

PARTICLE–CORE COUPLING MODEL APPROACH TO STRUCTURE OF THE LOW–LYING STATES OF ^{11}B AND THE EFFECT OF THE PAULI BLOCKING

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The particle–core coupling model with a restricted configurational space containing only 0_1^+ and 2_1^+ states is applied to describe the low–lying states of ^{11}B . Using only one parameter, a mixing coefficient for the ground state wave function, twelve observables, for which there are known the experimental data, are successfully described. It indicates on the simple correlation relations between data. It is shown that the Pauli blocking effect influences very strongly on the properties of the $7/2_1^-$ state of ^{11}B .

Key words: Particle–core coupling, Gamow–Teller transitions, Pauli blocking.

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1. INTRODUCTION

The 1p shell nuclei have been a testing ground for nuclear models since the beginning of studies in nuclear structure. Different approaches, namely, with a basis restricted by the 1p configurational space [1, 2], particle–core coupling model [3], cluster model [4–7] and *ab initio* shell model calculations [8] have been applied to describe the properties of these nuclei. In the framework of the shell model one type of the approaches was related to checking different forms of the nucleon–nucleon interaction, the other type of the approaches consider the matrix elements of the two–body interaction as the parameters which must be fitted by data.

The simple picture of the low–lying excited states, like that seen in ^{11}B , looks very suitable to treat the properties of these states in the framework of the particle–core coupling model. This approach being quite simple can be also used in combination with the cluster model ideas if the experimental data indicate the necessity to take into account the excited states of some clusters.

The simple models like the particle–core coupling model contains a small number of parameters. In fact this gives the possibility to obtain some relations which contain only the experimentally observed quantities. These relations will reflect only the basic structure of the assumed wave functions without using a concrete and detailed microscopic structure of the core states.

Concerning a justification of the particle–core coupling model we can indicate on the results obtained in [9]. Calculations performed in [9], based on the assumption of a weak particle–hole interaction compare to the interaction between particles

in the same shell [10], have shown that the calculated wave functions can be well interpreted as a single combination of simple states in neighboring nuclei. For ^{11}B it means a coupling of a proton hole in $p_{3/2}$ state to the ground and the first excited 2^+ states of ^{12}C .

At the same time using the particle–core coupling model we can see the effects of an addition of an odd particle or a hole to the concrete state. Because of the Pauli blocking of some configurations the core state is modified very strongly.

It is the aim of the present paper to apply the particle–core coupling model to description of the properties of the low–lying states of ^{11}B for which there is a sufficient amount of the experimental information. Comparing to the previous particle–core model calculations of the states of ^{11}B the amount of the experimental information has increased significantly.

2. MODEL AND A TEST OF THE WAVE FUNCTIONS

The general expression for the wave function in the particle–core coupling model is the following

$$|J_n M\rangle = \sum_{j, I_k} h_{j I_k}^{J_n} \left(\alpha_j^+ |I_k\rangle \right)_{JM} \quad (1)$$

where α_j^+ is an operator creating particle or hole in the single particle state with angular momentum j , $|I_k\rangle$ is a state of a core with angular momentum I . In the case of ^{11}B $\alpha_{j=3/2}^+$ creates a hole in the $p_{3/2}$ orbit, and only the ground state and the first excited 2^+ state of ^{12}C are taken into account as the states of the core. In this basis the wave vectors of the states with $J_k^\pi = 1/2_1^-, 5/2_1^-$ and $7/2_1^-$ contains only one component

$$|J_k^\pi = 1/2_1^-, 5/2_1^-, 7/2_1^-\rangle_M = \left(\alpha_{p_{3/2}}^+ |2_1^+\rangle \right)_{JM} \quad (2)$$

The wave vectors of the states with $J^\pi = 3/2^-$ have two components

$$|3/2_1^-\rangle = \frac{1}{\sqrt{1+w^2}} \left(\alpha_{p_{3/2}}^+ |0_1^+\rangle - w \cdot (\alpha_{p_{3/2}}^+ |2_1^+\rangle)_{3/2} \right), \quad (3)$$

$$|3/2_2^-\rangle = \frac{1}{\sqrt{1+w^2}} \left(w \cdot \alpha_{p_{3/2}}^+ |0_1^+\rangle + (\alpha_{p_{3/2}}^+ |2_1^+\rangle)_{3/2} \right), \quad (4)$$

