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DUALITY CONDITION FOR s- AND t-CHANNEL EXCHANGE IN NUCLEON-NUCLEON SCATTERING

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We specify conditions under which the nucleon-nucleon interaction, based on the t-channel meson-exchange mechanism, is equivalent to an interaction generated through an s-channel exchange of six-quark bags. The duality is possible provided the alternation of zeros and poles of the non-dispersive part of D function takes place in the normalization where the imaginary part of D is non-negative and the CDD poles are the only poles of D.

Key words: Duality, nucleon-nucleon interaction, exchange mechanisms.

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1. INTRODUCTION

The fact that elementary particles are associated with the poles of S matrix was realized in the 1950s. Stable particles correspond to the poles for real values of energy and are identified with the asymptotic states of quantum field theory. Most of the poles are located at complex energies. These states are interpreted as resonances. Resonances, if they are not too far from the unitary cut, are observed experimentally as peaks in the cross sections. The most complete relationship between the singularities of S matrix as an analytic function and the Feynman diagrams was established by Landau [1].

Quark models predict the existence of exotic hadrons. The experimental searches for exotic mesons and baryons have not been successful. In the late1970s, Jaffe and Low [2] proposed to look for signs of exotic hadrons in P matrix, but not S matrix. P matrix can be constructed from the known scattering phase and its properties can then be examined. In this sense, the P-matrix poles can be observed in about the same way as the S-matrix poles corresponding to the resonances, however, the difference being that P matrix is a model-dependent object. In any case, the understanding came out that the set of S-matrix poles is, perhaps, too limited to accommodate all of the (quasi-) particle states that play a dynamical role in the strong interaction. * The poles of P matrix specify stable particles, resonances, and the so-called "primitives". Jaffe and Low proposed to identify exotic hadrons with the primitives, *i.e.*, P-matrix poles that do not manifest themselves as poles of S matrix.

A dynamic scheme of P matrix as an extension of the Dyson model [4] was proposed by Simonov [5]. Compound states of the model are associated with simple zeros of D function [6]

$$D(s) = 0. \tag{1}$$

A fraction of the zeros specifies bound states and resonances. The zeros on the unitary cut are associated with the primitives. Primitives do not generate the singularities of S matrix,

$$S(s) = \frac{D(s-i0)}{D(s+i0)}, \tag{2}$$

because the zeros occur in pairs in the denominator and the numerator and cancel each other. The effect is similar to the disappearance of the singularity from the amplitude when the resonance width tends to vanish.

In the scattering problem, there are discrete values of energy, that seem to play special role in the dynamics, not being singular points of S as an analytic function. Those values identify states, the primitives, which are not asymptotic states and thus can not be registered by detectors straight out. In an attempt to isolate them experimentally, it would be futile to look for peaks in the cross sections as well. The experimental signature of the primitives is zeros modulo π of the scattering phase with a negative slope.

Summarizing, the following states are highlighted as zeros of D function:

(*i*) Stable particles, which we identify with asymptotic states of quantum filed theory and register experimentally with the use of detectors.

(*ii*) Resonances are observed as peaks in the cross sections.

(*iii*) Primitives are observed as zeros modulo π of the scattering phase with a negative slope.

We learn of the existence of resonances and primitives on the basis of observations of stable particles. The zeros modulo π of the scattering phase with a positive slope are identified as the CDD poles associated with compound states. Depending on the nature of the interaction, compound states are, in turn, associated with bound states, resonances, or primitives.

*On the other hand, the set of poles of S-matrix is too broad: Among the isolated poles there is a subset of "spurious poles" that do not correspond to particles (see *e.g.* [3]).

The concept of a primitive thus leads to a revision of the concept of an elementary particle. In the broad sense, elementary particles correspond to simple zeros of D function, and only sometimes to simple zeros of D function and isolated poles of S-matrix.

In the theory of potential scattering, the negative slope of phase shift indicates repulsion between the particles. Bound states also tend to reduce the scattering phase. We are interested in the nucleon-nucleon scattering, where deuteron is the only bound state. In other two-nucleon channels, bound states are absent. In what follows we consider, if the opposite is not specified explicitly, D functions without zeros below the threshold.

The s-channel exchange of $6q$-primitives was proposed to account for the repulsion between nucleons at short distances [5]. This mechanism was also found to be sufficient to model the long-distance interaction dominated by the attraction.

