Dedicated to Academician Aureliu Sandulescu's 80th Anniversary

RADIOACTIVE STATES IN R-MATRIX THEORY

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The Radioactive States are approached in framework of Lane-Robson theory of nuclear reactions. The Radioactive State is not described by a R- matrix pole but rather by the channel equation $1 - R_{nn}L_n = 0$ relating R- matrix element R_{nn} to decay channel logarithmic derivative L_n. The dependence of radioactive process on decay channel results into renormalization of reduced width. The renormalization factor is R- matrix compression factor, *i.e.* wave function normalization over internal region to normalization over all space including decay channel. Extension of results to multi-channel system results into replacement of channel R- matrix element by its reduced counterpart.

This paper is dedicated to outstanding scientist Aureliu Emil Sandulescu, whose achievements in the field of Radioactive Decay are worldwide recognized and highly appreciated, (Encyclopedia Britannica). Firstly, in the sixties, Sandulescu approached the topic of α decay in terms of α particle preformation (inside atomic nucleus) and penetration factor (barrier tunneling). The 'preformation factors' were calculated as R- matrix reduced widths, by using microscopic nuclear structure models. In forthcoming years Sandulescu approached α decay from different perspectives. In eighties Sandulescu predicted the Exotic Radioactivity, a new type of disintegration by emission of heavy nuclear clusters, within an unitary approach of α decay cluster radioactivity, cold fission and nuclear fission (Sandulescu, Poenaru and Greiner - EChAYa). Moreover Sandulescu discovered experimentally the ^{22}Ne Radioactivity (Dubna).

1. INTRODUCTION

The Radioactive states were firstly introduced by Gamow as complex-energy eigenstates in order to describe α decay in a quasi stationary formalism [1]. Alternative approach relates Gamow Functions to Green functions of the problem near its poles [2]. At present the problem of complex-energy eigenstates is subject of Gamow Shell Model [3]. This work does approach the Radioactive or Gamow state in R- matrix terms. The Radioactive state is not described by a R- matrix pole but rather by a 'decay channel equation', relating channel R- matrix element and channel logarithmic derivative.

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The channel equation $R_{nn}^{-1} = S_n$ defines the bound state in R-matrix theory: a single particle bound state appears at that energy at which the internal R_{nn}^{-1} and external S_n logarithmic derivatives match [4]. The channel logarithmic derivative S_n at negative energy becomes boundary condition at channel radius for a bound state.

The logarithmic derivative of outgoing wave L_n is the corresponding, at positive energy, of the shift function S_n defined for negative energy. The logarithmic derivative of internal wave function R_{nn}^{-1} has to equal, at channel radius, the logarithmic derivative of outgoing wave L_n; this condition corresponds to a quasi stationary radioactive state. The quasi stationary state is a pure outgoing wave; the outgoing wave at infinity corresponds to the quasi stationary state decay [5].

The single channel equation $R_{nn}^{-1} = L_n$ defines either the bound state (below threshold) or quasi stationary radioactive state (above threshold). The present approach to equation $1 - L_n R_{nn} = 0$ is in terms of quasi stationary state complex energy pole $\mathscr{E}_\lambda$ which is root of implicit equation $1 - L_n R_{nn}(\mathscr{E}_\lambda) = 0$. The boundary conditions for radioactive state are those of out waves at state energy $\mathscr{E}_\lambda$, not at prescribed energy. As in case of bound states, this condition yields a set of eigen energies which now are complex; the equation determines the energy and decay width of quasi stationary state. The energy and decay width of quasi stationary state are both constants. (The Kapur-Peierls approach to equation $1 - L_n R_{nn} = 0$, with L_n as boundary condition, assumes the energy dependence of logarithmic derivative L_n is a parametric one; then the root of resonance equation becomes energy dependent.)

In multichannel systems the equation becomes $1 - \mathscr{R}_{nn} L_n = 0$ where $\mathscr{R}_{nn}$ is channel reduced R- Matrix element. The single particle bound or quasi stationary state, from closed or open channel, induces resonance in competing open channels.