Thus, there is only one parameter, the mixing coefficient w , which is used to describe the eigenvectors. The best description of data is obtained with $w=0.9(0.05)$. With these wave functions we have calculated the magnetic dipole and electric quadrupole moments of the ground state, the M1 and E2 transition probabilities between the low–lying states and the Gamov–Teller transition strength $B(\text{GT})$ for the $^{11}\text{B} \rightarrow ^{11}\text{C}$ transitions. All together seventeen observables are considered. In the calculations

of the magnetic dipole moments and B(M1) reduced transition probabilities with the M1 transition operator given by

$$\mathcal{M}_\mu(M1) = \sqrt{\frac{3}{4\pi}} (g_l l_\mu + g_s s_\mu + g_R R_\mu) \quad (5)$$

we used the nonrenormalized values of the nucleon g -factors: +5.58 for protons and -3.82 for neutrons. The single hole matrix element of the quadrupole operator has been determined using harmonic oscillator radial wave functions of the $p_{3/2}$ state. The matrix elements of the E2 operator

$$Q_{2\mu} = \sum_{m,m'} \langle p_{3/2} m' | r^2 Y_{2\mu} | p_{3/2} m \rangle a_{p_{3/2} m'}^+ a_{p_{3/2} m} + \sum_{I,I'=0,2} \sum_{MM'} \langle I' M' | Q_{2\mu} | I M \rangle | I' M' \rangle \langle I M | \quad (6)$$

between the core states have been extracted from the experimental data on $B(E2; 0_1^+ \rightarrow 2_1^+)$ and spectroscopic quadrupole moment $Q(2_1^+)$ in ^{12}C . The resulting values of the reduced matrix elements are: $|\langle 2_1^+ || \mathcal{M}(E2) || 0_1^+ \rangle| = 6.16e \cdot fm^2$, $|\langle 2_1^+ || \mathcal{M}(E2) || 2_1^+ \rangle| / |\langle 0_1^+ || \mathcal{M}(E2) || 2_1^+ \rangle| = 1.3(7)$. The experimental data on ^{12}C used in the calculations are presented in the Table 1.

Table 1.

Experimental data on ^{12}C used in the calculations.

$E(2_1^+)$	4.439 MeV
$B(E2; 2_1^+ \rightarrow 0_1^+)$	$7.6(4) e^2 \cdot fm^4$
$Q(2_1^+)$	$+6(3) e \cdot fm^2$

The relative sign of the matrix elements is not determined by these data. By analogy with the well deformed axially symmetric nuclei we assume that the ratio $|\langle 2_1^+ || \mathcal{M}(E2) || 2_1^+ \rangle| / |\langle 0_1^+ || \mathcal{M}(E2) || 2_1^+ \rangle|$ is negative. The results of calculations are shown in the Tables 2–3. The agreement between the calculated and the experimental results is almost perfect for the magnetic dipole moments of the ground states of the ^{11}B and ^{11}C . The same value of w was used for the wave functions of ^{11}B and ^{11}C . The calculated value of the spectroscopic quadrupole moment of the ground state of ^{11}B is also in a good agreement with data. The deviation is less than 20%.

The results obtained for the M1 reduced transition probabilities are also in a good or reasonable agreement with the experimental data excluding the $B(M1; 7/2_1^- \rightarrow 5/2_1^-)$. In the last case a disagreement between the experimental and calculated values is very large. This problem is probably related to description of the $7/2_1^-$ state in the particle–core coupling model. This will be discussed in the next section.

The known strong E2 transition probability between the $5/2_1^-$ and $3/2_1^-$ states

Table 2.

The calculated and experimental values of the ground state magnetic dipole moment (in μ_N) and the M1 reduced transition probabilities (in μ_N^2).

μ and B(M1)	cal.	exp
$\mu(3/2_1^-, ^{11}\text{B})$	+2.70	+2.69
$\mu(3/2_1^-, ^{11}\text{C})$	-0.96	-0.96
$B(M1; 1/2_1^- \rightarrow 3/2_1^-)$	1.32	1.07(7)
$B(M1; 5/2_1^- \rightarrow 3/2_1^-)$	0.62	0.52(2)
$B(M1; 3/2_2^- \rightarrow 1/2_1^-)$	0.81	0.98(4)
$B(M1; 3/2_2^- \rightarrow 3/2_1^-)$	0.58	1.13(4)

is described very well. However, the calculated value of $B(E2; 7/2_1^- \rightarrow 3/2_1^-)$ is in 2.7 times larger than the experimental value. We see again that the larger deviations of the calculated results from the experimental data are observed when the characteristics of the $7/2_1^-$ state are involved in the calculations.

Table 3.

The calculated and experimental values of the ground state electric quadrupole moment (in units $e \cdot fm^2$) and the E2 reduced transition probabilities (in units $e^2 \cdot fm^4$).

