

NEUTRINO MASSES WITH A SUITABLE PARAMETRIZATION IN THE  
PPF 3-3-1 GAUGE MODEL

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Plausible phenomenological consequences of a particular 3-3-1 model are analyzed by using the algebraical approach for gauge models with high symmetries. We prove that a suitable parametrization in the Higgs sector and a redefinition of the three scalar triplets involved therein can lead to flexible and realistic predictions for the neutrino mass spectrum, while a minimal number of coupling parameters are employed in the Yukawa sector. The whole Standard Model phenomenology is recovered.

*Key words:* 3-3-1 gauge models, extended electroweak Higgs sector, neutrino masses and mixings.

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**1. INTRODUCTION**

One of the most investigated extensions of the Standard Model (SM) in the last decade has been the so called Pisano-Pleitez-Frampton (PPF) model [1, 2] based on the gauge group  $SU(3)_c \otimes SU(3)_L \otimes U(1)_Y$  (in short “3-3-1”) that – in its electroweak sector – undergoes a *spontaneous symmetry breakdown* (SSB) up to the  $U(1)_{em}$  electromagnetic one. In the literature, the scalar sector of such a model assumes an extended Higgs mechanism in two steps ( $331 \rightarrow 321 \rightarrow 31$ ). Three scalar triplets plus a scalar sextet [3] are employed in order to generate plausible masses for all the fermions and bosons in the model. A detailed analysis of the most general Higgs potential in 3-3-1 models that contains three scalar triplets and a scalar sextet is worked out in Ref. [4]. Note that PPF model is a particular version of the rich class of 3-3-1 models, namely the one that allows for exotic electric charges both in the boson sector ( $\pm 2e$ ) and the quark sector ( $\pm 4e/3; \pm 5e/3$ ) in addition to the well-known ones ( $0, \pm e$ ) and ( $\pm 2e/3; \pm e/3$ ) respectively. The fundamental fermion triplet is the leptonic one and its three positions are occupied as follows: left-handed charged lepton  $l_L$ , right-handed charged lepton ( $l^c$ )<sub>L</sub>, and corresponding left-handed neutrino  $\nu_{lL}$ . This structure is identically triplicated for the well-known generations:  $e$  family,  $\mu$  family, and  $\tau$  family respectively.

An exact algebraical approach for gauge models with high symmetries  $SU(n)_L \otimes U(1)_Y$  – subject to SSB – was proposed several years ago by Cotăescu [5]. The method displays a minimal Higgs mechanism (mHm) in one step (331  $\rightarrow$  31) that finally gives rise to the mass-generating Yukawa terms in unitary gauge, while only one neutral scalar – namely, the physical Higgs field – survives the SSB just like in the SM. The results essentially depend on a parametrisation in the Higgs sector of the model where a vector space structure is imposed and the required number of Higgs multiplets (vectors in different directions) is equal to the dimension of the fundamental irreducible representation (irrep) of the  $SU(n)$  group. Any other necessary scalar representations (including the required sextet) are obtained out of these multiplets by constructing certain tensor-like products among them. At the same time, the correction due to the mixing angle between the neutral gauge bosons is not needed any more at the end of the calculus, since this task is performed as a step of the method itself by means of a special *generalized Weinberg transformation* (gWt). Regarding the renormalizability of such a theory, it is achieved by restricting ourselves to dealing only with gauges (unitary gauge or equivalent ones) where Yukawa sector recovers the traditional form of the couplings, as the scalar sector contains no higher powers than  $\phi^4$ .

When applied to the PPF 3-3-1 model, the method supplies exact results regarding the boson and lepton mass spectrum, the charged and neutral currents in the model, as well as the possible neutrino mass patterns, all being presented in Ref. [6]. We strictly followed therein the prescriptions of the general procedure, so that the mHm exhibits one vev only. Therefore, the Yukawa sector called for a plethora of free coupling parameters. Here we develop those results in a more suitable direction, by taking into account a redefinition of the scalar fields that leads to a proper involvement of the scalar sector's parameters in the vev splitting and thus to a decrement of the number of the free coupling parameters in the Yukawa sector. All the previously obtained results regarding the boson sector and the charges in the model are not affected.

The paper is organized as follows. A brief review of the model is presented in Sec. 2 while Sec. 3 contains our proposal for the possible Yukawa terms to generate both Dirac and Majorana neutrino masses, dealing with only three coupling parameters (a distinct one for each lepton family). Sec. 4 is devoted to our conclusions and some phenomenological predictions.