2. RADIOACTIVE STATE AND SCATTERING THEORY

Scattering operators, as defined in [6] and [7],

$$S = \Omega(1 + 2iP^{\frac{1}{2}} \mathscr{R} P^{\frac{1}{2}})\Omega \tag{1}$$

with Ω and P as the phase and penetration diagonal matrices [4]. The $\mathscr{R}$ matrix is associated with Green operator $\mathscr{G}$

$$\mathscr{R}_{cc'} = < c' | \mathscr{G} | c > \tag{2}$$

where $|c>$ denotes channel wave function, *i.e.* the product of internal wave functions of the fragments and the angular wave function of their relative motion [8].

The Green operator, for Hamiltonian H and physical boundary conditions $\mathscr{L}$ [9], is defined by

$$\mathscr{G} = (H + \mathscr{L} - E)^{-1} \tag{3}$$

$$\mathscr{L} = \sum_c |c > \frac{\hbar^2}{2m_c}\delta(r_c - a_c)[\frac{d}{dr_c} - \frac{b_c - 1}{a_c}] < c| \tag{4}$$

$$b_c = a_c \frac{dO_c}{dr}/O_c \tag{5}$$

with O_c - channel *out* wave, a_c and m_c - channel radius and reduced mass and $(\hbar^2 a_c/2m_c)^{\frac{1}{2}}$ - channel normalization factor. The Bloch operator, $\mathscr{L}$, destroys *out* waves in all channels provided the set $|c>$ is orthogonal.

One defines the Green operator $\mathscr{G}^0$ for the system (H^0, $\mathscr{L}^0$)

$$\mathscr{G}^0 = (H^0 + \mathscr{L}^0 - E)^{-1} \tag{6}$$

with changes both in Hamiltonian and boundary conditions

$$h = (H - H^0) + (\mathscr{L} - \mathscr{L}^0) \tag{7}$$

The two Green operators are related by

$$\mathscr{G} = (1 + \mathscr{G}^0 h)^{-1}\mathscr{G}^0 \tag{8}$$

$$h = (H - H_0) + (\mathscr{L} - \mathscr{L}_0) = H^{'} - L \tag{9}$$

L - channels logarithmic derivative. In R-Matrix theories one takes into account only changes in boundary conditions, ($H^{'} = 0$), $h = -L = -\Delta\mathscr{L}$.

The $\mathscr{G}^0$ operator is split in two components

$$\mathscr{G}^0 = \mathscr{G}^0_A + \mathscr{G}^0_B \tag{10}$$

$\mathscr{G}^0_A$ referring to retained states $|\lambda>$ and $\mathscr{G}^0_B$ to complementary ones (background)

$$\mathscr{G}^0_A = \sum_\lambda \frac{|\lambda >< \lambda|}{E_\lambda - E} \tag{11}$$

In terms of the two components, the operator $\mathscr{G}$ [7] is

$$\mathscr{G} = \mathscr{G}_B + (1 - \mathscr{G}_B h)[1 + \mathscr{G}^0_A(h - h\mathscr{G}_B h)]^{-1}\mathscr{G}^0_A(1 - h\mathscr{G}_B) \tag{12}$$

$$\mathscr{G}_B = (1 + \mathscr{G}^0_B h)^{-1}\mathscr{G}^0_B \tag{13}$$

$$\mathscr{G} = \mathscr{G}_B + \sum_{\lambda\lambda'} |g_\lambda > D_{\lambda\lambda'} < \tilde{g}_\lambda| \tag{14}$$

$$(D^{-1})_{\lambda\lambda'} = (E_\lambda - E)\delta_{\lambda\lambda'} + < \lambda|h - h\mathscr{G}_B h|\lambda^{'} > \tag{15}$$

$$|g_\lambda >= (1 - \mathscr{G}_B h)|\lambda >, \qquad < \tilde{g}_\lambda| =< \lambda|(1 - h\mathscr{G}_B) \tag{16}$$

$$\omega_{c\lambda} =< c|g_\lambda >, \qquad \tilde{\omega}_{\lambda c} =< \tilde{g}_\lambda|c > \tag{17}$$

with D as Level Matrix and ω as reduced widths.

