Dedicated to Academician Aureliu Sandulescu's 80th Anniversary

THE NEUTRINOLESS DOUBLE BETA DECAY AND THE NEUTRINO MASS

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Neutrinoless Double Beta Decay ($0\nu\beta\beta$) is presently the only known experiment to distinguish between Dirac neutrinos, different from their antiparticles, and Majorana neutrinos, identical with their antiparticles. In addition $0\nu\beta\beta$ allows to determine the absolute scale of the neutrino masses. This is not possible with neutrino oscillations. To determine the neutrino masses one must assume, that the light Majorana neutrino exchange is the leading mechanism for $0\nu\beta\beta$ and that the matrix element of this transition can be calculated reliably. The experimental $0\nu\beta\beta$ transition amplitude in this mechanism is a product of the light left handed effective Majorana neutrino mass and of this transition matrix element. The different methods, Quasi-particle Random Phase Approximation (QRPA), Shell Model (SM), Projected Hartree-Fock-Bogoliubov (PHFB) and Interacting Boson Model (IBM2) used in the literature and the reliability of the matrix elements in these approaches are reviewed. In the second part it is investigated how one can determine the leading mechanism or mechanisms from the data of the $0\nu\beta\beta$ decay in different nuclei. Explicit expressions are given for the transition matrix elements. It is shown that possible interference terms allow to test CP (Charge and Parity conjugation) violation.

Key words: Double beta decay; Nuclear matrix element; Quasiparticle random phase approximation; Shell Model; Hartree-Fock-Bogoliubov; Interacting Boson Model.

1. INTRODUCTION

The neutrinoless double beta decay ($0\nu\beta\beta$) allows as only known experiment to distinguish between Dirac (neutrinos different from the antiparticles) and Majorana neutrinos (neutrinos identical with its antiparticles apart of a possible phase). It also provides a method to determine the absolute scale of of all three neutrino masses in connection with neutrino oscillation data. Neutrino oscillations give only the differences of the masses squared. The determination of the masses is possible, if one assumes, that the light left handed Majorana neutrino exchange is the leading mechanism for the neutrinoless double beta decay and one is also able to calculate reliably the transition matrix element. The experimental amplitude called normally $\mathcal{T}^{0\nu}$ is the product of the effective Majorana neutrino mass and a transition matrix element $\mathcal{M}_{\nu}^{0\nu}$. To determine the absolute masses the matrix element $\mathcal{M}_{\nu}^{0\nu}$ is as important as

the data for the $0\nu\beta\beta$ transition. The different methods used to calculate these matrix elements are presented and compared with their advantages and their drawbacks [1]. The methods are the Quasi-particle Random Phase Approximation (QRPA) for spherical (chapter 2) and deformed (chapter 3) nuclei [1–6], the Shell Model (SM) [7–9], the Projected Hartree-Fock-Bogoliubov (PHFB) approach [10–14] and the Interacting Boson Model (IBM2) [15]. (In [13] Pradfulla Rath *et al.* corrected a missing factor 2 and thus in all older publications the matrix element have to be multiplied in their work [11, 12] by a factor 2.)

In the second part the assumption, that the light left handed Majorana neutrino exchange is the leading mechanism is not assumed and possibilities to determine the leading mechanism are given [16–18] even in cases, where two equally strong mechanisms interfere. There the relative phase can test the CP (combined Charge conjugation and Parity) conservation or violation due to relative Majorana phases. For CP conservation the strength coefficients η must be real and thus the relative phase angle can only be zero or 180 degrees.

Figure 1 shows the diagram for the neutrinoless double beta decay of ${}^{76}_{32}\mathrm{Ge}_{44}$ to ${}^{76}_{34}\mathrm{Se}_{42}$ through the intermediate nucleus ${}^{76}_{33}\mathrm{As}_{43}$.

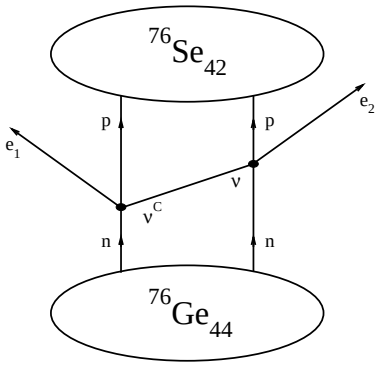

Fig. 1 – The neutrinoless double beta decay ${}^{76}_{32}\mathrm{Ge}_{44} \rightarrow {}^{76}_{34}\mathrm{Se}_{42}$ through the intermediate nucleus ${}^{76}_{33}\mathrm{As}_{43}$. The neutrino is emitted at the left vertex as an antineutrino ν^c with positive helicity due to the left handed interaction and must be absorbed at the right vertex as a neutrino with negative helicity. This is only possible for a massive Majorana neutrino, which violates helicity conservation and is a 50% mixture of neutrino and antineutrino. The two emitted electrons violate lepton number conservation. This is possible due to the lepton and antilepton mixture of the Majorana neutrino.

In addition to the light left handed Majorana neutrino exchange, one has other possible mechanisms as cause for the neutrinoless double beta decay: Grand Unification (GUT), Supersymmetry (SUSY) and extensions to extra dimensions. We shall discuss here extension to GUT's and SUSY.