The interaction of nucleons is, however, traditionally described in terms of the t-channel exchange of mesons. This approach got the greatest advancement in the one-boson exchange (OBE) models.

Both mechanisms successfully describe a wide range of experimental data. This fact is motivating an interest in the possible duality of the s-channel exchange of $6q$-primitives and the t-channel exchange of mesons [7]. The duality in the nucleon-nucleon scattering is illustrated in Fig. 1.

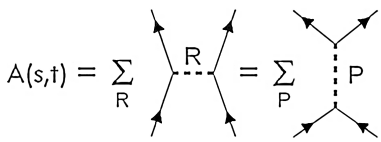

Fig. 1 – Representation of the scattering amplitude of nucleons as a sum of the t-channel meson-exchange diagrams. Duality means that the amplitude has an equivalent representation as a sum of primitives (P) in the s channel. R denotes meson resonances.

In this paper we investigate conditions under which the duality holds. From a formal point of view, it suffices to show that the OBE models can be reformulated in terms of an s-channel exchange model.

First, however, we discuss in Sect. 2 a hybrid model of Lee [8], which is not so radically different from the OBE models. The s-channel exchange of compound states is embedded in it, but primitives are explicitly absent. We examine factors that control the occurrence of compound states of the primitive type and provide reducibility to the model [6].

The inverse scattering problem for the s-channel exchange has many features in common with the problem of the reduction of the OBE models. We discuss this problem as well.

In the potential scattering theory and in the hybrid Lee models, primitives exist only as the peculiarities of scattering phases with no *ab initio* significance. In Sect. 3, we investigate conditions under which the potential scattering problem can be reformulated in terms of an s-channel exchange of primitives. In cases where this is possible, the primitives become key objects upon the reduction.

2. REDUCIBILITY OF HYBRID LEE MODEL

The hybrid Lee model [8] and the models of Refs. [4–7] are generalizations of the Lee model [9]. We formulate the hybrid Lee model in a language that is convenient for the subsequent reduction.

2.1. HYBRID LEE MODEL

The hybrid Lee model is formulated for n_c compound states with masses M_α and n_f contact four-fermion vertices. The summation of the s-channel loops gives rise to a $\mathcal{D}$ matrix

$$\mathcal{D}_{\alpha\beta}(s) = \mathcal{K}_{(\alpha)}\delta_{\alpha\beta} - \mathcal{P}_{\alpha\beta}(s) \tag{3}$$

of the dimension $n = n_c + n_f$. $\mathcal{K}_{(\alpha)} = s - M_\alpha^2$ are the inverse free propagators of the compound states $\alpha = 1 \ldots n_c$ and $\mathcal{K}_{(\alpha)} = C_\alpha$ are real constants entering the vertices of four-fermion interactions $\alpha = n_c + 1 \ldots n$. The self-energy operator has the form

$$\mathcal{P}_{\alpha\beta}(s) = -\frac{1}{\pi}\int_{s_0}^{+\infty} \Phi_2(s')\frac{\mathcal{F}_\alpha(s')\mathcal{F}_\beta(s')}{s'-s}ds'. \tag{4}$$

The integral converges provided $\mathcal{F}_\alpha(s) = O(1/s^\nu)$ at $s \to +\infty$ with $\nu > 0$. For $0 < \nu < 1/2$ $\mathcal{P}_{\alpha\beta}(s) = O(1/s^{2\nu})$ at $s \to -\infty$, and for $1/2 < \nu$ $\mathcal{P}_{\alpha\beta}(s) = O(1/s)$ at $s \to -\infty$. The matrix elements $\mathcal{D}_{\alpha\beta}(s)$ are analytic functions in the complex s-plane with the unitary cut $(s_0, +\infty)$. The discontinuity equals

$$\Im\mathcal{D}_{\alpha\beta}(s) = -\Im\mathcal{P}_{\alpha\beta}(s) = \Phi_2(s)\mathcal{F}_\alpha(s)\mathcal{F}_\beta(s). \tag{5}$$

It determines the off-shell decay widths of the compound states $\alpha = 1,\ldots,n_c$:

$$\sqrt{s}\Gamma_\alpha(s) = \Im\mathcal{D}_{\alpha\alpha}(s) \geq 0,$$

and a background for $\alpha = n_c + 1,\ldots,n$.