One level:

$$(D^{-1})_{\lambda\lambda} = (E_\lambda - E) + < \lambda|h - h\mathscr{G}_B h|\lambda >= E_\lambda - E + \Phi_\lambda \tag{18}$$

$$\mathscr{G} = \mathscr{G}_B + |g_\lambda > \frac{1}{E_\lambda + \Phi_\lambda - E} < \tilde{g}_\lambda| \tag{19}$$

$$\mathscr{G}|\lambda >= \mathscr{G}_B|\lambda > + |g_\lambda > \frac{1}{E_\lambda + \Phi_\lambda - E} < \tilde{g}_\lambda|\lambda > \tag{20}$$

$$\mathscr{G}_B|\lambda >= (1 + \mathscr{G}_B^0 h)^{-1}\mathscr{G}_B^0|\lambda > \tag{21}$$

If $\mathscr{G}_B^0|\lambda > = 0$

$$\mathscr{G}|\lambda >= \frac{1}{E_\lambda + \Phi_\lambda - E}|g_\lambda > \tag{22}$$

The 'Resonance' or 'Radioactive' State $|g_\lambda >$ is not an eigenstate of Schrödinger equation but rather is subject to equation [10],

$$(H + \mathscr{L} - E)|g_\lambda >= (E_\lambda + \Phi_\lambda - E)|\lambda > \tag{23}$$

the state $|\lambda >$ being an eigenstate of H^0

$$[H^0 + \mathscr{L}^0]|\lambda >= E_\lambda|\lambda > \tag{24}$$

The decay of Quasistationary Radioactive State involves only out wave. The Bloch operator $\mathscr{L}$ destroys out wave [11],

$$\mathscr{L}(E^{'})|g_\lambda >= 0 \tag{25}$$

Comparing with usual Schrödinger equation $[H + \mathscr{L} - E|\Psi = \mathscr{L}\Psi$

$$(H + \mathscr{L}(E^{'}) - E^{'})|g_\lambda >= \mathscr{L}(E^{'})|g_\lambda >= 0 \tag{26}$$

one obtains a constraint for energy of Radioactive State $|g_\lambda >$ *i.e.* $E^{'} = \mathscr{E}_\lambda = E_\lambda + \Phi_\lambda$. The Quasistationary Radioactive State equation is,

$$[H + \mathscr{L}(E_\lambda + \Phi_\lambda) - (E_\lambda + \Phi_\lambda)]|g_\lambda >= 0 \tag{27}$$

The Radioactive State equation, in Hamiltonian terms,

$$\mathscr{G}^{-1}|g_\lambda >= [H + \mathscr{L}(\mathscr{E}_\lambda) - \mathscr{E}_\lambda]|g_\lambda >= 0 \tag{28}$$

is rewritten in channel space by multiplying on left by $< \tilde{g}_\lambda|$ and by inserting the channel projector $\sum_c |c >< c|$, provided the channel wave functions are orthogonal. (This could be the case of unique fragmentation of compound system, *i.e.* only elastic and inelastic scattering channels.) One assumes that the channels c exhaust the decay of radioactive state $|g_\lambda >$, *i.e.* $< \tilde{g}_\lambda|o >$= 0, (o - other reaction channels).

$$\sum_{cc^{'}} < \tilde{g}_\lambda|c >< c|[H + \mathscr{L}(\mathscr{E}_\lambda) - \mathscr{E}_\lambda]|c^{'} >< c^{'}|g_\lambda >= 0 \tag{29}$$

$$\sum_{cc^{'}} < \tilde{g}_\lambda|c >< c|\mathscr{G}^{-1}(\mathscr{E}_\lambda)|c^{'} >< c^{'}|g_\lambda >= 0 \tag{30}$$

One obtains the Radioactive State equation in terms of Scattering Operator,

$$\sum_{cc'} \tilde{\omega}_{\lambda c} \mathscr{R}^{-1}_{cc'} \omega_{c'\lambda} = 0 \tag{31}$$

The 'energy' ($\mathscr{E}_\lambda$) is pole of $\mathscr{R}_{cc'} = < c' | \mathscr{G} | c >$ Kapur-Peierls scattering matrix.

