COMBINED DYNAMICS OF PHYSICAL AND GHOST TYPE FIELDS IN THE FREEDMAN-TOWNSEND MODEL

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The paper analyses the dynamics of the whole set of variables which generate the extended phase-space in the standard BRST Hamiltonian approach. The main interest is devoted to the investigation of the mutual influence that the real (physical) fields and the ghost-type variables determine upon each other when the ghosts are considered as being physical fields, too. The Freedman-Townsend model will be considered as a concrete application.

1. INTRODUCTION

The Freedman-Townsend model represents one of the most fruitful model of 1-reducible gauge theories and, in the same time, it is directly connected with the Witten's string theory. The model was intensively studied in the frame of the BRST formalism, both at the Lagrangean [1–3] and at the Hamiltonian level [4]. In this last context we mention the papers [5, 6] where an irreducible BRST procedure for our reducible model is applied. Using the equivalence between the Hamiltonian and the Lagrangean formalism, the sp(3) Lagrangean description of the model has been obtained in [8].

The present paper offers an approach which effectively takes into account the reducible status of the model. This means that the general algorithm for reducible theories presented in [7] will be applied. On the other hand, the study will be extended in the sense of considering the ghost fields as being real and trying to take into account the equations of motion for these "unphysical" variables. The main question we will look for answer will be if the initial model is equivalent with the one obtained when the whole set of generators, real and ghost types, are considered as being real variables. This is an important question taking into account that a direct comparision between the action of the initial model and the extended action obtained in the BRST frame is not possible, as long as the two actions are defined in different spaces.

The paper has the following structure: after this introductory part, the canonical analysis of the Freedman-Townsend model is reviewed. In the third section the standard BRST Hamiltonian formalism for our model is completely implemented. The possibility of recovering the main structure of the model when the equations of motion for the ghosts are taken into account is tackled out in the forth section. Some concluding remarks and comments concerning the advantages offered by the use of the equations of motion for ghosts will end the paper.

2. CANONICAL ANALYSIS OF THE MODEL

We will start with the canonical analysis of the model and we will present then how the standard BRST symmetry could be implemented in this case. The starting point is represented by the Lagrangian action

$$S_0[A_\mu^m, B_m^{\mu\nu}] = \frac{1}{2}\int d^4x\,(-B_m^{\mu\nu}F_{\mu\nu}^m + A_\mu^m A_m^\mu), \tag{1}$$

where $B_m^{\mu\nu}$ represent some antisymmetric tensor fields and the strength field has the form:

$$F_{\mu\nu}^m = \partial_\mu A_\nu^m - \partial_\nu A_\mu^m - f^m{}_{nr}A_\mu^n A_\nu^r$$
$$\mu, \nu = 0, 1, 2, 3; \quad m, n, r = 1, \ldots, d$$

The structure functions f_{nr}^m are real constants in our model. The canonical analysis of this model leads to the following set of constraints:

$$\Phi_i^{(1)m} \equiv \varepsilon_{0ijk}p^{jkm} \approx 0, \quad \Phi_i^{(2)m} \equiv \frac{1}{2}\varepsilon_{0ijk}(F^{jkm} - (D^{[j})^m{}_n p^{k]0n}) \approx 0, \tag{2}$$

$$\chi_i^{(1)m} \equiv p_{0i}^m \approx 0, \qquad \chi_i^{(2)m} \equiv p_i^m + B_{0i}^m \approx 0, \tag{3}$$

$$\chi^{(1)m} \equiv p_0^m \approx 0, \qquad \chi^{(1)m} \equiv A_0^m + f_{nr}^m B_{0i}^n p^{0ir} + (D_i)^m{}_n p^{in} \approx 0. \tag{4}$$

