in};\widetilde{m}'\overline{\Psi}(0)\gamma_o\gamma^5\Psi(0)|\widetilde{m};_{in}\rangle u(\mathbf{p_1}), \quad (4.5)$$

Thus the renormalizable Yukawa Lagrangian $\mathcal{L}_{ps}$ (4.2) generates the one-pion exchange diagrams in Fig. 5A, 5C and 5E and the non-renormalizable Lagrangian $\mathcal{L}_{pv}$ produces an additional overlapping (contact) term depicted in Fig. 5B, 5D and 5E. Using more complete Lagrangian models, one obtains heavy meson σ,ρ,ω exchange diagrams [18–20, 27]. Moreover, in the ref.[19, 20] the One Boson Exchange (OBE) Bonn model of the NN potential was analytically reproduced from the equal-time anticommutators. The contact (overlapping) terms arise from the ϕ^4 (four-point) part of Lagrangians or from the nonrenormalizable Lagrangians and they play an important role for the NN scattering. The numerical estimation of the contact (overlapping) terms for low energy NN phase shifts was done in [19, 20], This contribution generates non-negligible corrections up to 25%.

The simplest contact (overlapping) terms depicted in Fig. 5F is $\langle_{in};\mathbf{p_2}'\mathbf{p_3}'|\overline{\Psi}(0)\gamma_o\gamma^5\Psi(0)|\mathbf{p_2}\mathbf{p_3};_{in}\rangle$ (4.5) within the pseudo-vector Lagrangian model. This term has the same structure as the amplitude of the reaction $NN \to N'N'\pi'$. Using the reduction formulas for the nucleons $2'$ and 2 and keeping the contact (overlapping) terms only one obtains the diagram in Fig. 5F and 5G. The diagram in Fig. 5F without the propagators of the intermediate particles presents the pure 6N contact (overlapping) terms. The diagram in Fig. 5G contains one vertex in tree approximation and two vertices generates by Yukawa three-point interaction. This simplest two off mass shell boson exchange term is

$$Y_{1'2'3',123} = -i^3 g_\pi^3 \frac{\overline{u}(\mathbf{p_1'})\gamma^5 u(\mathbf{p_1})}{(P_{2'+3'} - P_{2+3})^2 - m_\pi^2}\langle\mathbf{p_2'}|j_\pi(0)|\mathbf{p_2}\rangle \frac{\overline{u}(\mathbf{p_3'})\gamma^5(\gamma^\sigma Q_\sigma + m_N)\gamma_5\tau^i u(\mathbf{p_3})}{Z_1 Z_3\Big((p_3' - p_3)^2 - m_\pi^2\Big)(Q^2 - M_N^2)}, \quad (4.6)$$

where $Q = p_1 + p_2 - p_1'$.

The other source for the overlapping (contact) terms in the quantum field theory is the quark-gluon degrees of freedom which is considered in the next section.

As "input" by construction of the effective three-body potentials are required the off shell amplitudes of the reactions $2N \to 2N$ and $2N \to 2N' + \pi$ and the three-point one-variable vertices. One can determine these vertex functions from the experimental data using the quark counting rules, dispersion relations, the Regge trajectories theory, or inverse scattering method[30]. Afterwards one has to construct the corresponding off shell amplitudes of the reactions $2N \to 2N$ and $2N \to 2N' + \pi$ based on the above vertices.

5. QUARK DEGREES OF FREEDOM

Considering hadrons as bound states of quarks we can build their creation and annihilation operators according to the field-theoretical treatment of the bound state problem [32–35]. Thus the composite nucleon field operator $\Upsilon_p(X)$ (which is non-local because it depends on the on mass shell nucleon four-momentum p) is constructed through the three Heisenberg quark fields $q_i(x_i)$ with corresponding mass m_i (i=1,2,3) in the following manner [32–34]

$$\Upsilon_p(X) = \lim_{\rho_{12}\to 0} \lim_{\rho_3 \to 0} \frac{T\big(q_1(x_1)q_2(x_2)q_3(x_3)\big)}{\chi_p(X=0,\rho_{12},\rho_3)}, \tag{5.1a}$$

where

$$\chi_p(X,\rho_{12},\rho_3) \equiv \chi_p(x_1,x_2,x_3) = \langle 0|T\big(q_1(x_1)q_2(x_2)q_3(x_3)\big)|\mathbf{p}\rangle \tag{5.2}$$

is the three quark-nucleon bound state wave function which in principle is unambiguously determined as the solution of the bound state Bethe-Salpeter equation with a quark-gluon interaction potential. $X = (m_1x_1 + m_2x_2 + m_3x_3)/(m_1 + m_2 + m_3)$, $\rho_{12} = x_1 - x_2$ and $\rho_3 = (m_1x_1 + m_2x_2)/(m_1 + m_2) - x_3 \equiv x_{12} - x_3$ are the center of mass and the two relative Jacobi four-coordinates.