$\mathcal{P}_{\alpha\beta}(s)$ describes the interaction of the channels with the continuum. The form factors $\mathcal{F}_\alpha(s)$ are real functions on the unitary cut. Compound states correspond to bound states and resonances. In the presence of primitives, the correspondence becomes ambiguous.

The scattering amplitude has the form

$$A(s) = -(\Im\mathcal{D}_{\beta\alpha}(s))\mathcal{D}^{-1}_{\alpha\beta}(s). \tag{6}$$

It is not difficult to see that this amplitude satisfies the unitarity:

$$\begin{aligned} A(s) - A^*(s) = 2i\Im A(s) \\ &= -\Phi_2(s)\mathcal{F}_\alpha(s)\mathcal{D}^{-1}_{\alpha\beta}(s)\mathcal{F}_\beta(s) + \Phi_2(s)\mathcal{F}_\alpha(s)\mathcal{D}^{*-1}_{\alpha\beta}(s)\mathcal{F}_\beta(s) \\ &= \Phi_2(s)\mathcal{F}_\alpha(s)\mathcal{D}^{*-1}_{\alpha\beta}(s)\left(\mathcal{D}_{\beta\gamma}(s) - \mathcal{D}^*_{\beta\gamma}(s)\right)\mathcal{D}^{-1}_{\gamma\delta}(s)\mathcal{F}_\delta(s) \\ &= 2i\Phi_2(s)\mathcal{F}_\alpha(s)\mathcal{D}^{*-1}_{\alpha\beta}(s)\left(\Phi_2(s)\mathcal{F}_\beta(s)\mathcal{F}_\gamma(s)\right)\mathcal{D}^{-1}_{\gamma\delta}(s)\mathcal{F}_\delta(s) \\ &= 2i\left|\Phi_2(s)\mathcal{F}_\alpha(s)\mathcal{D}^{-1}_{\alpha\beta}(s)\mathcal{F}_\beta(s)\right|^2 \\ &= 2i\left|A(s)\right|^2, \end{aligned} \tag{7}$$

and can therefore be represented as $A(s) = e^{i\delta(s)}\sin\delta(s)$. The S matrix has the form

$$S(s) = 1 + 2iA(s) = \mathcal{D}^*_{\beta\alpha}(s)\mathcal{D}^{-1}_{\alpha\beta}(s). \tag{8}$$

Here and everywhere below, for any function on the real axis of the complex s-plane $F(s)$ means $F(s+i0)$, $F^*(s)$ means $F(s-i0)$. As analytic function, $F(s+i0)$ is defined on the first sheet of the Riemann surface. Eq. (8) generalizes eq. (2). Using equation

$$\mathcal{D}^{-1}_{\beta\alpha}(s)\det||\mathcal{D}(s)|| = \frac{1}{(n-1)!}\epsilon_{\alpha\alpha_2..\alpha_n}\epsilon_{\beta\beta_2...\beta_n}\mathcal{D}_{\alpha_2\beta_2}(s)\mathcal{D}_{\alpha_3\beta_3}(s)...\mathcal{D}_{\alpha_n\beta_n}(s),$$

where $\epsilon_{\alpha_1\alpha_2..\alpha_n}$ is the totally antisymmetric tensor, $\epsilon_{12...n} = +1$, one can write the N/D representation:

$$A(s) = \frac{\mathcal{N}(s)}{\det||\mathcal{D}(s)||}. \tag{9}$$

The numerator function has the form

$$\mathcal{N}(s) = -\frac{1}{(n-1)!}\epsilon_{\alpha_1\alpha_2..\alpha_n}\epsilon_{\beta_1\beta_2...\beta_n}(\Im\mathcal{D}_{\alpha_1\beta_1}(s))\mathcal{D}_{\alpha_2\beta_2}(s)...\mathcal{D}_{\alpha_n\beta_n}(s). \tag{10}$$

One can verify that, by virtue of Eq.(5), $\Im\mathcal{N}(s) = 0$ on the unitary cut, and so $\mathcal{N}(s)$ is an analytic function in the complex s-plane except for singular points of the phase space function $\Phi_2(s)$ and the form factors $\mathcal{F}_\alpha(s)$. The singularities of $\mathcal{F}_\alpha(s)$ originate from the exchange of particles in the crossing channels. In Eq.(10) $\mathcal{D}_{\alpha\beta}(s)$ can be replaced with $\Re\mathcal{D}_{\alpha\beta}(s)$.