3. RADIOACTIVE STATES AND R- MATRIX

The Green operator is related to Kapur-Peierls Matrix and R- Matrix [6],

$$< c' | \mathscr{G} | c > = \mathscr{R}_{cc'} = [(1 - RL)^{-1} R]_{cc'}, \quad \mathscr{R} = (R^{-1} - L)^{-1} = (1 - RL)^{-1} R \tag{32}$$

with $L = \Delta \mathscr{L}$ as channel logarithmic derivative. The Radioactive State equation

$$\sum_{cc'} \tilde{\omega}_{\lambda c} \mathscr{R}^{-1}_{cc'} \omega_{c'\lambda} = 0$$

becomes, in R- Matrix terms [4],

$$\sum_{cc'c''} \tilde{\omega}_{\lambda c} R^{-1}_{cc''} (1 - RL)_{c''c'} \omega_{c'\lambda} = 0 \tag{33}$$

As both $\tilde{\omega}_{\lambda c}$ and $R^{-1}_{cc'}$ are finite at $E = \mathscr{E}_\lambda$ one obtains the following R- Matrix form for Radioactive State equation

$$\sum_{c'} (1 - RL)_{cc'} \omega_{c'\lambda} = 0 \tag{34}$$

or in matrix form

$$[1 - R(\mathscr{E}_\lambda) L(\mathscr{E}_\lambda)] \omega_\lambda = 0 \tag{35}$$

The poles $\mathscr{E}_\lambda$ are complex roots of determinant equation

$$|1 - R(\mathscr{E}_\lambda) L(\mathscr{E}_\lambda)| = 0 \tag{36}$$

Both R and L, energy dependent, are assumed to be analytically continuable from E to complex $\mathscr{E}_\lambda$. Both $\mathscr{E}_\lambda$ and $\omega_{\lambda c}$ are energy independent, provided L is considered analytic.

One proves the Level Matrix D matrix is diagonal provided is taken into account the radioactive state condition $1 - R(\mathscr{E}_\lambda) L(\mathscr{E}_\lambda) = 0$ and $< c | \mathscr{G}_B | \lambda > = 0$. (The reduced width $< c | \mathscr{G}_B | \lambda >$ is level decay width mediated by background scattering $\mathscr{G}_B$.)

$$D^{-1} = ||D_{\lambda\lambda} \delta_{\lambda\mu}||^{-1} = ||(E_\lambda + \Phi_\lambda - E) \delta_{\lambda\mu}|| = ||(\mathscr{E}_\lambda - E) \delta_{\lambda\mu}|| \tag{37}$$

$$\mathscr{R} = \sum_\lambda \omega_\lambda \frac{1}{\mathscr{E}_\lambda - E} \omega_\lambda \tag{38}$$

The Radioactive State equation in R- Matrix terms

$$\mathscr{R} = (1 - RL)^{-1}R = \sum_{\lambda} \frac{\omega_\lambda \times \omega_\lambda}{\mathscr{E}_\lambda - E} \tag{39}$$

with $\mathscr{E}_\lambda$ as complex eigenvalue and ω_λ as complex reduced width.

$$R = \sum_{\lambda} [(1 - RL)\omega_\lambda \times \omega_\lambda]/(\mathscr{E}_\lambda - E) \tag{40}$$

By multiplying on left R by $(\omega_\lambda \times \omega_\lambda)L$

$$(\omega_\lambda \times \omega_\lambda)LR = \sum_{\mu} (\omega_\lambda \times \omega_\lambda)[(L - LRL)\omega_\mu \times \omega_\mu]/(\mathscr{E}_\mu - E) \tag{41}$$

and by using an identity for direct product of matrices, one obtains

$$(\omega_\lambda \times \omega_\lambda)LR = \sum_{\mu} (\omega_\lambda, (L - LRL)\omega_\mu)(\omega_\lambda \times \omega_\mu)/(\mathscr{E}_\mu - E) \tag{42}$$

The radioactive state condition, $R(\mathscr{E}_\lambda)L = 1$, and the R-Matrix equation for Radioactive State in vicinity of $\mathscr{E}_\lambda$, result into

$$(\omega_\lambda, (L - LR(\mathscr{E}_\lambda)L)\omega_\lambda)/(\mathscr{E}_\lambda - E) = 1 \tag{43}$$

This equation results into normalization condition for complex reduced width ω_λ

$$1 = (\omega_\lambda, (\frac{dL}{dE} + L\frac{dR(\mathscr{E}_\lambda)}{dE}L)\omega_\lambda) \tag{44}$$

where R, L and dL/dE are evaluated at complex eigen-energy $\mathscr{E}_\lambda$.

