STUDY OF THE MAGNETIC RECONNECTION DURING THE SAWTOOTH INSTABILITY*

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It is widely accepted that the sawtooth oscillations of the plasma parameters (temperature or density) in the central region of a tokamak are closely related to the configuration of the magnetic field. On the basis of some theoretical analysis on the onset of the (1,1) internal MHD mode into the nonlinear regime, we justify the use of the magnetic stochasticity hypothesis in order to explain the sawtooth crash and discuss the complete or incomplete nature of the reconnection. Two magnetic configurations, with monotonous safety factor, respectively with reversed magnetic shear, are studied using the Hamiltonian description of the magnetic field lines and of the mapping technique. A perturbation that contains not only the (1,1) mode, but also higher harmonics is considered and the role of these additional perturbations during the sawtooth crash is observed. The influence of the minimum value of the safety factor, the flatness coefficient and the amplitude of the perturbations on the reconnection process is systematically studied. It is shown that the mathematical models are able to generate poloidal cross sections of the magnetic field lines similar with the experimental ones and a scenario for the reconstruction of the magnetic field during the sawtooth instability is proposed.

Key words: tokamak, fusion plasma, sawtooth oscillations.

1. INTRODUCTION

Sawtooth oscillations of the plasma parameters (for example a repetitive and rapid crash of the central electron temperature) were first observed in 1974 in

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parallel experiment on the ST and T-4 tokamaks. They are considered to be instabilities of the (1,1) mode which are observed when the safety factor, $q(r)$, falls below unity on the magnetic axis [1].

Despite the sustained experimental and theoretical research, there is no definitive explanation for the crash process.

In this paper we justify the use of the magnetic stochasticity hypothesis in order to explain the sawtooth crash. The sawtooth crash is a localized penetration of the hot core plasma into the colder outer region. The penetration region, described as a region of localized magnetic reconnection, is associated with the stochastization of the magnetic field because the increased turbulent transport in the plasma core causes the observed crash of the plasma temperature.

Our study generalizes (for arbitrary monotonous safety factors) the scenario proposed in [2]. We propose a different scenario for the reversed shear magnetic configurations.

2. THE MATHEMATICAL MODEL

In order to describe the magnetic configuration in a tokamak we use the generalized toroidal coordinates (ψ,θ,ζ), where $\psi = r^2/2$ is the toroidal magnetic flux, θ is the poloidal angle and ζ is the toroidal angle.

It is well-known that the equations describing the magnetic field lines have Hamiltonian structure (ψ and θ are canonically conjugate variables and ζ is the independent variable, playing the role of time) [3]. The poloidal magnetic flux (the Hamiltonian of the system) considered in our study is

$$H(\theta,\psi,\zeta) = \int W(\psi)\,d\psi + \varepsilon\frac{\psi}{\psi+1}\cdot\left[\sum_{m=1}^{3} a_m \cdot \sum_{n=-M}^{M}\cos(m\theta - n\zeta)\right]\cdot \qquad (1)$$

In (1) $W(\psi)=1/q(\psi)$ is the winding function. In order to observe the role of the safety factor $q(\psi)$ on the dynamics of the system we consider two winding functions: $W_1(\psi) = w_0(2-\psi)(2-2\psi+\psi^2)/4$, $W_2(\psi) = w_0 - 0.5(\psi-0.3)^2$.

The parameter w_0 indicates the maximal value of the winding function (*i.e.* $w_0 = 1/q(0)$ for W_1, respectively $w_0 = 1/q(0.3)$ for W_2).

The particular form of the perturbation is motivated by experimental observations made in ASDEX-Upgrade tokamak using the spectral SXR signal [2]: the primary mode (1,1) exists well before the crash, the modes (2,2) and (3,3) appear just before the crash. Between the crashes only the (1,1) component is present.