We consider first a left-right symmetric model of GUT.

$$W_1 = \cos\vartheta_{GUT} + \sin\vartheta_{GUT} \quad W_2 = -\sin\vartheta_{GUT} + \cos\vartheta_{GUT} \tag{1}$$

Here W_1 is the usual vector bosons of Rubbia and coworkers of 80.4 GeV mainly responsible for the left handed weak interaction. This allows at each vertex in figure 1 left and right handed interactions and one handedness at the leptonic, electron-neutrino side and an other handedness at the hadronic, neutron-proton side inside each four-vertex. In addition one has SUSY contributions, where mainly the three linear terms are responsible for the lepton number violation [16–19]. This will be discussed in chapter four of this contribution.

Fermi's Golden Rule of second order time dependent perturbation theory yields:

$$\mathcal{T}^{0\nu} = \int \mathrm{d}E_k(\nu) \sum_k \frac{< f|\hat{H}_W|k >< k|\hat{H}_W|i >}{E_{0^+}(^{76}\mathrm{Ge}) - [E_k(e_1^-) + E_k(\nu) + E_k(^{76}\mathrm{As})]} \tag{2}$$

$$\mathcal{T}^{0\nu} = \mathcal{M}_\nu^{0\nu} \cdot < m_\nu > + \mathcal{M}_\vartheta < \tan\vartheta > + \mathcal{M}_{W_R} < (\frac{M_1}{M_2})^2 > \\ + \mathcal{M}_{\mathrm{SUSY}} \cdot \lambda'^2_{111} + \mathcal{M}_{NR} < \frac{m_p}{M_{MR}} > + \ldots \tag{3}$$

The first term on the right hand side of (3) with the matrix element $\mathcal{M}_\nu^{0\nu}$ and the effective Majorana mass (3) with the matrix element $\mathcal{M}_\nu^{0\nu}$ and the effective Majorana mass

$$< m_\nu >= \sum_{k=1,2,3} (U_{ek})^2 \cdot m_{k\nu} = \sum_{k=1,2,3} e^{2i\alpha_k} \cdot |U_{ek}|^2 \cdot m_{k\nu} \tag{4}$$

with

$$\nu_e = \sum_{k=1,2,3} U_{ek}\nu_k \tag{5}$$

is often called the gold plated term. If one wants to determine the effective light left handed Majorana neutrino mass from the neutrinoless double beta decay, one assumes that this is the leading term and one can neglect the rest. But one needs apart of the data a reliable value for the matrix element. In the second chapter we will compare the different methods to calculate these matrix elements.

2. THE DIFFERENT MANY BODY APPROACHES FOR THE $0\nu\beta\beta$ MATRIX ELEMENTS

The groups in Tübingen, Bratislava and Jyväskylä [1–6] are using mainly the Quasiparticle Random Phase Approximation (QRPA), while the Strasbourg-Madrid group [7–9] uses the Shell Model (SM), Tomoda, Faessler, Schmid and Gruemmer [10] and Rath and coworkers [11–13] use the angular momentum projected Hartree-Fock-Bogoliubov method (HFB) (See the correction for the missing factor 2 in the

work of Pradfulla Rath *et al.* in [13]), and Barea and Iachello [15] use the Interacting Boson Model (IBM2, which distinguishes between protons and neutrons).

The QRPA [1–4] has the advantage to allow to use a large single-particle basis. Thus, one is able to include to each single nucleon state in the QRPA model space also the spin-orbit partner, which guarantees that the Ikeda sum rule [20] is fulfilled. This is essential to describe correctly the Gamow-Teller strength. The SM [7] is presently still restricted to a nuclear basis of four to five single-particle levels for the description of the neutrinoless double beta decay. Therefore, not all spin-orbit partners can be included and, as a result, the Ikeda sum rule is violated by 34 to 50% depending on the single particle basis used. On the other side, the shell model can in principle take into account all many-body configurations in a given single-particle basis. The excited states in the QRPA for spherical even-mass nuclei include 'seniority' (the number of broken quasiparticles) states two, six, ten, ... and for the ground state correlations 'seniority' zero, four, eight, ... configurations. The SM takes for the ground state seniority zero, four, six, eight, ..., and for the excited states seniority two, four, six, ... into account. But the numerical results of the shell model show [1, 7], that in agreement with the philosophy of RPA the contributions of seniority 6 configurations are small and can be neglected [1]. In QRPA one starts from the transformation to Bogoliubov quasiparticles :

$$a_i^\dagger = u_i c_i^\dagger - v_i c_{\bar{i}}. \tag{6}$$

The creation $c_i^\dagger$ and annihilation operators of time reversed single-particle states $c_{\bar{i}}$ are usually defined with respect to oscillator wave functions [2]. The single-particle energies are calculated with a Woods Saxon potential [2]. The single-particle basis can include up to 23 nucleon levels (all single-particle states without a core up to the $i_{13/2}$ level and even much larger, if deformation is allowed [5, 6]) for the protons and also for the neutrons. But the QRPA results for the $0\nu\beta\beta$ matrix elements turn out to be stable as a function of the basis size already for smaller basis sets (from 6 or 7 levels and larger, respectively) in lighter systems.