The imaginary part of the determinant,

$$\det||\mathcal{D}(s)|| = \frac{1}{n!}\epsilon_{\alpha_1\alpha_2..\alpha_n}\epsilon_{\beta_1\beta_2...\beta_n}\mathcal{D}_{\alpha_1\beta_1}(s)\mathcal{D}_{\alpha_2\beta_2}(s)...\mathcal{D}_{\alpha_n\beta_n}(s), \tag{11}$$

gives the numerator function on the unitary cut

$$\Im\det||\mathcal{D}(s)|| = -\mathcal{N}(s). \tag{12}$$

Suppose we have on the real s-axis above s_0, n_+ simple zeros of $\mathcal{N}(s)$ with positive $\delta'(s)$ and n_- simple zeros of $\mathcal{N}(s)$ with negative ones. The zeros of the first and second kinds are denoted by $s_{+\beta}$ with $\beta = 1, \ldots, n_+$ and $s_{-\beta}$ with $\beta = 1, \ldots, n_-$, respectively. Hence we have $\delta(s_{\pm\beta}) = 0 \mod (\pi)$ and $\delta'(s_{\pm\beta}) \gtrless 0$.

Levinson's theorem establishes a connection between the asymptotic value of the scattering phase and the number of bound states n_B:

$$\delta(+\infty) - \delta(s_0) = -\pi n_B. \tag{13}$$

The values of n_+ and n_- are related to n_B. By continuity, there are three possibilities:

$$n_- - n_+ - n_B = \begin{cases} -1, & \text{if} \quad \delta'(s_0) < 0, \\ 0, & \text{if} \quad \delta'(s_0) \gtrless 0, \\ 1, & \text{if} \quad \delta'(s_0) > 0, \end{cases} \tag{14}$$

the prime denotes differentiation with respect to the momentum.

2.2. SIMONOV-DYSON MODEL

Noticeable simplifications arise when the form factors coincide up to normalization

$$\mathcal{F}_\alpha(s) = g_\alpha \mathcal{F}(s). \tag{15}$$

In such a case, the determinant of $\mathcal{D}_{\alpha\beta}(s)$ can easily be found. Its polynomial part is the product of $\mathcal{K}_{(\alpha)}$, while the dispersion part is linear in

$$\mathcal{P}_{\alpha\beta}(s) = g_\alpha g_\beta \Pi(s). \tag{16}$$

The result can be written in the form

$$\det ||\mathcal{D}(s)|| = \prod_\alpha \mathcal{K}_{(\alpha)} \left(1 - \Pi(s)\Lambda^{-1}(s)\right), \tag{17}$$

where

$$\Lambda^{-1}(s) = \sum_{\alpha=1}^{n_c} \frac{g_\alpha^2}{s - M_\alpha^2} + f. \tag{18}$$

The constant f is expressed through the constants C_α of the four-fermion interaction and g_α:

$$f = \sum_{\alpha=n_c+1}^{n} \frac{g_\alpha^2}{C_\alpha}. \tag{19}$$

The S matrix is the ratio of D function on two different sheets of the Riemann surface (see Eq. (2)), so multiplication or division of D by a polynomial is a valid operation. The canonical D function can be taken in the form

$$D(s) = \Lambda(s) - \Pi(s). \tag{20}$$

Dispersion integral is determined up to a polynomial. By the replacement $\Pi(s) \to \Pi(s) + C$ and $\Lambda(s) \to \Lambda(s) + C$, that leaves the D function invariant, one can redefine masses of the compound states and shift the P-matrix poles. The masses of the compound states are therefore model dependent. They are, however, located strictly between the consecutive CDD poles.

From a formal point of view, the distinction of the model [5] from the model [4] is very subtle. In the latter $\mathcal{F}^2(s) > 0$, whereas in the former $\mathcal{F}^2(s) \geq 0$.

In the model of Simonov, the zeros of $\mathcal{F}(s)$ on the unitary cut are thus permitted. Their existence is a necessary condition for the existence of the primitives. In terms of the s-channel exchange, repulsion is generated by the primitives and is related to the zeros of $\mathcal{F}(s)$.