## 2. BRIEF REVIEW OF THE MODEL

The *Lagrangian density* (Ld) of any gauge model that undergoes a SSB must consist of several distinct terms, each describing one of the following sectors: (i) the fermion sector, (ii) the gauge boson sector and (iii) the Higgs sector, respectively. In addition, (iv) the Yukawa sector must be employed in order to

generate fermion masses, once the SSB took place. For our purpose here, the latter one is of great interest. Therefore, we begin by briefly presenting its ingredients – namely, the fermion families and the scalar triplets – as irreps of the 3-3-1 gauge group. Consequently, we focus on the possible ways of constructing the mass-generating terms. In this respect, a sextet is constructed in the scalar sector. The gauge sector consists of two neutral bosons ( $Z, Z'$ ), the photon ( $\gamma$ ), two singly charged bosons ( $W^\pm, V^\pm$ ) and a doubly-charged one ( $U^{\pm\pm}$ ). However, all the details regarding the gauge sector – such as the boson mass spectrum and the charges of the particles with respect to these bosons – are definitely established in Ref. [6].

### Gauge sector

The two diagonal generators of the  $su(3)$  Lie algebra are considered

$$D_1 = T_3 = \frac{1}{2} \text{Diag}(1, -1, 0), \quad D_2 = T_8 = \frac{1}{2\sqrt{3}} \text{Diag}(1, 1, -2) \quad (1)$$

while the  $E_j^i = H_j^i / \sqrt{2}, i \neq j$ , related to the off-diagonal real generators  $H_j^i$ . Therefore the elements  $\xi = D_i \xi^i + E_j^i \xi_j^i \in su(n)$  (in our case  $su(3)$ ) are now parameterized by  $n - 1 = 2$  real parameters,  $\xi^i$ , and by  $n(n - 1)/2 = 3$   $c$ -number ones,  $\xi_j^i = (\xi_i^j)^*$ , for  $i \neq j$ . The advantage of this choice is that the parameters  $\xi_j^i$  can be directly associated to the  $c$ -number gauge fields due to the factor  $1/\sqrt{2}$  which gives their correct normalization.

In this basis the gauge fields are  $A_\mu^0$  and  $A_\mu \in su(3)$  algebra. That is

$$A_\mu = \frac{1}{2} \begin{pmatrix} A_\mu^3 + A_\mu^8 / \sqrt{3} & \sqrt{2} U_\mu & \sqrt{2} V_\mu \\ \sqrt{2} U_\mu^* & -A_\mu^3 + A_\mu^8 / \sqrt{3} & \sqrt{2} W_\mu \\ \sqrt{2} V_\mu^* & \sqrt{2} W_\mu^* & -2A_\mu^8 / \sqrt{3} \end{pmatrix}. \quad (2)$$

Apart from the charged Weinberg bosons,  $W$ , there are new charged bosons,  $V$  and  $U$ , among them  $U$  is doubly charged – the so called “bilepton” – connecting different chiral states of the same charged lepton.

The PPF 3-3-1 model displays the following anomaly-free particle content:

### Lepton families

$$f_{\alpha L} = \begin{pmatrix} e_\alpha^c \\ e_\alpha \\ \nu_\alpha \end{pmatrix}_L \sim (1, \mathbf{3}, 0) \quad (e_{\alpha L})^c \sim (\mathbf{1}, \mathbf{1}, 1) \quad (3)$$

### Quark families

$$Q_{iL} = \begin{pmatrix} J_i \\ u_i \\ d_i \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}^*, -1/3) \quad Q_{3L} = \begin{pmatrix} J_3 \\ -b \\ t \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}, +2/3) \quad (4)$$

$$(b_L)^c, (d_{iL})^c \sim (\mathbf{3}, \mathbf{1}, 1/3) \quad (t_L)^c, (u_{iL})^c \sim (\mathbf{3}, \mathbf{1}, -2/3) \quad (5)$$

$$(J_{3L})^c \sim (\mathbf{3}, \mathbf{1}, +5/3) \quad (J_{iL})^c \sim (\mathbf{3}, \mathbf{1}, -4/3) \quad (6)$$

with  $\alpha = 1, 2, 3$  and  $i = 1, 2$ . In the gauge group's irreps displayed above we assume, like in majority of the papers in the literature, that the third generation of quarks transforms differently from the other two ones. This could explain the unusual heavy masses of the third generation of quarks, and especially the uncommon properties of the top quark. The capital letters  $J$  denote the exotic quarks included in each family. The subscript  $L$  means left-handed component, while the numbers in parentheses are the representations and characters with respect to the gauge group of the model. These fermion families must be coupled through the scalar irreps in order to form mass generating terms.