The excited states $|m\rangle$ with angular momentum J in the intermediate odd-odd mass nucleus are created from the correlated initial and final 0^+ ground states by a proton-neutron phonon creation operator:

$$|m\rangle = Q_m^\dagger |0^+\rangle; \;\; \hat{H} Q_m^\dagger |0^+\rangle = E_m Q_m^\dagger |0^+\rangle. \tag{7}$$

$$Q_m^\dagger = \sum_\alpha [X_\alpha^m A_\alpha^\dagger - Y_\alpha^m A_\alpha], \tag{8}$$

which is defined as a linear superposition of creation operators of proton-neutron quasiparticle pairs:

$$A_\alpha^\dagger = [a_i^\dagger a_k^\dagger]_{JM}, \tag{9}$$

For the present presentation the complication of angular momentum coupling,

which must and is included in the quantitative calculations, is not shown.

The inverse $0\nu\beta\beta$ lifetime for the light Majorana neutrino exchange mechanism is given as the product of three factors,

$$\left(T_{1/2}^{0\nu}\right)^{-1} = G^{0\nu}\left|\mathcal{M}_{\nu}^{0\nu}\right|^2 \cdot < m_\nu >^2, \tag{10}$$

where $G^{0\nu}$ is a calculable phase space factor, $\mathcal{M}_{\nu}^{0\nu}$ is the $0\nu\beta\beta$ nuclear matrix element and $< m_\nu >$ is the (nucleus-independent) "effective Majorana neutrino mass" 4.

The expressions for the matrix elements $\mathcal{M}_{\nu}^{0\nu}$ and the corresponding $0\nu\beta\beta$ transition operators are given, *e.g.*, in [2]:

$$\mathcal{M}_{\nu}^{(0\nu)} = \mathcal{M}_{GT}^{0\nu} - (\frac{g_V}{g_A})^2\mathcal{M}_{F}^{0\nu} - \mathcal{M}_{T}^{0\nu} \tag{11}$$

The SM approach has been applied by the Strasbourg-Madrid group [9] to neutrinoless double beta decay [7] using the closure relation with an averaged energy denominator. In this way one does not need to calculate the states in the odd-odd intermediate nuclei. The quality of the results depends then on the description of the 0^+ ground states in the initial and final nuclei of the double beta decay system, *e.g.* ^{76}Ge $\rightarrow^{76}$Se, on the nucleon-nucleon interaction matrix elements fitted by the Oslo group in neighboring nuclei and on the average energy denominator chosen (fitted) for closure. The $0\nu\beta\beta$ transition matrix element (11) simplifies as shown in equations (5) to (11) of [8]. Since the number of many body configurations is increasing drastically with the single-particle basis, one is forced to restrict for mass numbers $A = 76$ and $A = 82$ in the SM the single-particle basis to $1p_{3/2}$, $0f_{5/2}$, $1p_{1/2}$ and $0g_{9/2}$. In [8] the ^{82}Se nucleus is calculated in the SM for five basis single-particle levels including also $0f_{7/2}$. For the mass region around $A = 130$ the SM basis is restricted to $0g_{7/2}$, $1d_{3/2}$, $1d_{5/2}$, $2s_{1/2}$ and $0h_{11/2}$ levels. The problem with these small basis sets is that the spin-orbit partners $0f_{7/2}$ and $0g_{7/2}$ have to be omitted [8]. The SM results then automatically violate the Ikeda Sum Rule (ISR) [20], while the QRPA satisfies it exactly. The Ikeda sum rule is:

$$S_- - S_+ = 3(N - Z), \tag{12}$$

$$S_- = \sum_{\mu}\langle 0_i^+|[\sum_{k}^{A}(-)^{\mu}\sigma_{-\mu}(k)t_+(k)][\sum_{l}^{A}\sigma_{\mu}(l)t_-(l)]|0_i^+\rangle, \tag{13}$$

For S_+ the subscripts at the isospin rising and lowering operators are exchanged.

Figure 2 shows the QRPA contributions of different angular momenta of the neutron pairs, which are changed in proton pairs with the same angular momenta. In figure 2 the left bar is the result for ^{82}Se obtained with the single-particle basis $1p_{3/2}$, $0f_{5/2}$, $1p_{1/2}$ and $0g_{9/2}$ used in the SM. The ISR is exhausted by 50% only.