In an other approach [35], the composite nucleon field operator is defined as

$$\Upsilon_p(X) = \int d^4\rho_{12} d^4\rho_3 \chi_p^\dagger(X=0,\rho_{12},\rho_3) T\Bigg(q_1(x_1)q_2(x_2)q_3(x_3)\Bigg). \tag{5.1b}$$

The nucleon annihilation operator $\mathcal{B}^{in(out)}(\mathbf{p})$ for an on mass-shell nucleon with four momentum $p = (\sqrt{\mathbf{p}^2 + m_N^2}, \mathbf{p})$ can be constructed in the following way as a three-quark bound state annihilation operator

$$\mathcal{B}^{in(out)}(\mathbf{p}) = \lim_{X^0 \to \pm\infty} \mathcal{B}_{\mathbf{p}}(X^0), \tag{5.3a}$$

where the Heisenberg operator $\mathcal{B}_{\mathbf{p}}(X^0)$ is given as

$$\mathcal{B}_{\mathbf{p}}(X^0) = \int d^3\mathbf{X} \exp(ipX) \overline{u}(\mathbf{p}) \gamma_0 \Upsilon_p(X). \tag{5.3b}$$

In the same manner one can construct also composite meson fields through the quark-anti-quark operators.

In [32–35] it was shown that asymptotic field operators (5.3a) satisfy the same anti-commutation relations as the ordinary local field operators of the nucleons in the conventional quantum field theory. These relations make it possible to construct any "in" or "out" states with an arbitrary number of on-mass shell non-interacting, free particle states. The completeness condition for the asymptotic "in" or "out" fields remain hereby valid in the hadron Fock space. However, in contrast to the local quantum field theory, the Heisenberg fields $\mathcal{B}_{\mathbf{p}}(X^0)$ (5.3b) do not satisfy the corresponding equal-time anti-commutation relations. Nevertheless, in this field-theoretical approach for the composite field operators all basic requirements of relativistic quantum field theory are valid. Therefore, one can reproduce exactly the reduction formulas and all other relations in the sections 1,2,3 and 4 exactly using the nonlocal source operator of the nucleon $\mathcal{J}_{\mathbf{p}'_\mathbf{N}}(X) = Z_{N'}^{-1/2}\overline{u}(\mathbf{p}'_\mathbf{N})(i\gamma_\mu\partial_X^\mu - m_N)\Upsilon_p(X)$ and the nonlocal field $\mathcal{B}_{\mathbf{p}}(X^0)$ (5.3b).

Consequently, one obtains the S-matrix element (2.1a), the three-body amplitudes (2.1b), the off shell unitarity conditions (2.15a,b) and their representation in the equivalent three-body Lippmann-Schwinger-type equations (3.3a,b) for the connected amplitudes.

6. SUMMARY AND CONCLUSION

This paper is devoted to the alternative field-theoretical time-ordered equations for the coupled channels $3N - Nd$. Unlike to the 4D Bethe-Salpeter equations and their quasipotential reductions, the present formulation is free from the ambiguities of the 3D reductions and there are not required the input three-variable vertices, because the present three-body and two-body equations are three-dimensional from the beginning and required input for construction of potential of the these equations are the one-variable vertex functions.

The considered field-theoretical formulation is not less general as the 4D Bethe-Salpeter equations. The final form of the equations (3.3a,b) and (2.13) are not dependent on the choice of the Lagrangian and these equations are valid for any QCD motivated models with the quark-gluon degrees of freedom. But the suggested equations are simpler as the analogical Bethe-Salpeter and other field-theoretical equations and they can be numerically solved with the present computers. In particular, the suggested equation are more convenient in practical applications as other field-theoretical equations because of the following reasons:

1) In the considered formulations the on mass shell and off mass shell degrees of freedom are separated. The off mass shell particle exchange and overlapping (contact) terms are included into the part of the potential with the equal-time commutators.

On mass shell exchange diagrams are included in the generalized unitarity condition which unambiguously determines the final Lippmann-Schwinger equation. Therefore, the practical applications of these equation does not require a truncation of the off shell degrees of freedom. Moreover, this truncation does violate the unitarity, current conservation and other first principles of the theory.

2) The quark-gluon degrees of freedom does not change the form of the suggested equations and their potential. Moreover, the intermediate propagators of the quarks and gluons does not contribute into unitarity condition of the hadron amplitudes.