The derivative, dR_{cc}/dE, of R-Matrix element R_{cc} is implied in definition of single-channel reduced width [4],

$$\frac{dR_{cc}}{dE} = \frac{R_{cc}^2}{\gamma_{\lambda c}^2} \tag{45}$$

The normalization condition for the reduced width $\omega_{\lambda c}$, of level λ overlapping channel c

$$1 = (\omega_{\lambda c}, \frac{dL_c}{dE}\omega_{\lambda c}) + (\omega_{\lambda c}, L_c\frac{R_{cc}^2}{\gamma_{\lambda c}^2}L_c\omega_{\lambda c}) \tag{46}$$

$$1 = (\omega_{\lambda c}, \frac{dL_c}{dE}\omega_{\lambda c}) + (\omega_{\lambda c}, \frac{1}{\gamma_{\lambda c}^2}\omega_{\lambda c}) = \omega_{\lambda c}^2(\frac{dL_c}{dE} + \frac{1}{\gamma_{\lambda c}^2}) \tag{47}$$

results into spectroscopic aspects of radioactive states.

$$\omega_{\lambda c}^2 = \frac{1}{\frac{dL_c}{dE} + \frac{1}{\gamma_{\lambda c}^2}} = \frac{\gamma_{\lambda c}^2}{1 + \gamma_{\lambda c}^2\frac{dL_c}{dE}} \tag{48}$$

i.e. renormalization of decay reduced width in terms of decay channel logarithmic derivative.

The R-Matrix reduced width dependence on on channel wave functions is given by

$$\gamma_{\lambda c}^2 = \frac{\omega_{\lambda c}^2}{1-\omega_{\lambda c}^2 \frac{dL_c}{dE}} \tag{49}$$

The dependence of R-Matrix reduced width on channel wave function was studied too, but from another point of view, in the work [12].

According to R-Matrix Theory [4], the probability integral for the compound system is

$$\int \Psi^2 d\tau = (D, \frac{dR}{dE} D) \tag{50}$$

where V and D are 'value quantity' and 'derivative quantity' of the wave functions at channel radius. For the case of single channel c the value V and derivative D quantities are related to logarithmic derivative L when evaluated at pole energy $\mathscr{E}_\lambda$

$$D_c = L_c V_c \tag{51}$$

The probability integral for the single level λ becomes,

$$\int \Psi^2 d\tau = (L_c V_c, \frac{R_{cc}^2}{\gamma_{\lambda c}^2} L_c V_c) = \frac{V_c^2}{\gamma_{\lambda c}^2} \tag{52}$$

The logarithmic derivative L_c for *out* waves, if evaluated at pole $\mathscr{E}_\lambda$, equals the ratio D_c/V_c and the reduced width $\omega_{\lambda c}$ is just V_c [4],

$$V_c = \omega_{\lambda c} \tag{53}$$

The volume and channel terms of probability integral are

$$\int_0^a \Psi^2 d\tau = (\omega_{\lambda c}, L_c \frac{R_{cc}^2}{\gamma_{\lambda c}^2} L_c \omega_{\lambda c}) = \frac{\omega_{\lambda c}^2}{\gamma_{\lambda c}^2} = \frac{1}{1+\gamma_{\lambda c}^2 \frac{dL_c}{dE}} = \beta_{\lambda c}(E) \tag{54}$$

$$\int_a^\infty \Psi^2 d\tau = (\omega_{\lambda c}, \frac{dL_c}{dE} \omega_{\lambda c}) = \omega_{\lambda_c}^2 \frac{dL_c}{dE} = \frac{\gamma_{\lambda c}^2 \frac{dL_c}{dE}}{1+\gamma_{\lambda c}^2 \frac{dL_c}{dE}} = 1-\beta_{\lambda c}(E) \tag{55}$$

with β as R-Matrix compression factor. The physical meaning of compression factor is wave function normalization over internal region to the normalization over all space including decay channel [13].