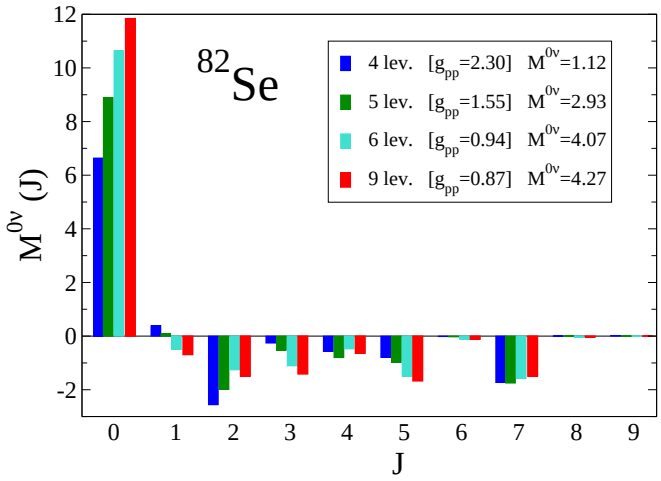

Fig. 2 – (Color online) Contributions of the transforming neutron pairs with different angular momenta J^π to the total $M_\nu^{0\nu}$ calculated within the QRPA and different basis sizes for the $0\nu\beta\beta$ decay ^{82}Se→^{82}Kr. The left bar is calculated with the same basis of four levels, $1p_{3/2}, 0f_{5/2}, 1p_{1/2}$ and $0g_{9/2}$, used in the shell model calculations [7–9]. The Ikeda Sum Rule (ISR) [20] is exhausted by 50%. The second bar from the left includes in addition the $1f_{7/2}$ level, one of the two missing spin-orbit partners given for the ^{82}Se nucleus in [8] for the shell model. The ISR is exhausted by 66%. The third bar from the left includes both missing spin-orbit partners $0f_{7/2}$ and $0g_{7/2}$ amounting in total to 6 single-particle levels. The ISR is fulfilled by 100%. This leads to the increase in the neutrinoless matrix element from 1.12 to 4.07. The right bar represents the QRPA result with 9 single-particle levels ($1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $1g_{9/2}$, $2d_{5/2}$, $3s_{1/2}$, $2d_{3/2}$, $1g_{7/2}$.). The matrix element gets only slightly increased to 4.27. The spin-orbit partners are essential to fulfill the Ikeda Sum Rule (ISR). In all four QRPA calculations the QRPA "renormalization" factor g_{pp} (given in the figure) of the particle-particle strength of the Bonn CD nucleon-nucleon interaction is adjusted to reproduce the experimental $2\nu\beta\beta$ decay rates.

The second bar from the left represents the result with addition of the $1f_{7/2}$ level. The ISR is exhausted by 66%. The third bar from the left shows the result obtained by inclusion of both spin-orbit partners $0f_{7/2}$ and $0g_{7/2}$ missing in the four level basis of the SM. The ISR is 100% fulfilled. For the right bar the basis is increased to 9 single-particle levels for neutrons and protons ($0f_{7/2}$, $1p_{3/2}$, $0f_{5/2}$, $1p_{1/2}$, $0g_{9/2}$, $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, $0g_{7/2}$).

In the last ten years P. K. Rath and coworkers [11–13] have published a whole series of papers (see references in [12]) on $2\nu\beta\beta$ decay and, since 2008, also on $0\nu\beta\beta$ decay, in which they used a simple pairing plus quadrupole many body Hamiltonian of the Kumar and Baranger type [21] to calculate the neutrinoless double beta decay transition matrix elements with angular momentum projection from a Hartree-Fock-Bogoliubov (HFB) wave function after variation. Schmid [22] did show, that with the assumption of a real Bogoliubov transformation (real coefficient A and B), axial

symmetry

$$a_\alpha^\dagger = \sum_{i=1}^{M} (A_{i\alpha} c_i^\dagger + B_{i\alpha} c_i) \tag{14}$$

and no parity mixing, only 0^+, 2^+, 4^+, ... nucleon pairs and excited states are allowed (see eq. (4.2.3) on page 603 of [22]). Rodriguez and Martinez-Pinedo start with the projected HFB approach but allow admixtures of different deformations using the Generator Coordinate Method (GCM) and the Gogny force [23]. The QRPA and the SM do not have this restriction like PHFB and also in the PHFB with the deformation GCM extension (GCM-PNAMP) [14].

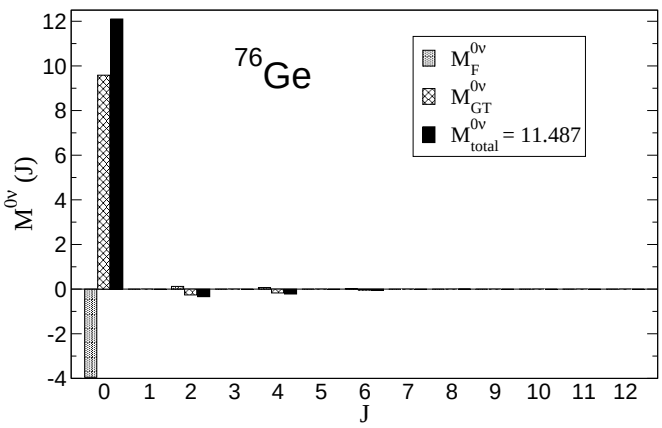

Fig. 3 – Contributions of neutron pairs with different angular momenta to the neutrinoless double beta decay transition matrix elements for ^{76}Ge $\rightarrow^{76}$Se calculated from a HFB wave function with angular momentum and proton and neutron particle number projection before variation. The Fermi, the Gamow-Teller and the total contribution including the tensor part as defined in eq. (11) are separately given. The nucleon-nucleon interaction is an improved Gogny type force [23]. The deformations $\beta_{Ge} = -0.08$ and $\beta_{Se} = 0.11$ correspond to the minima of the projected HFB total energy. The results for the transition matrix elements are qualitatively and almost quantitatively the same for the experimental deformation from the Coulomb reorientation effect: $\beta_{Ge} = 0.16, \beta_{Se} = 0.10$ and also for different forces. The angular momenta of the neutron pairs are in the PHFB approach with axial symmetry, real coefficients and no parity mixing restricted to $0^+, 2^+, 4^+, \ldots$. In addition the contributions of higher angular momentum neutron pairs $2^+, 4^+, \ldots$ are drastically reduced compared to the QRPA and the SM.