3) The three-body equations have a form of the three-body Lippmann-Schwinger or the three-body Schrödinger equations with the connected potential. Therefore, there is not necessary to use an auxiliary amplitudes for calculation of the contributions of the disconnected parts of the three-body potentials. On the other hand these equations can be represented in the form of the Faddeev equations as it is done in equations (3.10)-(3.12).

4) The suggested formulation allow to obtain the complete set of the three-body forces and the corresponding potentials where the contact (overlapping) terms are separated from the off mass shell and on mass shell particle exchange potentials. In particular it is demonstrated, that the Yukawa or three-point interaction does not generate the contact (overlapping) terms i.e the contact (overlapping) term arise due to four-point or more complicated nonrenormalizable interactions.

Therefore, in the nonrelativistic limit without four-point and nonrenormalizable interactions one has only the one off mass shell particle exchange potential like OBE NN potential.

5) The quark-gluon exchange off shell potential constructed from the equal-time commutators, consists also from the off mass shell particle exchange and the contact (overlapping) parts. [22]. But in this case one can not neglect the contact (overlapping) terms in the nonrelativistic limit.

The only principal approximation, that is necessary to do in the considered approach is the truncation of the intermediate multi-particle states. But here, unlike to the Bethe-Salpeter equations, one has to cut down only the *on mass shell intermediate states*. In any case for the self-consistent calculation of the two-body and the three-body reactions in the low and intermediate energy region, it is advisable to work out the scheme of a suppression mechanism of transition into multi-particle intermediate states.

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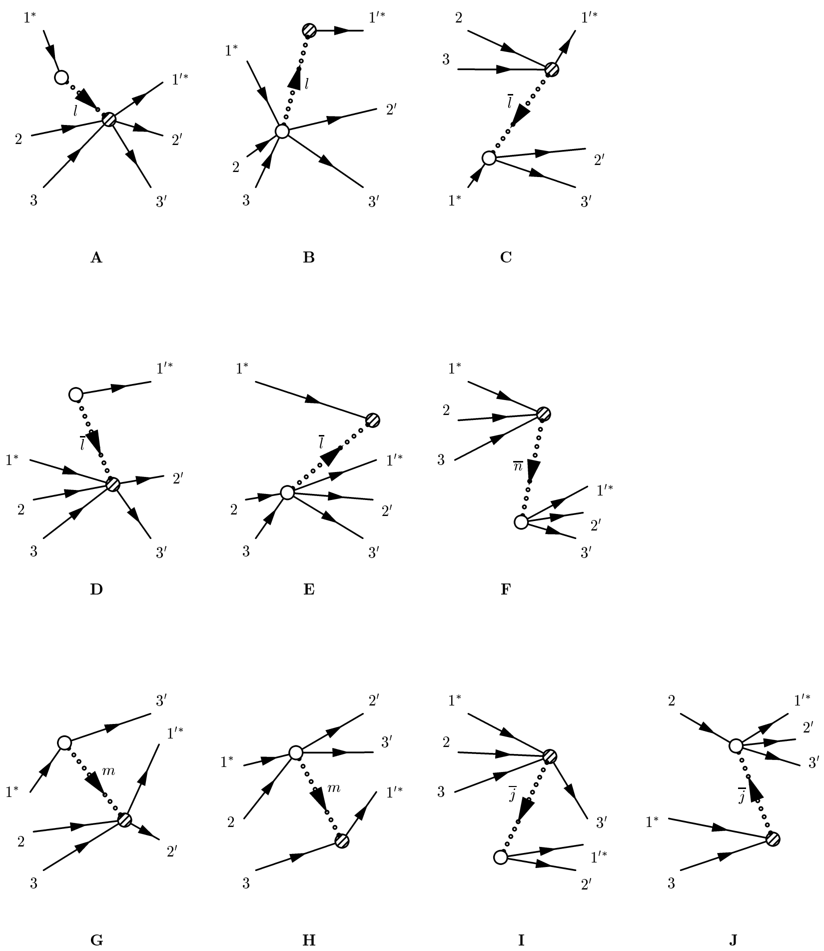

Fig. 3 – Diagrams obtained after crossing of the pair of the nucleons $(2,3)$ and $(2',3')$ transposition from the s-channel diagram in Fig.2A and from the diagram in Fig.2E.