A direct check shows that (2) are first class constraints and (3), (4) second class; moreover, $\Phi_i^{(2)m}$ are 1-reducible. The first class constraints, the first class Hamiltonian and the reducibility relations will be:

$$G_i^{(1)m} \equiv \varepsilon_{0ijk}p^{jkm} \approx 0, \qquad G_i^{(2)m} \equiv \frac{1}{2}\varepsilon_{0ijk}F^{jkm} \approx 0, \tag{5}$$

$$H' = \frac{1}{2}\int d^3x\,(B_m^{ij}F_{ij}{}^m - A_i^m A_m^i + ((D^i)^m{}_e B_{0i}^e)(D_j)_m{}^g B_g^{oj}), \tag{6}$$

$$Z^{im}{}_n G_i^{(2)n} \equiv (D^i)^m{}_n G_i^{(2)n} = 0. \tag{7}$$

3. THE BRST HAMILTONIAN FORMALISM

The canonical analysis of the Freedman-Townsend model shows that the secondary constraints only are reducible. The reducibility relations ask [7] for supplementary variables associated to the reducibility functions. The problem that occurs is: the number of the ghost generators corresponding to the (irreducible) primary constraints is less than that for the secondary constraints. This "symmetry breaking" on the number of generators is solved either by using an irreducible mechanism for the reducible constraints [5, 6] or, by contrary, using the reducible technique for the set of irreducible constraints. This second alternative will be applied here: we will equalize the number of the ghost variables used for each type of constraints by introducing supplementary variables which will oblige the irreducible primary constraints to satisfy some relations and, by that, to become reducible. The Koszul-Tate complex of the theory will be generated by the following set of real and ghost momenta:

$$P_A \equiv \{p_\mu^m, p_{\mu\nu}^m, P_\mu^{(\Delta)m}, P^{(\Delta)m}, \Delta = 1, 2, m = 1, \ldots, d\}. \tag{8}$$

The canonical structure of the extended phase space is assured by a similar set of ghost type variables:

$$Q^A \equiv \{A_m^\mu, B_m^{\mu\nu}, Q_m^{(\Delta)\mu}, Q_m^{(\Delta)}, \Delta = 1, 2, \ m = 1, \ldots, d\}. \tag{9}$$

Using the homological perturbation theory we obtain the following expression for the BRST charge, Ω:

$$\Omega = \int d^3x \left(\varepsilon_{0ijk} p^{jkm} Q_m^{(1)i} + \frac{1}{2} \varepsilon_{0ijk} F^{jkm} Q_m^{(2)i} + P_i^{(2)m} (D^i)_m{}^e Q_e^{(2)} \right). \tag{10}$$

The extended BRST Hamiltonian can be put in the form:

$$\begin{aligned} H = \frac{1}{2} \int d^3x \, (& B_m^{ij} F_{ij}{}^m - A_i^m A_m^i + P_i^{(2)m} Q_m^{(1)i} + \\ & + [(D^i)^n{}_e B_{0i}^e + f_{mn}^r (P_i^{(2)m} Q_r^{(2)i} + P^{(2)m} Q_r^{(2)})]^2). \end{aligned} \tag{11}$$

The existence of the quadratic term in the Hamiltonian will be used later on for recovering the temporal component of A_μ^m, loosed by passing to the Dirac bracket.

The main problem to be solved from now on consists in obtaining a gauge fixed Hamiltonian, without unphysical degrees of freedom. This could be done by choosing a suitable gauge fixing function *K*. The form of the gauge fixed Hamiltonian is [9]:

$$H_K = H + [K, \Omega]. \tag{12}$$

The complexity of our model will lead to the following possible expression for *K*:

$$K=\int d^3x\left(\varepsilon^{0ijk}(\partial_j B_{0km})P_i^{(1)m}+\frac{1}{2}\varepsilon^{0ijk}(\partial_i B_{jkm})P_0^{(2)m}-P^{(1)m}(D_i)_m{}^n Q_n^{(2)ia}\right). \quad (13)$$

The Hamiltonian (12) with the gauge fixing function (13) will lead to the covariant gauge fixing action:

$$\begin{aligned}S_K=\int d^4x\Big(&-\frac{1}{4}B^m_{\mu\nu}F_m^{\mu\nu}+\frac{1}{2}A_\mu^m A_m^\mu+\frac{1}{2}\varepsilon^{\mu\nu\sigma\rho}b_\mu^m(\partial_\nu B_{\sigma\rho m})-\\&-\frac{1}{2}(\partial_{[\mu}P^m_{\nu]})(D^{[\mu})_m{}^n Q_n^{\nu]}-\frac{1}{2}(\partial^\mu P^m_\mu)(D_\nu)_m{}^n Q_n^\nu-\\&-(D_\mu)^m{}_n P^n(D^\mu)_m{}^r Q_r+\frac{1}{8}\varepsilon^{\mu\nu\sigma\rho}f^r_{mn}(\partial_{[\mu}P^m_{\nu]})(\partial_{[\sigma}P^n_{\rho]})Q_r\Big)\end{aligned} \quad (14)$$

where

$$b_\mu^m=(G_0^{(2)m},G_i^{(1)m};i=1,2,3)$$

and

$$Q_m^\mu\equiv(Q_m^{(1)0},Q_m^{(2)i}),\qquad P_\mu^m\equiv(P_i^{(1)m},P_0^{(2)m}).$$