2.3. REPRESENTATION $\Im D(s) \geq 0$

Consider the amplitude (6) in the vicinity of $s = s_{+\beta} > s_0$. Let $\mathcal{N}(s) \propto s - s_{+\beta}$, then $\det||\mathcal{D}(s)|| \approx a - i(s - s_{+\beta})$. Dividing $\mathcal{N}(s)$ and $\det||\mathcal{D}(s)||$ by $-s + s_{+\beta}$, one gets

$$A(s) \approx -\frac{1}{-\dfrac{a}{s - s_{+\beta}} + i}. \tag{21}$$

The phase shift behaves like $\delta(s) \approx (s - s_{+\beta})/a \mod (\pi)$. Using the condition $\delta'(s_{+\beta}) = 1/a > 0$ we conclude that a is positive. As compared to the imaginary part, the residue of the pole term in the denominator of $A(s)$ has the correct sign for a CDD pole.

Now, consider $A(s)$ in the vicinity of $s = s_{-\beta} > s_0$. Let $\mathcal{N}(s) \propto -s + s_{-\beta}$, then $\det||\mathcal{D}(s)|| \approx a + i(s - s_{-\beta})$. Using condition $\delta'(s_{-\beta}) < 0$ we conclude that a is positive and $A(s) \approx -1/(\frac{a}{s - s_{-\beta}} + i)$. The phase shift $\delta(s) \approx -(s - s_{-\beta})/a \mod (\pi)$ has a negative slope. As compared to the imaginary part, the residue of the pole term in the denominator of $A(s)$ has the wrong sign for a CDD pole. One can, however, multiply $\mathcal{N}(s)$ and $\det||\mathcal{D}(s)||$ by $(s - s_{-\beta})/a$ to give

$$A(s) \approx -\frac{1}{\dfrac{a}{s - s_{+\beta}} + i} \equiv -\frac{(s - s_{-\beta})^2/a}{s - s_{-\beta} + i(s - s_{-\beta})^2/a}. \tag{22}$$

The amplitude is proportional to the propagator of particle with mass $M_\beta = \sqrt{s_{-\beta}}$ and energy dependent width $\sqrt{s}\Gamma(s) \approx (s - s_{-\beta})^2/a$. The width vanishes on the mass shell. This is signature of primitive. A quantity $\sim (s - s_{-\beta})^2$ in the numerator can be regarded as an expansion of the primitive form factor squared near its mass where the form factor vanishes.

The $s_{+\beta}$ and $s_{-\beta}$ zeros should therefore be treated differently. The canonical

$N(s)$ and $D(s)$ functions can be taken to be

$$N(s) = \eta \frac{\pi_-(s)}{\pi_+(s)} \mathcal{N}(s), \quad (23)$$

$$D(s) = \eta \frac{\pi_-(s)}{\pi_+(s)} \det ||\mathcal{D}(s)||, \quad (24)$$

where

$$\pi_\pm(s) = \prod_{\beta=1}^{n_\pm} (s - s_{\pm\beta}). \quad (25)$$

The analytical properties of $\Im D(s) = -N(s)$ are the same as of $\mathcal{N}(s)$, $\Im D(s)$ does not change the sign on the unitary cut. Above the threshold $\mathcal{N}(s)$ is real, but not a sign-definite function. The expression $\pi_-(s)\mathcal{N}(s)/\pi_+(s)$ is a sign-definite function. Yet it can be both positive and negative. We multiplied the right-hand sides of Eqs. (23) and (24) by $\eta = \pm 1$ to make $N(s)$ non-positive. According to the convention, $\Im D(s) = -N(s) \geq 0$.

We thus made the identity transformation, which resulted in the non-negative imaginary part of $D(s)$. $\Im D(s)$ can therefore be interpreted in terms of the decay width.

$D(s)$ has n_+ poles at $s = s_{+\beta}$. The residues of $D(s)$ at $s = s_{+\beta}$ are real, since $\Im \det ||\mathcal{D}(s_{+\beta})||$ vanishes. $\Re D(s)$ and $\Im D(s)$ have n_- first- and second-order zeros at $s = s_{-\beta}$. In addition, $\Re D(s)$ is zero everywhere, where $\Re \det ||\mathcal{D}(s)||$ vanishes.