Let us evaluate the pole parameters $\mathscr{E}_\lambda = E_\lambda + \Phi_{\lambda\lambda}$, *i.e.* $\Phi_{\lambda\mu}$, in R- Matrix terms.

$$(D^{-1})_{\lambda\mu} = (E_\lambda - E)\delta_{\lambda\mu} + <\lambda|h - h\mathscr{G}_B h|\mu> = (E_\lambda - E)\delta_{\lambda\mu} + \Phi_{\lambda\mu} \tag{56}$$

$$\begin{aligned} &\Phi_{\lambda\mu} = <\lambda|h - h\mathscr{G}_B h|\mu> = -<\lambda|L + L\mathscr{G}_B L|\mu> = \\ &-\sum_{cc'} <\lambda|c><c|L + L\mathscr{G}_B L|c'><c'|\mu> = -\sum_{cc'} \gamma_{\lambda c}(L + L\mathscr{R}_B L)_{cc'}\gamma_{c'\mu} \end{aligned} \tag{57}$$

One defines, up to Ω phase shift, the background scattering matrix,

$$S_B = 1 + 2iP^{\frac{1}{2}}\mathscr{R}_B P^{\frac{1}{2}} \tag{58}$$

in terms of Kapur-Peierls $\mathscr{R}_B$ - Matrix

$$\mathscr{R}_B = || < c^{'}|\mathscr{G}_B|c > ||, \tag{59}$$

as well as the Wigner R_B - Matrix form

$$S_B = 1 - 2iP^{\frac{1}{2}}L^{-1}P^{\frac{1}{2}} + 2iP^{\frac{1}{2}}L^{-1}(L^{-1} - R_B)^{-1}L^{-1}P^{\frac{1}{2}}, \tag{60}$$

resulting into

$$L + L\mathscr{R}_B L = (L^{-1} - R_B)^{-1} = (1 - LR_B)^{-1}L = L(1 - R_B L)^{-1} = L(B) \tag{61}$$

$L(B)$ is renormalized logarithmic derivative due to background. In Level matrix form of R-Matrix [4], one defines the complex level shift $\chi_{\lambda\mu}$

$$\chi_{\lambda\mu} = (\beta_\lambda, \gamma_\mu) = (L(1 - R_B L)^{-1}\gamma_\lambda, \gamma_\mu) \to \sum_c \gamma_{\lambda c} L_c \gamma_{c\mu} \tag{62}$$

$$\begin{aligned} \chi_{\lambda\mu} = \sum_{cc'} \gamma_{\lambda c} L_c (1 - R_B L)^{-1}_{cc'} \gamma_{c'\mu} = \sum_{cc'} \gamma_{\lambda c} (L^{-1} - R_B)^{-1}_{cc'} \gamma_{c'\mu} = \\ \sum_{cc'} \gamma_{\lambda c} L(B)_{cc'} \gamma_{c'\mu} = \sum_{cc'} \gamma_{\lambda c} (L + L\mathscr{R}_B L)_{cc'} \gamma_{c'\mu} = -\Phi_{\lambda\mu} \end{aligned} \tag{63}$$

$$\mathscr{E}_\lambda = E_\lambda + \Phi_\lambda = E_\lambda - \chi_{\lambda\lambda} \to E_\lambda - \sum_c L_c \gamma^2_{\lambda c} \tag{64}$$

Thomas approximation for logarithmic derivative ($d/dE ='$ - energy derivative),

$$L_c(\mathscr{E}_\lambda) = L_c(E_\lambda) + (\mathscr{E}_\lambda - E_\lambda) L^{'}_c(E_\lambda) \tag{65}$$

$$\mathscr{E}_\lambda = E_\lambda - \sum_c L_c(E_\lambda)\gamma^2_{\lambda c} - \mathscr{E}_\lambda \sum_c \gamma^2_{\lambda c} L^{'}_c(E_\lambda) + E_\lambda \sum_c \gamma^2_{\lambda c} L^{'}_c(E_\lambda) \tag{66}$$