The parameter ε indicates the general amplitude of Hamiltonian perturbation and a_1, a_2, a_3 are the amplitudes of the partial perturbations. The mode (m,m) is

excited if $a_m \neq 0$. In [2] it is observed that the relative amplitude of the mode $(2,2)$ before the crash is by a factor of 2 smaller than the amplitude of the mode $(1,1)$, so we will consider $a_1 = 1$, $a_2 = 1/2$ and $a_3 = 1/3$ when the corresponding modes are activated.

In order to better understand the dynamics of the magnetic field lines we use the Poincare map associated to the Hamiltonian system and to the poloidal section $\zeta = 0$. It is obtained using the mixed generating function and the symmetric mapping technique [4] if $2M + 1 >> 1$. This condition is physically acceptable because the considered Hamiltonian is the superposition of many plane waves.

3. THE MAGNETIC RECONNECTION

The region where the sawtooth crash occurs was precisely localized in experiments around the surface $q = 1$ and it is associated with the reconnection of the magnetic field lines [5]. This reconnection is the cumulative effect of two factors: the configuration of the safety factor and the magnetic perturbations. It is influenced by the number and the position of the low-order rational surfaces.

Stochastization requires the existence of several low-order surfaces which can be excited by overlapping the $q = 1$ surface. Without these resonant surfaces the system is not stochastic. In the case of monotonous safety factors, these surfaces appear for smaller values of q_0 (Figure 1a). It means that lower values of q_0 are favorable for the stochastization.

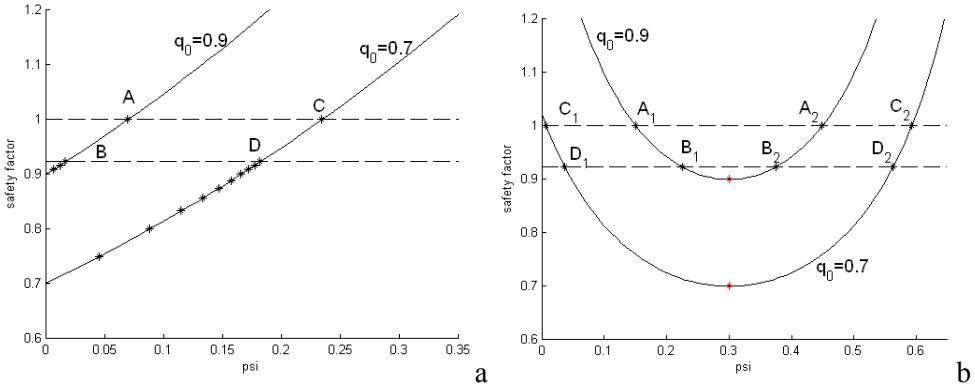


Fig. 1 - Monotonous safety factors corresponding to W_1 (a) and non-monotonous safety factors corresponding to W_2 (b), for $q_0 = 0.7$ and $q_0 = 0.9$. The position of the $q = 1$ and $q = 12/13$ surfaces is shown on the horisontal dashed lines. Some other low order ration al surfaces are marked with * in the left figure.

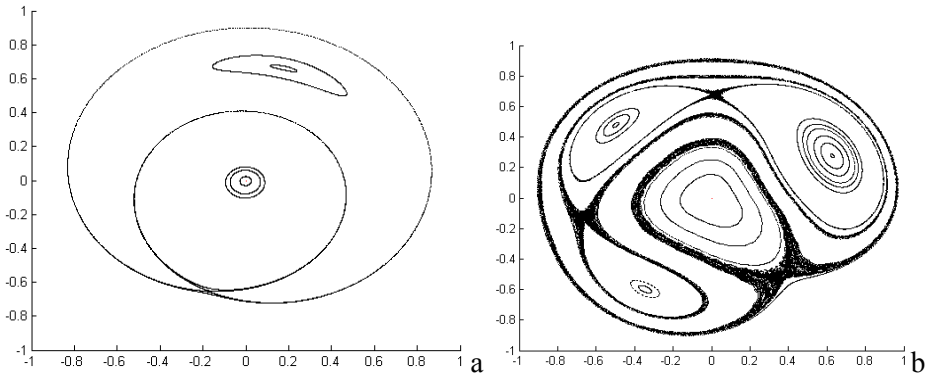
Because the stochastization occurs just before the sawtooth crash, the general scenario in the monotonous safety factor case could be: in the rise time the value of q_0 *decreases slowly (for some threshold the stochastization is obtained) and increases rapidly in the crash time (for some threshold the regularization of the orbits is obtained).*