### Higgs triplets

The Higgs sector consists of three scalar triplets which obey the following irreps  $\phi^{(1)} \sim (\mathbf{1}, \mathbf{3}, -1)$ ,  $\phi^{(2)} \sim (\mathbf{1}, \mathbf{3}, 1)$  and  $\phi^{(3)} \sim (\mathbf{1}, \mathbf{3}, 0)$ . The mHm prescribed by the general method [5] and formerly applied to the PPF model [6] involves the parameters

$$\eta^2 = (1 - \eta_0^2) \text{Diag} \left( 1 - a, \frac{a(1 - 3 \tan^2 \theta_W)}{2}, \frac{a(1 + 3 \tan^2 \theta_W)}{2} \right) \quad (7)$$

only in the boson mass spectrum, since the single remaining vev of the model  $\langle \phi \rangle$  requires a lot of new parameters  $A, B, C, A', B', C'$  – Yukawa coupling coefficients in the lepton Yukawa sector – in order to generate adequate masses. The aim of this paper is proving that a more suitable approach can reduce at least three of the above Yukawa couplings. Namely, the parameters  $A', B', C'$  can be erased.

In this respect one redefines the scalar triplets in the following manner:

$$\phi^{(i)} \rightarrow \eta^{(i)} \phi^{(i)} \quad (8)$$

keeping at the same time the orthogonality condition (Eq. (27) in Ref. [5]) for the new scalar triplets in order to avoid the Goldstone bosons successively the SSB.

It is easy to check that this redefinition does not alter anyhow the minimum condition (Eq.(33) in Ref.[5]) for the scalar potential (Eq. (32) in Ref. [5]), since  $\lambda$ 's are still arbitrary and they will account only for the Higgs mass (but this is not our task here).

The advantage of the present method is that of splitting the vev  $\langle\phi\rangle$  into three vevs  $\langle\phi^{(i)}\rangle = \eta^{(i)}\langle\phi\rangle$ , thus getting closer to the traditional approaches to 3-3-1 models with SSB in two steps. The trace condition in the parameter matrix Eq. (29) in Ref. [5] ensures that  $\langle\phi\rangle^2 = \langle\phi^{(1)}\rangle^2 + \langle\phi^{(2)}\rangle^2 + \langle\phi^{(3)}\rangle^2$ .

With this assignments, the mass spectrum of the gauge bosons – according to Eqs. (53)–(55) in Ref. [5] – reads

$$\begin{aligned} m^2(W) &= m^2(Z)\cos^2\theta_w = m^2a, \\ m^2(Z') &= \frac{m^2}{1-4\sin^2\theta_w} \left\{ \frac{4}{3}\cos^2\theta_w - a \left[ 1 - (1-4\sin^2\theta_w)\tan^2\theta_w \right] \right\}, \\ m^2(V) &= m^2 \left[ 1 - \frac{a}{2}(1-3\tan^2\theta_w) \right], \\ m^2(U) &= m^2 \left[ 1 - \frac{a}{2}(1+3\tan^2\theta_w) \right]. \end{aligned} \tag{9}$$

The boson mass spectrum is now a matter of tuning the sole free parameter  $a$  once the mass scale  $m$  (or, in other words the vev) is established. Obviously,  $Z$  is the neutral boson of the SM, while  $Z'$  is the new neutral boson of this model. The first line in Eqs. (9) expresses the well known SM condition  $m(Z) = m(W)/\cos\theta_w$ . Moreover, one computes the currents with respect to this physical bosons using the general method. They can be found in the Table in Ref. [6]. The method itself considered the mixing angle between  $Z$  boson and  $Z'$  one as an inner step and, thus, the results given in the Table therein are the exact ones.

With these minimal preliminaries which confirm all the phenomenological features of the SM we go further on in order to obtain the fermion masses in the PPF 3-3-1 model.