$Q(J_{gr})$ and B(E2)	cal.	exp
$Q(3/2_1^-, ^{11}\text{B})$	3.64	4.06
$B(E2; 1/2_1^- \rightarrow 3/2_1^-)$	12.3	–
$B(E2; 5/2_1^- \rightarrow 3/2_1^-)$	12.3	14(3)
$B(E2; 3/2_2^- \rightarrow 3/2_1^-)$	1.8	–
$B(E2; 3/2_2^- \rightarrow 1/2_1^-)$	0.03	–

In the Table 4 are shown the results of calculations of B(GT) for the $^{11}\text{B} \rightarrow ^{11}\text{C}$ transition determined by the expression

$$B(GT; J_i, T_i, M_T \rightarrow J_f, T_f, M_T \pm 1) = \frac{g_A^2}{4\pi} \sum_{\mu, m_i} |\langle J_f, m_f, T_f, M_T \pm 1 | t_{\pm} \sigma_{\mu} | J_i, m_i, T_i, M_T \rangle|^2. \quad (7)$$

They are compared with the recent experimental data [11–13]. The agreement is quite satisfactory.

Thus, using only one parameter, namely, an amount of mixing in the ground state wave function, to characterize the wave functions of the ground and the excited states we describe twelve observables (magnetic dipole and electric quadrupole moments, B(M1), B(E2) and B(GT)).

Table 4.

The calculated and experimental B(GT) values for the ^{11}B to ^{11}C transitions (in units $g_A^2/4\pi$).

B(GT)	cal.	exp
$B(GT; 3/2_1^- \rightarrow 3/2_1^-)$	0.687	0.35(1)
$B(GT; 3/2_1^- \rightarrow 1/2_1^-)$	0.298	0.44(2)
$B(GT; 3/2_1^- \rightarrow 5/2_1^-)$	0.418	0.53(3)
$B(GT; 3/2_1^- \rightarrow 3/2_2^-)$	0.264	0.53(3)
SUMMED	1.67	1.85

3. DESCRIPTION OF THE ENERGIES AND THE EFFECT OF THE PAULI PRINCIPLE ON THE WAVE FUNCTION OF THE $7/2_1^-$ STATE

One of the procedures used to describe the properties of the light nuclei was to consider the matrix elements of the two-body interaction as parameters and to fit them so as to describe the nuclear excitation spectra in nuclei belonging to the same oscillator shell [1, 14, 15]. Later the three-body term has been included in a fitting procedure [2, 8]. Following to this logic we should introduce a particle-core interaction with different multipole-multipole terms, i.e. not only a quadrupole-quadrupole interaction as it is adapted in the case of heavy nuclei, and determine the interaction parameters so as to achieve a better description of the energies and the wave functions, performing calculations in the selected basis.

In the configurational space restricted by the proton hole in the $p_{3/2}$ single particle state and 0_1^+ and 2_1^+ states of the core the most general form of the Hamiltonian is the following

$$H = E(2_1^+) \sum_M |2_1^+ M\rangle \langle 2_1^+ M| + \sum_{\lambda=0}^3 (-1)^{\lambda-\mu} q_{\lambda\mu} Q_{\lambda-\mu}^c \quad (8)$$

where $q_{\lambda\mu}$ is a single hole multipole moment operator

$$q_{\lambda\mu} = \sum_{m,m'} C_{3/2m\lambda\mu}^{3/2m'} a_{p_{3/2}m'}^+ a_{p_{3/2}m}, \quad (9)$$

and $Q_{\lambda\mu}^c$ is a core multipole moment operator

$$Q_{\lambda\mu}^c = \sum_{I,I'=0,2} \sqrt{\frac{5}{2I'+1}} Q_{\lambda}^{II'} \sum_{M,M'} C_{IM\lambda\mu}^{I'M'} |I'M'\rangle \langle IM|. \quad (10)$$

In principle, we should include in (8) the single particle term. However, since proton hole occupies only one state $p_{3/2}$ the single particle term is a constant, in fact. In the expressions above the core multipole moment operators $Q_{\lambda\mu}^c$ include interaction constants as the multipliers and this is a reason why we did not indicate in (8) the interaction constants. They are included into definition of $Q_{\lambda}^{II'}$.

The equation (8) can be rewritten in the following way

$$H = E(2_1^+) \sum_M |2_1^+ M\rangle \langle 2_1^+ M| + \sum_{J,M} g_{22}^J \left(a_{p_{3/2}}^+ |2_1^+\rangle \right)_{JM} \left(\langle 2_1^+ | a_{p_{3/2}} \right)_{JM} \\ + g_{02}^{3/2} \sum_M \left(\left(a_{p_{3/2}}^+ |2_1^+\rangle \right)_{3/2M} \langle 0_1^+ | a_{p_{3/2}M} + h.c. \right), \quad (11)$$

where

$$g_{22}^J = (-1)^{3/2-J} 2\sqrt{5} \sum_{\lambda=0}^3 Q_\lambda^{22} \left\{ \begin{matrix} 3/2 & 2 & J \\ 2 & 3/2 & \lambda \end{matrix} \right\}, \quad (12)$$

and

$$g_{02}^{3/2} = Q_2^{22} \langle 0_1^+ \| \mathcal{M}(E2) \| 2_1^+ \rangle / \langle 2_1^+ \| \mathcal{M}(E2) \| 2_1^+ \rangle. \quad (13)$$