We thus arrive at the representation (20) of the D function. The dispersion part takes the form

$$\Pi(s) = -\frac{s^\gamma}{\pi} \int_{s_0}^{+\infty} \Phi_2(s') \frac{\mathcal{F}^2(s')}{s' - s} \frac{ds'}{s'^\gamma}. \quad (26)$$

The leading term $\sim s^{n_c}$ of $\det ||\mathcal{D}(s)||$ at infinity arises in calculating the determinant because of the multiplication of the diagonal elements. The imaginary part of $\det ||\mathcal{D}(s)||$ equals $O(s^{n_c - 1} \Im \mathcal{D}_{\alpha\beta}(s))$ and $O(s^{n_c} \Im \mathcal{D}_{\alpha\beta}(s))$ at $s \to +\infty$ for $n_f = 0$ and $1 \leq n_f$. Consequently, we write the dispersion integral with the subtraction. If the coefficient γ is chosen as $\gamma = n_c - 1 + n_- - n_+$ and $\gamma = n_c + n_- - n_+$ for $n_f = 0$ and $1 \leq n_f$, respectively, the convergence of the dispersion integral in Eq. (26) is the same as in Eq. (4).

The form factor $\mathcal{F}(s)$ satisfies the equation

$$\Phi_2(s)\mathcal{F}^2(s) \equiv \Im D(s) = -\eta \frac{\pi_-(s)}{\pi_+(s)} \mathcal{N}(s) \geq 0. \quad (27)$$

$\mathcal{F}(s)$, together with its first derivative, can be taken a continuous function on the unitary cut. The form factor defined this way becomes an analytic function in the complex s-plane, whose singularities are the singularities of $\Phi_2(s)$ and $\mathcal{N}(s)$. $\mathcal{F}(s)$ has simple zeros at $s = s_{-\beta}$.

$\Lambda(s)$ is a non-dispersive part of D and a rational function. Its simple poles are located at $s = s_{+\beta}$. The residues have negative sign, so the poles are the CDD poles.

Under certain conditions, which we have to clarify, zeros of $\Lambda(s)$ are associated with masses of compound states. In our case, these include the values of $s_{-\beta}$ and perhaps additional roots of equation $\Lambda(s) = 0$. n_* zeros of the second type are also denoted by $s_{-\beta}$, the index β takes the values $n_- + 1, \dots, n_- + n_*$. The polynomial part of $\det ||\mathcal{D}(s)||$ has the degree n_c, so one can expect that $n_* = n_c$.

$\Lambda(s)$ is thus the ratio of two polynomials of the degrees $n_- + n_*$ and n_+. This function may be provided the representation of (18) for

$$n_- - n_+ + n_* = \begin{cases} 0, & \text{if} \quad f \neq 0, \\ 1, & \text{if} \quad f = 0. \end{cases} \tag{28}$$

Equations (14) and (28) being combined give strong limit $n_c = n_* \leq 2$.

The structure of $\Lambda(s)$ from Eq. (18) is very simple. Zeros and poles of $\Lambda(s)$ alternate. This restriction should be imposed on the non-dispersive part of the D function (24). If between two consecutive CDD poles only one zero of $\Lambda(s)$ exists, the reducibility of the hybrid Lee model to the Simonov-Dyson model is guaranteed.

2.4. WHAT IF THERE IS NO REDUCIBILITY?

It is not difficult to imagine a situation in which the relations (28) are not fulfilled. For example, the phase shift can behave as $\delta(s) = -kb$. In potential scattering theory, this phase describes the scattering of a particle by an infinitely high potential wall located at a distance b from the origin. We leave aside the question of whether it is possible to generate such a phase shift in the Lee model. In the case of $\delta(s) = -kb$, we have an infinite number of intersections of the levels $-n\pi$ with a negative slope. In the vicinity of each such point, the scattering amplitude is given by Eq. (22). We conclude, therefore, that the amplitude there is dominated by the s-channel exchange of a primitive with mass $2\sqrt{m^2 + (n\pi/b)^2}$, where m is the nucleon mass. Until now it was assumed that the number of zeros of the amplitude with a negative slope is finite. For $\delta(s) = -kb$, however, $n_- = +\infty$ and $n_+ = n_* = 0$. We thus went beyond assumptions of the models discussed. †

When the interaction with the continuum is switched off, $\Pi(s) \equiv 0$, each compound state becomes zero of the D function. In this sense, compound states can be regarded as bare particles of the s-channel.