### 3. FERMION MASSES

#### 3.1. CHARGED LEPTON MASSES

In the PPF 3-3-1 model, the charged leptons acquire their masses by means of a scalar sextet [3], which is a compulsory ingredient in the Yukawa Ld. We build here this scalar sextet out of the scalar triplets already existing in the Higgs sector of the model as a tensor-like product in the following manner:

$$S = \phi^{-1} \left( \phi^{(1)} \otimes \phi^{(2)} + \phi^{(2)} \otimes \phi^{(1)} \right) \tag{10}$$

It plays the same role as the tensor blocks  $\chi^{\rho\rho'}$  in Eq. (16) in Ref. [5]. Evidently,  $S \sim (1, \mathbf{6}, 0)$  and hence the generating mass term in the charged leptons sector reads

$$G_\alpha \bar{f}_{\alpha L} S f_{\alpha L}^c + H.c. \quad (11)$$

Hence, consequently the SBB, only positions (12) and (21) in Eq. (11)) will remain non-zero. For the redefined scalar fields, that is

$$\langle S \rangle = \begin{pmatrix} 0 & \eta^{(1)} \eta^{(2)} & 0 \\ \eta^{(1)} \eta^{(2)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \langle \phi \rangle. \quad (12)$$

All the charged fermions acquire their masses through the above presented coupling terms – Eq. (11), since all couplings due to  $S$  get in the unitary gauge the traditional Yukawa form:  $G_\alpha \langle \phi \rangle \bar{e}_{\alpha L} e_{\alpha L}^c$  (according to a Dirac Lagrangian density put in the pure left form – see Appendix B in Ref. [5]). Therefore, one can identify the mass of the charged lepton as

$$m(e_\alpha) = G_\alpha \eta^{(1)} \eta^{(2)} \langle \phi \rangle \quad (13)$$

Note that Eqs. (13) introduce 3 more parameters  $G_\alpha$  in the model. Let them be:  $A$  for  $e$ ,  $B$  for  $\mu$  and  $C$  for  $\tau$ .

In the literature on the 3-3-1 models, the Higgs triplets irreps are commonly denoted by  $\rho \sim (1, \mathbf{3}, 1)$ ,  $\eta \sim (1, \mathbf{3}, 0)$  and  $\chi \sim (1, \mathbf{3}, -1)$ . One has now to perform a bijective mapping  $(\langle \chi \rangle, \langle \rho \rangle, \langle \eta \rangle) \rightarrow (\langle \phi^{(1)} \rangle, \langle \phi^{(2)} \rangle, \langle \phi^{(3)} \rangle)$  in order to identify the scalar triplets from our method. Consequently, one actually deals with 3 possible distinct cases after SSB, since Eq. (37) in Ref. [5] allows us to perform a boost  $U_j^i$  towards a new gauge that is equivalent to the unitary one. Therefore, a simple permutation could well be taken into consideration (the boost itself can perform it!), since in the unitary gauge Eq. (36) in Ref. [5] one could take  $\delta_{i,j-1}$  or  $\delta_{i,j+1}$  instead of  $\delta_{i,j}$ .

At this point, our method offers three distinct cases for the possible masses in the charged lepton sector, while keeping unchanged the order in the parameter matrix  $\eta$ . The charged lepton mass can have, with respect to the parameter order, the following possible values:

- $m(e_\alpha) = \frac{1}{2\sqrt{2}} G_\alpha \sqrt{(1-a)a(1-3\tan^2\theta_w)} \langle \phi \rangle$  (case I)
- $m(e_\alpha) = \frac{1}{2\sqrt{2}} G_\alpha \sqrt{(1-a)a(1+3\tan^2\theta_w)} \langle \phi \rangle$  (case II)

$$\bullet m(e_\alpha) = \frac{1}{4} G_\alpha a \sqrt{(1 - 9 \tan^4 \theta_W)} \langle \phi \rangle \text{ (case III).}$$

Up to this stage each of these cases has two subcases for the choice of the remaining two scalar triplets. As further steps, one has to investigate the phenomenological aspects of each choice and rule out the unsuitable ones.

It is worth to note that this kind of approach led to plausible phenomenological predictions [7, 8] in the case of the 3-3-1 models with right-handed neutrinos.

### 3.2. NEUTRINO MASS MATRIX

Since the neutrino oscillations are an undisputable evidence [9]–[15], all the extensions of the SM must incorporate realistic theoretical mechanisms for generating tiny masses in the neutrino sector. There are two main lines in the literature to obtain these tiny masses: (a) **see-saw mechanism** [16]–[18] (see, for instance, Refs. [19]–[24] for its particular realisations in 3-3-1 models) and (b) **radiative corrections** (widely exploited in several variants of 3-3-1 models [25]–[31]).