One assumes energy variation only for n-channel logarithmic derivative, $L^{'}_n \neq 0$, $L^{'}_{c\neq n} = 0$,

$$\mathscr{E}_\lambda = E_\lambda - \sum_c L_c(E_\lambda)\gamma^2_{\lambda c} - \mathscr{E}_\lambda \gamma^2_{\lambda n} L^{'}_n(E_\lambda) + E_\lambda \gamma^2_{\lambda n} L^{'}_n(E_\lambda) \tag{67}$$

$$\begin{aligned} \mathscr{E}_\lambda(1 + \gamma^2_{\lambda n} L^{'}_n(E_\lambda)) = E_\lambda - \sum_c L_c(E_\lambda)\gamma^2_{\lambda c} + E_\lambda \gamma^2_{\lambda n} L^{'}_n(E_\lambda) = \\ E_\lambda(1 + \gamma^2_{\lambda n} L^{'}_n(E_\lambda)) - \sum_c L_c(E_\lambda)\gamma^2_{\lambda c} \end{aligned} \tag{68}$$

$$\mathscr{E}_\lambda = E_\lambda - L_n(E_\lambda)\frac{\gamma_{\lambda n}^2}{1+\gamma_{\lambda n}^2 L_n'(E_\lambda)}$$

$$-\sum_{c\neq n} L_c(E_\lambda)\gamma_{\lambda c}^2 \frac{1}{1+\gamma_{\lambda n}^2 L_n'(E_\lambda)} = E_\lambda - \omega_{\lambda n}^2 L_n(E_\lambda) - \beta_{\lambda n}\sum_{c\neq n} L_c(E_\lambda)\gamma_{\lambda c}^2 \quad (69)$$

$$\mathscr{E}_\lambda - E = E_\lambda - \omega_{\lambda n}^2 L_n(E_\lambda) - \beta_{\lambda n}\sum_{c\neq n} L_c(E_\lambda)\gamma_{\lambda c}^2 - E =$$

$$E_\lambda - \omega_{\lambda n}^2 L_n(E_\lambda) - \beta_{\lambda n}\Delta_\lambda - i\beta_{\lambda n}\Gamma_\lambda - E \quad (70)$$

with Δ_λ and Γ_λ - levels shift and width due to its coupling to complementary channels, $c \neq n$. The levels shift and width due to very channel n are both in $\omega_{\lambda n}^2 L_n = \beta_{\lambda n}\gamma_{\lambda n}^2 L_n$. Both levels shifts and widths are subject to renormalization by compression factor.

4. PROBLEM OF CHANNELS TRUNCATION; RELATION TO REDUCED R- MATRIX

One channel:

The single channel particle state is defined [10], as a state with large overlap to only one channel. We consider now the case of single particle state π_n in a single channel n, taking into account multichannel couplings in terms of reduced R- Matrix. The single channel collision matrix elements $< n|\mathscr{G}|n >\sim \mathscr{R}_{nn}$ in Radioactive State equation

$$\tilde{\omega}_{\lambda n}\mathscr{R}_{nn}^{-1}\omega_{n\lambda} = 0$$

are expressed in terms of n-channel reduced R- Matrix $\mathscr{R}_{nn}$

$$\mathscr{R}_{nn} = [(1-RL)^{-1}R]_{nn} = (1-\mathscr{R}_{nn}L_n)^{-1}\mathscr{R}_{nn} \quad (71)$$

Accordingly, the Radioactive State equation for the single-particle π_n in channel n becomes

$$[1-\mathscr{R}_{nn}(\mathscr{E}_{\pi n})L_n(\mathscr{E}_{\pi n})]\omega_{\pi n} = 0 \quad (72)$$

The pole $\mathscr{E}_{\pi n}$ is given by implicit equation

$$1-\mathscr{R}_{nn}(\mathscr{E}_{\pi n})L_n(\mathscr{E}_{\pi n}) = 0 \quad (73)$$

Lane' formula for channel n and quasi stationary state π:

The Kapur-Peierls Scattering Matrix element

$$\mathscr{R}_{nn} = \tilde{\omega}_{n\pi}\frac{1}{\mathscr{E}_\pi - E}\omega_{\pi n} \quad (74)$$