Thus, in (11) we have four free parameters $Q_\lambda^{22} (\lambda = 0, 1, 2, 3)$ to determine four excitation energies and the mixing coefficient w in the ground state wave function. The value of the mixing coefficient is already fixed in the previous section by description of the spectroscopic electromagnetic moments and the reduced M1 and E2 transition probabilities. Therefore we have only three parameters to describe four excitation energies. We have fixed these parameters so as to put the excitation energies of the $1/2_1^-$, $3/2_1^-$ and $5/2_1^-$ states to be equal to the experimental values. Then we have calculated the excitation energy of the $7/2_1^-$ state. As a result we have obtained a very low value of the excitation energy of this state. To understand a reason we have considered the wave function of the $7/2_1^-$ state in more details. The main components of the 2_1^+ state are the proton and neutron configurations $\left(p_{3/2}^{-1} p_{1/2} \right)_2$. However, in the case of the $7/2_1^-$ of ^{11}B the proton component $\left(p_{3/2}^{-1} p_{1/2} \right)_2$ is blocked because of the Pauli principle. The maximum angular momentum of the two holes in the $p_{3/2}$ state is 2. Adding to this value an angular momentum of a particle in the $p_{1/2}$ state we can obtain maximum $J = 5/2$. Indeed,

$$|J^-\rangle \sim \left(p_{3/2}^{-1} \left(p_{3/2}^{-1} p_{1/2} \right)_2 \right)_J^{(proton)} + \dots \sim \left(\left(p_{3/2}^{-1} p_{3/2}^{-1} \right)_{I=0,2} p_{1/2} \right)_{J=1/2,3/2,5/2}^{(proton)} + \dots \quad (14)$$

Thus, in the $7/2_1^-$ state the 2_1^+ state of the core is modified very strongly because the proton $\left(p_{3/2}^{-1} p_{1/2} \right)_2$ component of this state is blocked by the Pauli principle and only the neutron component $\left(p_{3/2}^{-1} p_{1/2} \right)_2$ survive in the case of ^{11}B . It means that the correlation energy in the 2_1^+ state is decreased roughly by factor 2 and the energy of the $7/2_1^-$ is shifted up by this amount. Let us estimate approximately the value of this shift. Using the standard formula for the energy gap produced by the pair

correlations [16]

$$2\Delta_p = 2B(N, Z) - B(N, Z - 1) - B(N, Z + 1) \quad (15)$$

we obtain $2\Delta_p \approx 14$ MeV. It means that the two quasi particle states should appear not lower than at 14 MeV. However, the energy of the 2_1^+ state is 4.3 MeV. So, correlations shift the energy of this state down by 9.7 MeV. A half of this value is just a shift up of the energy of the $7/2_1^-$ due to the Pauli blocking. Together with the value 1.2 MeV which we obtained in our calculations it gives for the excitation energy of the $7/2_1^-$ state the value 6.05 MeV. The experimental value is 6.74 MeV. Mention that the component of the particle–core multipole–multipole interaction with $\lambda = 3$ gives a very small contribution to the energies.

The Pauli blocking effect in the $7/2_1^-$ state, which is not taken into account in our calculations of $B(E2; 7/2_1^- \rightarrow 3/2_1^-)$, explains qualitatively why the calculated value of this quantity, which is equal to $5.4 e^2 fm^4$, exceeds almost in three times the experimental result $1.9(4)e^2 fm^4$.

Essential modification of the core wave function in the $7/2_1^-$ state by the Pauli blocking effect can increase a contribution of the other components of the wave function, which do not belong to our very restricted configurational space, to the $B(M1; 7/2_1^- \rightarrow 5/2_1^-)$. This can be a reason of a contradiction between the calculated result $0.84 \mu_N^2$ and the experimental value $0.006(2) \mu_N^2$.

4. SUMMARY

The particle–core coupling model with a very restricted configurational space containing only 0_1^+ and 2_1^+ states of the core and $p_{3/2}$ single hole state for an odd particle is applied to description of the properties of the low lying states of ^{11}B . Having only one parameter – the mixing coefficient for the ground state wave function – we describe successfully twelve observables (magnetic dipole and electric quadrupole moments, B(M1), B(E2) and B(GT)) for which there are known the experimental data. This indicates on the simple correlation relations between data. The ground state mixing parameter can be expressed through one of the B(M1) ratios. Then all other experimental data can be presented as functions of this ratio.

We have shown also that a strong disagreement between the calculated results and the experimental data related to the $7/2_1^-$ state can be explained by a strong influence of the Pauli blocking effect on the core 2_1^+ state in the case of the $7/2_1^-$ state.

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