Figure 3 shows on the other side, that the projected HFB approach is restricted to contributions of neutron pairs with angular momenta 0^+, 2^+, 4^+, ... In addition, one sees that the contributions of transition of higher angular momentum neutron to proton pairs 2^+, 4^+, ... are drastically reduced compared to the QRPA and the SM see fig. 3. The reason for this is obvious: in a spherical nucleus the HFB solution contains only seniority zero and no stronger higher angular momentum pairs. The

double beta decay system $^{76}_{32}\mathrm{Ge}_{44} \rightarrow ^{76}_{34}\mathrm{Se}_{42}$ has only small deformations and thus a projected HFB state is not able to describe an appreciable admixture of higher angular momentum pairs for $0^+ \rightarrow 0^+$ transitions as can be seen in [22]. The higher angular momentum contributions increase drastically with increasing intrinsic quadrupole and hexadecapole deformations of the HFB solution.

The results in figure 3 are calculated in Tuebingen by K.W. Schmid [22] within the HFB with angular momentum and particle number projection before variation with an improved Gogny force [23] adjusted in a global fit to properties of many nuclei.

In order to have also 1^+, 3^+, 5^+, ... neutron pairs contributing one has to use a Bogoliubov transformation with complex coefficients A and B (14). To have also 0^-, 1^-, 2^-, 3^-, 4^-, 5^-, ... one has to allow parity mixing in the Bogoliubov transformation (14). But even allowing all different types of angular momentum and parity pairs one would still have an unnatural suppression of the higher angular momenta especially for smaller deformations. This handicap could probably be overcome by a multi-configuration HFB wave function [22] with complex coefficients and parity mixing in the Bogoliubov (14) transformation.

The IBM (Interacting Boson Model) [15] can only change 0^+ (S) and 2^+ (D) fermionic pairs from two neutrons into two protons. In the bosonization to higher orders this leads to the creation and annihilation of up to three "s" and "d" boson annihilation and creation operators in [15]. But all these terms of equation (18) of reference [15] originate from the annihilation of a 0^+ (S) or a 2^+ (D) neutron pair into a corresponding proton pair with the same angular momentum. The higher boson terms try only to fulfill the Fermi commutation relations of the original nucleon pairs up to third order. The IBM can therefore change only a 0^+ or a 2^+ neutron pair into a corresponding proton pair.

3. INCLUDING THE NUCLEAR DEFORMATION IN QRPA.

We have also calculated [5, 6] the transition matrix elements of the light left handed Majorana neutrino exchange $\mathcal{M}^{0\nu}_{\nu}$. Different deformations for the initial and the final nuclei are allowed. The BCS overlaps are listed in table 1. The quadrupole deformations are taken from the reorientation Coulomb excitation of the 2^+ states.

$$\beta_2 = \sqrt{\frac{\pi}{5}} \frac{Q_{reorientation}}{Z < r^2 >_{\mathrm{charge}}} \tag{15}$$

The deformation reduces the matrix elements [5, 6] in ^{76}Ge slightly from 5.30 to 4.69 by 10% only. This is within the error of the matrix elements (see fig. 5). But the reduction of the matrix elements is severe in strongly deformed systems with different deformations for the initial and the final nuclei. In the system ^{150}Nd$\rightarrow^{150}$Sm

Table 1.

The values of the deformation parameter of the Woods-Saxon mean field β_2 for initial (final) nuclei fitted to reproduce the experimental quadrupole moment (labeled as "1"). The spherical limit is labeled as "0". The particle-particle strength parameters g_{pp} are listed. They multiply the Brückcner particle-particle nucleon-nucleon G matrix elements of the BONN CD force. They are fitted to the $2\nu\beta\beta$ decay half lives. The axial charge is assumed to be the vacuum value $g_A = 1.25$. The particle-hole strength parameter, with which the nucleon-nucleon BONN CD Brückner particle-hole matrix elements are multiplied, is adjusted to the excitation energy of the Gamow-Teller resonance in the intermediate nucleus $g_{ph} = 0.90$. The BCS overlap factor $\langle BCS_f|BCS_i\rangle$ between the initial and final BCS vacua is given in the last column.

initial (final) nucleus	β_2		g_{pp}	$\langle BCS_i\|BSC_f\rangle$
^{76}Ge (^{76}Se)	0.10 (0.16)	"1"	0.71	0.74
	0.0 (0.0)	"0"	0.68	0.81
^{150}Nd (^{150}Sm)	0.240 (0.153)	"1"	1.05	0.52
	0.0 (0.0)	"0"	1.01	0.85
^{160}Gd (^{160}Dy)	0.303 (0.292)	"1"	1.00	0.74

the matrix element is reduced from 6.12 to 3.34 (see table 2) and in the strongly deformed system ^{160}Gd$\rightarrow^{160}$Dy (see table 1) one obtains a matrix element of 3.76 (see table 2). The single nucleon basis in these deformed calculations are determined in a deformed Woods-Saxon potential. The results are then expanded into a deformed oscillator basis with the same deformation parameter and the appropriate oscillator length in seven oscillator shells [5, 6]. The deformed result for ^{150}Nd$\rightarrow^{150}$Sm is included in figure 5.