However, there may exist zeros of the D function which do not correspond to compound states. These are due solely to the interaction. In general, only a fraction of zeros of the D function corresponds to compound states and hence the P-matrix

†In the model [5] with infinite number of the zeros, $\delta(s) = -kb$ can be reproduced asymptotically for $s \to +\infty$.

poles. Another part disappears when the interaction is switched off. Such primitives match neither compound states nor P-matrix poles.

In all cases, however, we have the decomposition (22), so that there are reasons to highlight contributions of primitives, regardless of whether the scattering problem admits solution in terms of an s-channel exchange model.

2.5. BENJAMINS-VAN DIJK MODEL

Benjamins and van Dijk [10] used a hybrid model of Lee with $n_c = n_f = 1$ to describe the S-wave nucleon-nucleon interaction with the aim of constraining the admixture of the elementary particle component of the deuteron. The compound state masses are listed in Table 1. The numerator function $\mathcal{N}(s)$ has two zeros on the unitary cut. The first low-energy one with $\delta'(s) < 0$ is of the primitive type. The second high-energy one with $\delta'(s) > 0$ corresponds to the CDD pole. We thus have $n_- = n_+ = 1$. The zeros occur at $T_{lab} = 354$ MeV and at about 700 MeV in the $^3S_1$ channel, and at $T_{lab} = 265$ MeV and at about 1 GeV in the $^1S_0$ channel. This case corresponds to $\gamma = 0$ and thereby does not require subtraction.

Upon the reduction, the lower zeros of $\mathcal{N}(s)$ determine the masses of primitives, while the higher zeros determine the CDD poles. These zeros lead to the existence of two compound states in each of the channels. Their masses are determined from the equation $\Lambda(s) = 0$. The second mass M_2 is, however, very large and can not be determined reliably. The physical states correspond to solutions of Eq. (1). They are a primitive and a high-mass resonance. In Table 1 we compare parameters of the model [10] before and after the reduction with parameters of the model [6]. In the original model, $\det||D(s)||$ does not have CDD poles, while the equation $\det||D(s)|| = 0$ does not have roots on the unitary cut. The CDD poles and the primitives appear upon the reduction.

The hybrid Lee model in the version of Benjamins and van Dijk can therefore be reduced to and interpreted physically in terms of the Simonov-Dyson model.

3. ONE-BOSON EXCHANGE AND INVERSE SCATTERING PROBLEM FOR s-CHANNEL

The specific properties of the Lee model still were used very little. Any scattering amplitude can be written as the N/D ratio.

In our reduction scheme, zeros of $\mathcal{N}(s)$ on the unitary cut play a special role . They define polynomials $\pi_\pm(s)$ according to Eq. (25). By multiplying the initial N and D functions with a rational function $\pi_-(s)/\pi_+(s)$, we obtain $D(s)$ with a non-negative imaginary part, CDD poles, and zeros in a neighborhood of which $D(s)$ looks like denominator of the Breit-Wigner formula with energy-dependent width (see Eq. (22)). $\Im D(s) \geq 0$ can be interpreted as the probability.

Table 1.

Parameters of the hybrid Lee model [10], the reduced hybrid Lee (RHL) model, and the model [6] in the $^3S_1$ and $^1S_0$ channels of the nucleon-nucleon scattering. $M_i^{[0]}$ and M_i are masses (in MeV) of the compound states and the physical states, respectively. M is position of the CDD pole (in MeV).

$^3S_1$	compound states		physical states		CDD pole
Model	$M_1^{[0]}$	$M_2^{[0]}$	M_1	M_2	M
[10]	2228		high		
RHL	2167	$\sim 10^7$	2047	high	2204
[6]	2047		2047		3203
$^1S_0$	compound states		physical states		CDD pole
Model	$M_1^{[0]}$	$M_2^{[0]}$	M_1	M_2	M
[10]	2328		high		
RHL	2310	$\sim 10^5$	2006	high	2321
[6]	2006		2006		2916

Reduction scheme used in Sec. 2.3 is general enough to be applied to the OBE models, or even to redefine the phenomenological N and D of functions constructed on the basis of the experimental phase shifts.