The pole $\mathscr{E}_\pi$is complex root of implicit equation

$$R_{nn}(\mathscr{E}_\pi)L_n(\mathscr{E}_\pi) = 1 \quad (75)$$

which is R-Matrix equation for radioactive state.

$$R_{nn}(\mathscr{E}_\pi) = \frac{\gamma^2_{\pi n}}{E_\pi - \mathscr{E}_\pi} \tag{76}$$

Thomas approximation for logarithmic derivative ($d/dE ='$ - energy derivative),

$$L_n(\mathscr{E}_\pi) = L_n(E_\pi) + (\mathscr{E}_\pi - E_\pi)L^{'}_n(E_\pi) \tag{77}$$

$$R_{nn}(\mathscr{E}_\pi)L_n(\mathscr{E}_\pi) = \frac{\gamma^2_{\pi n}}{E_\pi - \mathscr{E}_\pi}L_n(E_\pi) + \frac{\gamma^2_{\pi n}}{E_\pi - \mathscr{E}_\pi}(\mathscr{E}_\pi - E_\pi)L^{'}_n(E_\pi) = \frac{\gamma^2_{\pi n}}{E_\pi - \mathscr{E}_\pi}L_n(E_\pi) - \gamma^2_{\pi n}L^{'}_n(E_\pi) \tag{78}$$

The condition $R_{nn}(\mathscr{E}_\pi)L_n(\mathscr{E}_\pi) = 1$ implies

$$\frac{\gamma^2_{\pi n}}{E_\pi - \mathscr{E}_\pi}L_n(E_\pi) = 1 + \gamma^2_{\pi n}L^{'}_n(E_\pi) \tag{79}$$

$$E_\pi - \mathscr{E}_\pi = \frac{\gamma^2_{\pi n}L_n(E_\pi)}{1+\gamma^2_{\pi n}L^{'}_n(E_\pi)} = \omega^2_{\pi n}L_n(E_\pi), \quad \mathscr{E}_\pi = E_\pi - \omega^2_{\pi n}L_n(E_\pi) \tag{80}$$

but now the pole $(E_\pi - \mathscr{E}_\pi)$ of Reduced R-Matrix element $\mathscr{R}_{nn}$ includes 'shifts' $\sum_{a\neq n} L_a\gamma^2_{\pi a}$ due to complementary reaction channels. The derivation can be extended to Radioactive State equation of eliminated n-channel, in terms of Reduced R- Matrix element $\mathscr{R}_{nn}$. The energy $\mathscr{E}_\pi$ for resonance π from n-channel is complex root of implicit equation

$$\mathscr{R}_{nn}(\mathscr{E}_\pi)L_n(\mathscr{E}_\pi) = 1 \tag{81}$$

The Kapur-Peierls dynamical term for eliminated n- channel becomes

$$\mathscr{R}_{nn} = \frac{\omega^2_{\pi n}}{\mathscr{E}_\pi - E} = \frac{\omega^2_{\pi n}}{E_\pi - \omega^2_{\pi n}L_n(E_\pi) - E} \tag{82}$$

It is Lane's approach [13] to threshold effects induced by zero-energy neutron single channel resonance.

5. CONCLUSIONS

The matching of R- matrix element R_{nn} to channel logarithmic derivative L_n results into channel equation $R^{-1}_{nn} = L_n$. Below threshold, ($L_n = S_n$), it is R- matrix equation for bound state, $1 - R_{nn}S_n = 0$. Extended to positive energy channel, the corresponding state is the quasi stationary one. The boundary conditions for quasi stationary state are those of *out* waves at state energy $\mathscr{E}_\lambda$, not at prescribed energy. The quasi stationary level is no more defined by a R- matrix pole but rather by channel equation $1 - R_{nn}(\mathscr{E}_\lambda)L_n(\mathscr{E}_\lambda) = 0$.

The bound or quasi stationary state equation implies dependence of reduced widths on channel logarithmic derivative. The channel renormalization of reduced widths is effective near channel threshold due to abrupt change in spatial extension of wave function in reaction channel.

Implication of channel single particle bound or quasi stationary state on multi-channel resonances and threshold effects is subject of forthcoming work.

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