4. HOW TO FIND THE LEADING MECHANISMS FOR THE NEUTRINOLESS DOUBLE BETA DECAY?

Normally one assumes, that the first term of (3) is the leading one and with the experimental data and the matrix element for the light left handed Majorana neutrino exchange $\mathcal{M}_\nu^{0\nu}$ one can determine the effective Majorana neutrino mass (4). But in Grand Unification (GUT) and Supersymmetry (SUSY) additional mechanisms for the neutrinoless Double Beta Decay $(0\nu\beta\beta)$ are possible. The matrix element of this gold plated term is proportional to:

$$\mathcal{M}_\nu^{0\nu}(\text{light } \nu_L) \propto \sum_{k=1,2,3} U_{ek}^\nu \cdot P_L \frac{1}{\not{q} - m_{k\nu}} P_L \cdot U_{ek}^\nu = \frac{1}{q^2} \sum_{k=1,2,3} e^{2i\alpha_k^\nu} \cdot |U_{ek}^\nu|^2 m_{k\nu} \quad (16)$$

The exchange of a heavy left handed Majorana neutrino:

$$N_e = \sum_{k=1,\ldots 6} U_{ek}^N N_k \approx \sum_{k=4,5,6} e^{i\alpha_k^N} |U_{ek}^N| \cdot N_k \quad (17)$$

Table 2.

The total calculated nuclear matrix elements (NME) $\mathcal{M}_\nu^{0\nu}$ for $0\nu\beta\beta$ decays ^{76}Ge$\rightarrow^{76}$Se, ^{150}Nd$\rightarrow^{150}$Sm, ^{160}Gd$\rightarrow^{160}$Dy. The BCS overlaps from table 1 are taken into account. In the last two columns the $0\nu\beta\beta$ matrix element $\mathcal{M}_\nu^{0\nu}$ and the half-lives for assumed $< m_\nu >= 50$ meV are shown.

A	Def.	g_A	$M_\nu^{0\nu}$	$T_{1/2}^{0\nu} \cdot [10^{26} y]$ $< m_\nu >$=50 meV
76	"1"	1.25	4.69	7.15
	"0"	1.25	5.30	5.60
150	"1"	1.25	3.34	0.41
	"0"	1.25	6.12	0.12
160	"1"	1.25	3.76	2.26

with α_k^N the Majorana phases for these heavy left handed Majorana neutrinos.

$$\mathcal{M}^{0\nu}(\text{heavy } N_L) \propto \sum_{k=4,5,6} U_{ek}^N \cdot P_L \frac{1}{\not{q} - M_{kN}} P_L \cdot U_{ek}^N = -\sum_{k=4,5,6} e^{2i\alpha_k^N} \cdot |U_{ek}^N|^2 / M_{kN} \quad (18)$$

The lepton number and R-parity violating contributions in SUSY are the trilinear terms and the coupling of the lepton superfields to the Higgs particle.

$$W_{\not{R}} = \lambda_{ijk} \cdot L_i \cdot L_j \cdot E_k^c + \lambda'_{ijk} \cdot L_i \cdot Q_j \cdot D_k^c + \mu_i \cdot L_i \cdot H_2 \quad (19)$$

The lepton L, E and quark Q, D left (L) and right (R) handed superfields are defined as:

$$L_k = \begin{pmatrix} \nu \\ e \\ \tilde{\nu} \\ \tilde{e} \end{pmatrix}_{kL} ; \; E_k = \begin{pmatrix} e \\ \tilde{e} \end{pmatrix}_{kR} ; \; Q_k = \begin{pmatrix} u \\ d \\ \tilde{u} \\ \tilde{d} \end{pmatrix}_{kL} ; \; D_k = \begin{pmatrix} d \\ \tilde{d} \end{pmatrix}_{kR} . \quad (20)$$

The indices i, j, k run over the three families for leptons: e, μ, τ and for quarks: d, s, b. The subscripts L and R characterize left and right handed superfields. The tilde indicates SUSY particles like selectrons, sneutrinos and squarks.

The inverse half life is given by:

$$\frac{1}{T_{1/2}^{0\nu}} = \frac{w^{0\nu}}{\ln 2} \approx G^{0\nu}(E_0, Z) \cdot \Big| \big[\eta_\nu \mathcal{M}_\nu^{0\nu} + \eta_{NL} \mathcal{M}_{NL}^{0\nu} + \eta_{\lambda'} \mathcal{M}_{\lambda'}^{0\nu}\big]^2 + |\eta_{NR}|^2 |\mathcal{M}_{NR}^{0\nu}|^2 \Big| , \quad (21)$$

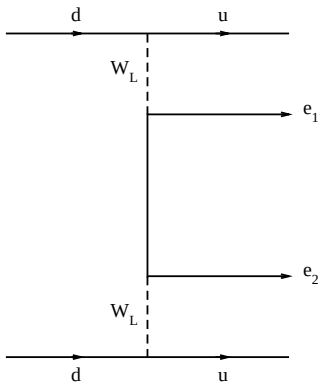

Fig. 4 – Diagram for the light Majorana neutrino exchange. The two neutrons in the initial nucleus in the ground state 0^+, which change into two protons are are coming from the left and are characterized by the two down quarks d in the two neutrons, which change into two up quarks u in two protons in the final nucleus. The vector bosons W_L mediating the left handed weak interaction are coupled by the expansion coefficients U_{ek} (5) to the neutrino mass eigenstates $m_{k\nu}$.