Let the scattering phase be known experimentally in the energy range $(s_0, +\infty)$. We construct the Jost function

$$D_J(s) = \exp(-\frac{s}{\pi}\int_{s_0}^{+\infty}\frac{\delta(s')}{s'(s'-s)}ds'). \tag{29}$$

It is assumed that the dispersion integral converges well enough so that asymptotically $D_J(s) = O(1)$. Analyzing the imaginary part, we isolate its zeros $s_{+\beta}$ and $s_{-\beta}$ with positive and negative slopes of the phase shift. These zeros are the zeros of the scattering amplitude. After that, we redefine $D_J(s)$ as in Eq. (24) and introduce

$$D(s) = \eta\frac{\pi_-(s)}{\pi_+(s)}D_J(s). \tag{30}$$

As a result, $\Im D(s)$ is non-negative. The difference between $D(s)$ and its dispersive part is a rational function $\Lambda(s)$ whose poles are the CDD poles. The zeros of $\Lambda(s)$ could determine the masses of compound states associated with resonances and primitives (as well as bound states and virtual states not discussed here).

It is known that any scattering data set can be associated with a smooth potential whose physical origin lies in the t-channel exchange.

A sufficient condition for the existence of an s-channel exchange model, in which the scattering phase $\delta(s)$ is reproducible, is the representation of $\Lambda(s)$ in the form of the function (18). This is possible if zeros and poles of $\Lambda(s)$ alternate. Under these conditions the inverse scattering problem can successfully be solved in the sense of finding physically meaningful parameters of an s-channel exchange model.

An example of such a solution is the reduction of hybrid Lee model discussed in Sect. 2.5 provided we consider the scattering phase as empirical quantity. The D function constructed there has the correct analytical properties and, up to normalization, coincides therefore with the function (30).

In Ref. [6] empirical phases of the nucleon-nucleon scattering in the $^3S_1$ and $^1S_0$ channels are described using a version of the model [5] after embedding correct analytic properties. We further describe the nucleon-nucleon scattering in the $^3P_1$, $^1P_1$ and $^3P_0$ channels [11]. In the cases considered, the correct analytical properties are satisfied, so that the constructed D functions essentially coincide with the function (30), up to normalizations, neglecting inelasticity, and the incomplete knowledge of the experimental phase shifts. In Refs. [6,11], the inverse scattering problem is not posed literally because of involvement of *a priori* form factor entering the vertices of compound states and the contact four-fermion vertices. Masses of compound states and their couplings with the continuum, as well as the four-fermion couplings, are however determined completely in the spirit of the inverse scattering procedure for an s-channel exchange interaction.

4. CONCLUSION

The primitives are simple zeros of D function on the unitary cut in the normalization where the imaginary part of D is non-negative and the only poles of D are the CDD poles. In the vicinity of the primitive the scattering amplitude is described by the Breit-Wigner formula. Specificity of primitives is the disappearance of their width on the mass shell.

In this paper, we found conditions under which OBE models with the t-channel meson exchange are dual to the Simonov-Dyson model where the nucleon-nucleon interaction is provided by the s-channel exchange of $6q$-primitives. The analysis is based on the N/D representation of the scattering amplitude and the possibility to redefine the N and D functions so as to meet the requirement of the non-negativity of $\Im D(s)$. The proposed technique with no significant changes applies to the inverse scattering problem of the s-channel exchange in the general case.

We showed that the duality takes place provided the non-dispersive part of D function has alternating zeros and poles. This condition also is sufficient for solving the inverse scattering problem for the s-channel exchange.

There are good phenomenological reasons to believe that the nucleon-nucleon interaction is of the type of interactions for which the duality holds at least approximately.

The arguments can be much detailed when considering reduction of the Benjamins and van Dijk version of the hybrid Lee model. This model, as well as the Simonov-Dyson model, provides reasonable description of the elastic nucleon-nucleon scattering data. Although sets of the compound states are different, the models were found to be equivalent.

A solution of the inverse scattering problem for the s-channel exchange mechanism was presented for a limited set of systems. The problem of constructing an s-channel exchange model, in which the inverse scattering problem admits solution for any data set, deserves additional study.

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