We propose here neutrino mass terms at three level in the Yukawa Ld of the PPF model. The order of magnitude for these masses will be a matter of tuning the free parameter  $a$  of the model. We examine in the following two distinct possibilities with different phenomenological implications. The PPF 3-3-1 model allows for either Dirac or Majorana masses in the neutrino sector, and even for both at the same time (since the see-saw mechanism can occur).

In order to minimize the number of free parameters in the Yukawa sector one can assign a unique coupling to each lepton family.

**Dirac neutrinos** Assuming the existence of the right-handed neutrinos one can add in the leptonic Yukawa sector of the PPF model presented above a supplementary term of the form

$$G_{\alpha\beta} \bar{f}_{\alpha L} \eta^* v_{\beta R} + H.c. \quad (14)$$

After SSB such a canonical Yukawa term generates a pure Dirac neutrino mass matrix:

$$M(\nu) = \frac{1}{2} \begin{pmatrix} A & D & L \\ E & B & F \\ K & G & C \end{pmatrix} \eta^{(\eta)} \langle \phi \rangle. \quad (15)$$

Obviously, the coupling constants are in our notation:  $A = G_{ee}$ ,  $B = G_{\mu\mu}$ ,  $C = G_{\tau\tau}$ ,  $D = G_{e\mu}$ ,  $E = G_{\mu e}$ ,  $F = G_{\mu\tau}$ ,  $G = G_{\tau\mu}$ ,  $L = G_{e\tau}$ ,  $K = G_{\tau e}$ .

At this point the three possible cases (determined by the vev alignment) are expressed as a function depending on the sole parameter  $a$ .

$$\begin{aligned}
\bullet \quad m(\nu_\alpha) &= \frac{1}{\sqrt{(1-a)}} \sqrt{\frac{(1+3 \tan^2 \theta_W)}{1-3 \tan^2 \theta_W}} m(e_\alpha) \quad (\text{case I}) \\
\bullet \quad m(\nu_\alpha) &= \frac{1}{\sqrt{(1-a)}} \sqrt{\frac{(1-3 \tan^2 \theta_W)}{1+3 \tan^2 \theta_W}} m(e_\alpha) \quad (\text{case II}) \\
\bullet \quad m(\nu_\alpha) &= 2 \frac{\sqrt{1-a}}{a} \frac{1}{\sqrt{(1-9 \tan^4 \theta_W)}} m(e_\alpha) \quad (\text{case III}).
\end{aligned}$$

**Majorana neutrinos** A second possibility is to introduce pure Majorana terms for neutrinos. Under these circumstances, the leptonic Yukawa sector is completed by a special term:

$$G_{\alpha\beta} \bar{f}_{\alpha L} \left[ \phi^{-1} \left( \phi^{(n)} \otimes \phi^{(n)} \right) \right] S f_{\beta L}^c + H.c. \quad (16)$$

which develops the well-known Yukawa shape in unitary gauge, successively the SSB. We notice that for the Majorana case, matrix  $M$  is a symmetric real one, with  $D = E$ ,  $F = G$ ,  $L = K$ . Hence, the Majorana neutrino mass matrix reads:

$$M(\nu) = \frac{1}{4} \begin{pmatrix} A & D & E \\ D & B & F \\ E & F & C \end{pmatrix} \left( \eta^{(n)} \right)^2 \langle \phi \rangle. \quad (17)$$

The three possible cases (depending on the vev alignment) exhibit the following mass spectrum for Majorana neutrinos.

$$\begin{aligned}
\bullet \quad m(\nu_\alpha) &= \frac{1}{2} \sqrt{\frac{a}{(1-a)}} \frac{(1+3 \tan^2 \theta_W)}{\sqrt{(1-3 \tan^2 \theta_W)}} m(e_\alpha) \quad (\text{case I}) \\
\bullet \quad m(\nu_\alpha) &= \frac{1}{2} \sqrt{\frac{a}{(1-a)}} \frac{(1-3 \tan^2 \theta_W)}{\sqrt{(1+3 \tan^2 \theta_W)}} m(e_\alpha) \quad (\text{case II}) \\
\bullet \quad m(\nu_\alpha) &= \frac{1-a}{a} \frac{1}{\sqrt{(1-9 \tan^4 \theta_W)}} m(e_\alpha) \quad (\text{case III}).
\end{aligned}$$

**Phenomenological restrictions on parameter  $a$ .** Now, for either Dirac or Majorana species of neutrinos, one can examine each of the resulting expressions and compare them to the available experimental data [32] regarding the order of magnitude in the neutrino mass spectrum and to other particular features the neutrino phenomenology exhibits [33]–[38]. Of course, some of the expressions displayed above for the neutrino masses will be ruled out by certain restrictive

conditions imposed by phenomenological reasons. We focus in the following on a single criterion, namely the order of magnitude for the neutrino masses.