with

$$\eta_\nu = \frac{< m_\nu >}{m_e} = (|U^\nu_{e1}|^2 \cdot m_1 + e^{2i\alpha_{21}} \cdot |U^\nu_{e2}|^2 \cdot m_2 + e^{2i\alpha_{31}} \cdot |U^\nu_{e3}|^2 \cdot m_3)/m_e, \quad (22)$$

$$\eta_{NL} = |U^N_{e4}|^2 \cdot \frac{m_p}{M_{4L}} + e^{2i\alpha_{54}} \cdot |U^N_{e5}|^2 \cdot \frac{m_p}{M_{5L}} + e^{2i\alpha_{64}} \cdot |U^N_{e6}|^2 \cdot \frac{m_p}{M_{6L}}, \quad (23)$$

$$\alpha_{ik} = \alpha_i - \alpha_k. \quad (24)$$

We restrict here for SUSY to the trilinear terms (19).They are lepton number and R parity violating. They can contribute by gluino or by neutralino exchange. The strength parameter $\eta_{\lambda'}$ for gluino exchange contains the coupling constant λ'_{211} the gluino mass $m_{\tilde{g}}$ and the SUSY left and right handed up and down squark masses.

$$\eta_{\lambda'} = \frac{\pi \cdot \alpha_S \cdot (\lambda'_{211})^2 \cdot m_p}{6 \cdot G_F^2 \cdot m^4_{\tilde{d}_R} \cdot m_{\tilde{g}}} \cdot \left[1 + (\frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}})^2\right]^2 \quad (25)$$

For neutralino exchange one obtains a corresponding expression [16, 17, 19]. Both exchanges can be summarized under the parameter $\eta_{\lambda'}$ for the phenomenological analysis.

To test, if the light left handed Majorana neutrino exchange is the leading mechanism, one meeds at least experimental data of the neutrinoless double beta decay and reliable transition matrix elements in two systems. The light Majorana neutrino exchange is represented by the gold plated term, which is the first on the right hand side of eq.(3). If the measurements and the matrix element are reliable enough [18] and the light Majorana neutrino exchange is indeed the leading mechanism, one should in both and also in all other systems obtain the same effective Majorana neutrino

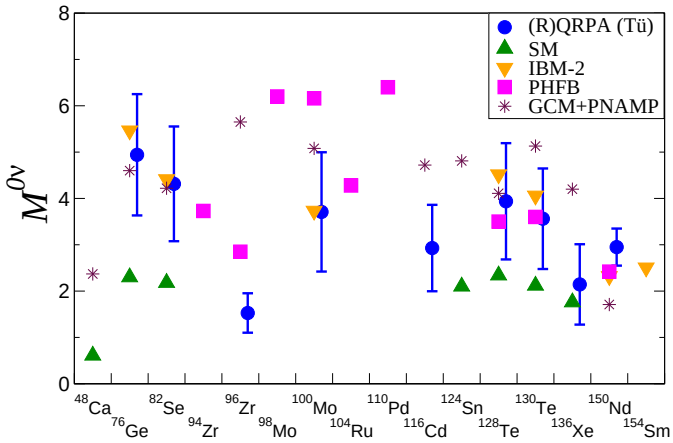

Fig. 5 – (Color online) Neutrinoless double beta decay transition matrix elements for the different approaches: QRPA [2, 3], the SM [7–9], the projected HFB method [13], the projected HFB with the Generator Coordinate Method (GCM) with deformations [14] (GCM-PNAMP) and the IBM2 [15]. The error bars of the filled circles for the QRPA are calculated as the highest and the lowest values for three different single-particle basis sets, two forces (Bonn CD and Argonne V18) two different axial charges $g_A = 1.25$ and the quenched value $g_A = 1.00$ and two different treatments of short range correlations (Jastrow-like [25] and the Unitary Correlator Operator Method (UCOM) [26]). The radius parameter is as in this whole work $r_0 = 1.2$ fm. The triangle with the tip up are the SM results [7–9]. The triangle with the tip down represent the transition matrix element of the Interacting Boson Model 2 (IBM2) [15]. The squares have been calculated by Pradfulla Rath and coworkers [13] with the correction of the factor 2 from December 2010 included, with which all previous results of Rath *et al.* have to be multiplied [11–13]. The star (GCM-PNAMP) is a projected HFB calculation with the Gogny force [23] by Rodriguez and Martinez-Pinedo [14] allowing for different deformations with the Generator Coordinate Method (GCM).

mass. If two mechanisms are at the same time contributing, one must distinguish between non-interfering and between interfering mechanisms. The light left handed Majorana neutrino and the heavy right handed neutrino exchange have negligible interference [17].

$$\frac{1}{T^{0\nu}_{i,1/2} \cdot G^{0\nu}_i(E_0, Z)} = |\eta_\nu|^2 (\mathcal{M}^{0\nu}_{i,\nu})^2 + |\eta_{NR}|^2 (\mathcal{M}^{0\nu}_{i,NR})^2 \tag{26}$$

To determine the absolute values of the two strength parameters η_ν and η_{NR} one needs at least two decay systems i. To verify, that these are indeed the leading mechanisms one needs at least a measurement in one additional decay system i. But if one forms ratios of half lives using the Tuebingen matrix elements for the light Majorana neutrino exchange and the heavy right handed neutrino exchange one

obtains a very restricted allowed interval for these ratios [17].