This relation between the existence of many low-order rational surfaces and stochastization is confirmed by some experiments in ASDEX-Upgrade tokamak [2], which show that the critical value to create a stochastic zone is $q_0 = 0.7 \pm 0.1$ and the critical value for avoiding stochastization is $q_0 = 0.9 \pm 0.05$. It was conjectured that during the sawtooth cycle q_0 changes slowly between 0.9 and 0.7 in the rise time and very rapidly between 0.7 and 0.9 in the crash time [2].

Our observations show that qualitatively similar results are obtained for any monotonous safety factor. For the reversed shear configuration the stochastization is obtained through the overlapping process of the separatrices of the twin island chains, when they become closer, *i.e.* q_0 increases (Figure 1b).

For reversed-shear configurations the value of q_0 *increases slowly in the rise time (for some value the stochastization is produced) and decreases rapidly in the crash time (for some threshold the regularization of the orbits is obtained).* The influence of the higher harmonics on the reconnection process is also important.

The absence of (2,2) and (3,3) modes significantly increases the threshold of stochasticity. Even small amplitudes of the (2,2) and (3,3) modes considerably increase the stochastic region, due to the overlapping phenomena. It works in the same way for monotonous and reversed-shear magnetic configurations (Figure 2).



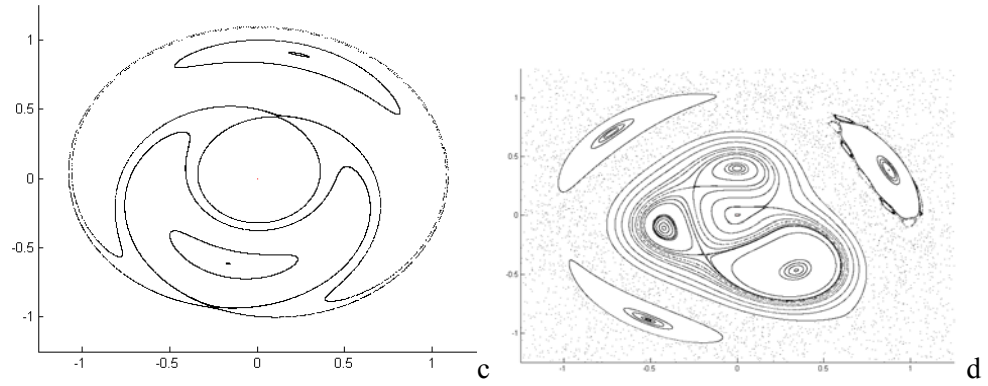


Fig. 2 – Influence of the higher harmonics. Monotone safety factor: (a) only the (1,1) mode is active, (b) the modes (1,1), (2,2), (3,3) are active. Reversed-shear safety factor: (c) only the (1,1) mode is active, (d) the modes (1,1), (2,2), (3,3) are active.

In our interpretation, the sawtooth oscillations are related to the modification of the minimum value of the safety factor and to the excitation of high modes through two different scenarios:

- for monotonous safety factor q_0 decreases slowly in the rise time, then q_0 rapidly increases in the crash time;
- for reversed shear safety factor q_0 increases slowly in the rise time, then q_0 rapidly decreases in the crash time.

In both cases the modes (2,2), (3,3) are excited just before the crash and are deactivated in the crash time. The stochastization of magnetic field lines [6] appears only for a short time period that is the crash phase itself. The islands are not destroyed and the (1,1) mode remains in the same position after the crash, but all of the temperature is lost in the stochastic phase.

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