A great deal of experimental data confirm that phenomenological values [37, 38] of neutrino masses  $m(\nu_\alpha)$  are severely limited to a few eVs. Therefore, one remains finally with only a few acceptable cases out of all possible ones that our method on theoretical grounds allows. Let us compute the sum of the neutrino masses. It is nothing but the trace of the neutrino mass matrix.

$$\sum_{\alpha} m(\nu_{\alpha}) = \text{Tr}M(\nu) = m(\tau) \left[ 1 + \frac{m(\mu)}{m(\tau)} + \frac{m(e)}{m(\tau)} \right] f(a, \theta_W). \quad (18)$$

First of all, we observe that one can neglect the small ratios  $m(\mu)/m(\tau) \sim 0.05$  and  $m(e)/m(\tau) \sim 0.0002$  in Eq. (15). Hence, the required sum will be well approximated by:

$$\sum_{\alpha} m(\nu_{\alpha}) \simeq m(\tau) f(a, \theta_W). \quad (19)$$

Note that Dirac neutrinos exclude the cases (I) and (II), since these cases supplies unacceptable values for the neutrino mass spectrum which now is lower bounded by  $m(e_{\alpha}) \sqrt{(1-3 \tan^2 \theta_W)/(1+3 \tan 2\theta_W)}$  which is far above the eVs domain. Therefore, in the case of pure Dirac neutrinos only the case (III) has to be further investigated. It favours values in the vicinity of 1 for parameter  $a$ . For such a parameter  $a$  also Majorana neutrinos accept only the case (III).

If one wants to keep the free parameter  $a$  in the vicinity of 0 one observes that pure Dirac neutrinos are not compatible with any of the above cases, while Majorana neutrinos could be compatible with cases (I) and (II).

Therefore, at this point one can say that by just tuning the parameter  $a$ , the neutrino masses – either they are pure Dirac or pure Majorana fields – could come out at viable values.

The see-saw mechanism can be – with this assignment – naturally implemented in the model. However, this issue will be analyzed in a future work.

#### 4. CONCLUDING REMARKS

In this paper we have proved that the well-known PPF 3-3-1 model can be investigated from an exact algebraical viewpoint by just tuning a single free parameter  $a$ . All the phenomenological consequences of the model occur due to this parameter.

For instance, if we take  $\sin^2 \theta_W \simeq 0.231$  and  $m(\tau) = 1.777$  GeV (Particle Data Group [32]), then plausible Dirac neutrino masses – in the range  $\sim 1$  eV – occur only if  $a \simeq 1$ , more precisely, a value  $a \simeq (1-10^{-16})$  is necessary. The case (III) is the

only one compatible with such a setting. Under these circumstances, the “old” bosons recover their SM mass values *i.e.*  $m(Z) \approx 91.1\text{GeV}$  and  $m(W) \approx 80.4\text{GeV}$  [32] while the “new” ones now become – according to Eqs. (26) in Ref. [6] – lighter:  $m(Z') \approx 63.76\text{GeV}$ ,  $m(U) \approx 17.8\text{GeV}$  and  $m(V) \approx 78.1\text{GeV}$ . Quite the same boson mass spectrum is obtained if neutrinos acquire Majorana masses, also in the case (III) presented in previous section. These possibilities arise at not a very high breaking scale  $\langle\phi\rangle \approx 500\text{GeV}$ .

For the Majorana neutrinos, case (II) – and even case (I) – also seem to be viable if and only if  $a \approx 10^{-20}$  or less. In these cases the boson mass spectrum – the same formulas (26) in Ref. [6] – looks like:  $m(Z') \approx 0.03 \times 10^{16}\text{GeV}$  and  $m(U) \approx m(V) \approx 0.0001 \times 10^{16}\text{GeV}$  and the vev  $\langle\phi\rangle$  of the model lies in the GUT energies region ( $10^{16}\text{GeV}$ ). This seems to be the price paid in order to have good phenomenological results for all the known particles in this 3-3-1 model, using only one free parameter.

Further experimental investigations at LHC will precisely determine the value of the new bosons masses. Only then one can decide which case of the three provided by our method will be favoured. However, one can imagine a suitable see-saw mechanism (to be presented in a future work) or radiative mechanisms designed to supply good neutrino masses without resorting to such a fine tuning.

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