$$0.15 \leq \frac{T^{0\nu}_{1/2}(^{100}\mathrm{Mo})}{T^{0\nu}_{1/2}(^{76}\mathrm{Ge})} \leq 0.18;$$
$$0.17 \leq \frac{T^{0\nu}_{1/2}(^{130}\mathrm{Te})}{T^{0\nu}_{1/2}(^{76}\mathrm{Ge})} \leq 0.22; \quad 1.14 \leq \frac{T^{0\nu}_{1/2}(^{130}\mathrm{Te})}{T^{0\nu}_{1/2}(^{100}\mathrm{Mo})} \leq 1.24. \tag{27}$$

The dependence of this ratios on the different parameters of the nuclear structure calculation is very minor. Due to the ratios the dependence on most changes drop approximately out. The ratios (27) are calculated for the axial charge $g_A = 1.25$. But the quenching of this value to $g_A = 1.00$ has only a minor effect [17].

If the two leading mechanisms like the light Majorana neutrino exchange and the SUSY mechanism with gluino or neutralino exchange can interfere, the situation is a bit more complicated: Let us assume the relative phase angle of the complex strength parameters η_ν and $\eta_{\lambda'}$ is $\vartheta_{\nu,\lambda'}$. The inverse of the half life time the phase space factor $G^{0\nu}_{1/2}(E_0, Z)$ is then:

$$\frac{1}{T^{0\nu}_{i,1/2} \cdot G^{0\nu}_i(E_0, Z)} = |\eta_\nu|^2 (\mathcal{M}^{0\nu}_{i,\nu})^2 + |\eta_{\lambda'}|^2 (\mathcal{M}^{0\nu}_{i,\lambda'})^2 + \cos\vartheta_{\nu,\lambda'} \cdot |\eta_\nu| \cdot |\eta_{\lambda'}| \cdot \mathcal{M}^{0\nu}_{i,\nu} \cdot \mathcal{M}^{0\nu}_{i,\lambda'}. \tag{28}$$

One needs three decay systems to determine the absolute values of the parameters $\eta_\nu, \eta_{\lambda'}$ and the relative phase angle $\vartheta_{\nu,\lambda'}$. At least one additional system is needed to verify, that indeed these two mechanisms are the leading ones. Again the ratios of the half lives are allowed to lie only in narrow regions [17]. If this is not the case, the chosen mechanisms are not the leading ones [17]. With CP conservation the strength parameters η must be real and thus the relative phase angle is zero or 180 degrees. So the determination of $\vartheta_{\nu,\lambda'}$ allows to test CP conservation or violation.

5. THE EFFECTIVE MAJORANA NEUTRINO MASS

Before we summarize the results let us assume Klapdor-Kleingrothaus *et al.* [24] have indeed measured the neutrinoless double beta decay in ^{76}Ge, although the general belief is, that this still needs confirmation. From the half life given by Klapdor *et al.* [24] one can derive with our matrix elements the effective Majorana neutrino mass (4).

$$T^{0\nu}_{1/2}(^{76}\mathrm{Ge}, \text{exp. Klapdor}) = (2.23 + 0.44 - 0.31) \cdot 10^{25} [\text{years}]; \tag{29}$$

With our matrix elements one obtains the effective light left handed Majorana neutrino mass under the assumption, that the light Majorana exchange is the leading

mechanism.

$$\langle m_\nu \rangle = 0.24[\text{eV}](\text{exp.} \pm 0.02; \text{theor.} \pm 0.01)[\text{eV}] \tag{30}$$

The uncertainty (error) from experiment is 0.02 [eV], while the theoretical error originates from the uncertainties of the QRPA matrix elements as indicated in figure 5. The theoretical error is 0.01 [eV].

6. CONCLUSIONS

Let us now summarize the results of this contribution:

The Shell Model (SM) [7–9] is in principle the best method to calculate the nuclear matrix elements for the neutrinoless double beta decay. But due to the restricted single-particle basis it has a severe handicap. The matrix elements in the ^{76}Ge region are by a factor 2 smaller than the results of the Quasiparticle Random Phase Approximation (QRPA) [1–4], the projected Hartree Fock Bogoliubov approach [13, 14] and the Interacting Boson Model (IBM2) [15]. With the same restricted basis as used by the SM the QRPA obtains roughly the same results as the SM (figure 2), but the Ikeda sum rule [20] gets strongly violated due to the missing spin-orbit partners in the SM single-particle basis.

The angular momentum projected Hartee-Fock-Bogoliubov (HFB) method [11] is restricted in its scope. With a real Bogoliubov transformation without parity mixing and with axial symmetry (14) one can only describe neutron pairs with angular momenta and parity 0^+, 2^+, 4^+, 6^+, $\ldots$ changing into two protons for ground state-to-ground state transitions. The restriction for the Interacting Boson Model (IBM) [15] is even more severe: one is restricted to 0^+ and 2^+ neutron pairs changing into two protons.

A comparison of the $0\nu\beta\beta$ transition matrix elements calculated recently in the different many body methods: QRPA with realistic forces (CD Bonn, Argonne V18), SM with nucleon-nucleon matrix elements fitted in neighboring nuclei, projected HFB [13] with pairing plus quadrupole force [21], projected HFB with the deformation as Generator Coordinate (GCM+PNAMP) [14] and with the Gogny force [23] and IBM2 [15] is shown in fig. 5.

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