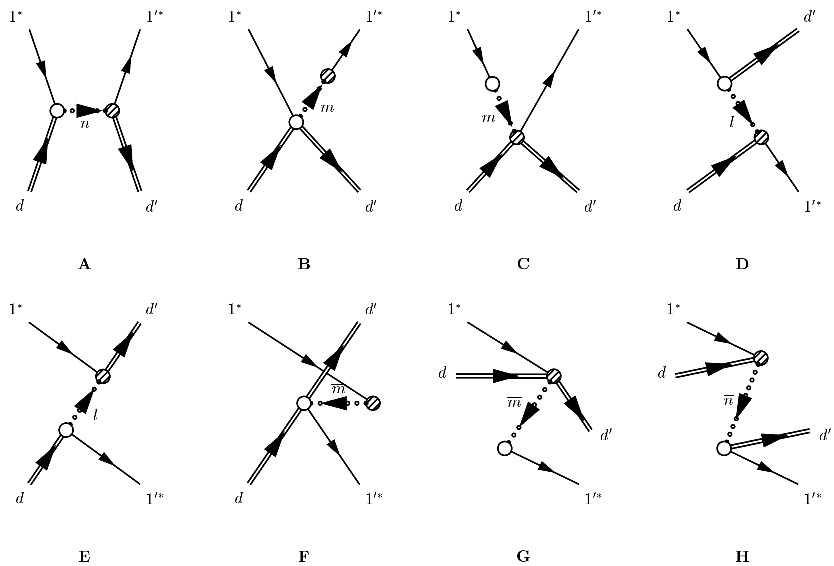

Fig. 4 – The graphical representation of the on mass shell particle exchange potential (2.13) for the $N+d \Longrightarrow N'+d'$ amplitude after cluster decomposition. The double line denotes the deuteron, $n=\pi+3N'',\pi+N+d,\dots$ stands for the s-channel multi-particle intermediate states, $m=\pi+N'',\dots$ and $l=\overline{N},\pi\overline{N},\dots$ corresponds to the d and d' transpositions. The diagrams E,F,G,H are obtained after crossing of $N \Longleftrightarrow N'$ from the diagrams A,B,C,D correspondingly

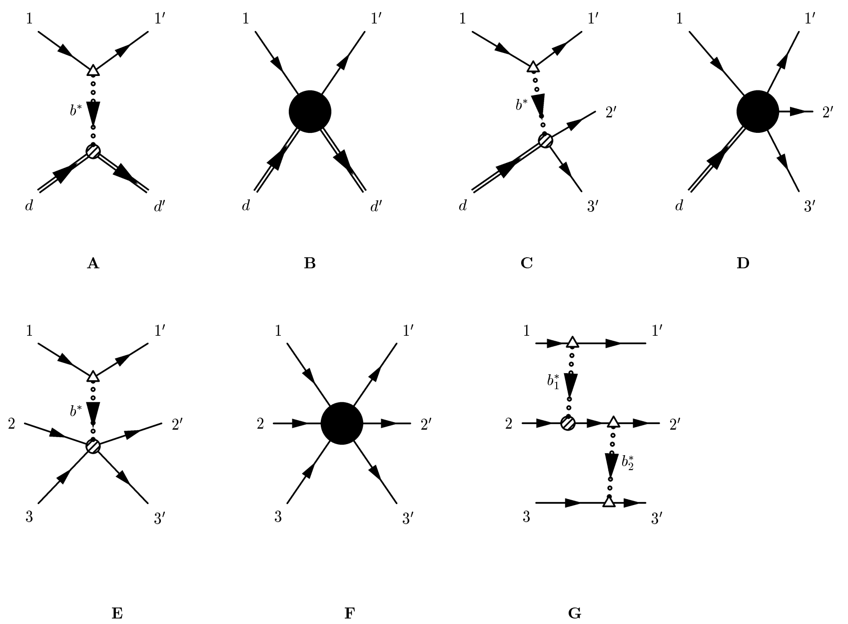

Fig. 5 – The graphical representation of the equal-time anticommutators (4.1a,b,c). These terms are depicted correspondingly for the Nd scattering in diagrams A, B; for the deuteron decay $Nd \Longrightarrow 3N'$ in diagrams C, D and for the three nucleon scattering $3N \Longleftrightarrow 3'N'$ in diagrams E, F, G. Diagrams A, C, E correspond to the one off-mass shell particle b^*-exchange interactions which are appearing in the ϕ^3-theory for the Yukawa-type interactions. The triangle denotes the vertex functions in the tree approximation. Diagrams B, D, F describe the contact (overlapping) interaction which does not contain the intermediate hadron propagator. Diagram 5G corresponds to the simplest one off mass shell boson fermion and two off mass shell boson exchange interaction which is obtained from the equal-time anti-commutators (4.1c) within the ϕ^3 theory.