4. THE EQUATIONS OF MOTION IN THE EXTENDED PHASE SPACE

The presence of the other variables helped us in obtaining a covariant form of the action but we still have to consider the equations of motion for these second part of variables. We will write them down and we will see that, on the basis of their use, a gauge fixed action without unphysical degrees of freedom can be obtained and there is not necessary to impose supplementary vanishing conditions for the ghosts in the asymptotic states. Moreover, we will see that there are some dependencies among the fields, so that a limited number of fields can be considered as being fundamental in our model.

Starting from the gauge fixing action (14) we obtain the following equations of motion:

$$A_0^m+(D^i)^m_n B^n_{0i}+f^m_{nr}\partial_{[0}P^r_{i]}Q^{in}+f^m_{nr}(\partial^\mu P^r_\mu)Q^{0n}-2f^m_{nr}(\partial_0 P^r)Q^n=0 \quad (15)$$

$$\begin{aligned}A_i^m+(D^0)^m_n B^n_{0i}+(D^j)^m_n B^n_{ji}-f^m_{nr}\partial_{[i}P^r_{0]}Q^{0n}-2f^m_{nr}\partial_{[i}P^r_{j]}Q^{jn}-\\-f^m_{nr}(\partial^\mu P^r_\mu)Q^{in}-2f^m_{nr}(\partial_i P^r)Q^n=0\end{aligned} \quad (16)$$

$$\varepsilon^{0ijk}(\partial_{[j}b_{k]m})=F_m^{0i} \quad (17)$$

$$\varepsilon^{0ijk}(\partial_{[0}b_{k]m})=\frac{1}{2}F_m^{ij} \quad (18)$$

$$\ddot{P}_i^m-\partial_i\dot{P}_0^m+f^m_{nr}A_0^n\partial_{[0}P^r_{i]}+\partial_i(\partial^\mu P^m_\mu)-f^m_{nr}A_i^n(\partial^\mu P^r_\mu)=0 \quad (19)$$

$$\ddot{P}_0^m + \partial^i \partial_{[i} P_{0]}^m + f_{nr}^m A^{ir} \partial_{[0} P_{i]}^n + \partial^0 (\partial^i P_i^m) - f_{nr}^m A^{0r} (\partial^\mu P_\mu^n) = 0 \tag{20}$$

$$\ddot{P}^m - f_{nr}^m A_0^r \partial_\mu P^n + \partial_i \partial^i P^m - f_{nr}^m A_i^r (\partial_\mu P^n) + \frac{1}{2} \varepsilon^{0ijk} f_{nr}^m \partial_{[0} P_{i]}^n \partial_{[j} P_{k]}^r = 0 \tag{21}$$

$$\begin{gathered}\ddot{Q}_m^i + f_{mn}^r (\partial_0 A^{0n}) Q_r^i + f_{mn}^r A^{0n} (\partial_0 Q_r^i) + 2\partial_j (D^{[j})_m^r Q_r^{i]} + \partial^i (D_\mu)_m^r Q_r^\mu - \\ + \varepsilon^{0ijk} f_{mn}^r \partial_{[j} P_{k]}^n (\partial_0 Q_r) + 2\varepsilon^{0ijk} f_{mn}^r \partial_{[0} P_{k]}^r (\partial_j Q_r) = 0\end{gathered} \tag{22}$$

$$\begin{gathered}\ddot{Q}_m^0 + f_{mn}^r (\partial_0 A^{0n}) Q_r^0 + f_{mn}^r A^{0n} (\partial_0 Q_r^0) - \partial_i (D^0)_m^r Q_r^i + \partial_i (D^i)_m^r Q_r^0 + \\ + \partial_0 (D_i)_m^r Q_r^i + \varepsilon^{0ijk} f_{mn}^r \partial_{[j} P_{k]}^n (\partial_i Q_r) = 0\end{gathered} \tag{23}$$

$$\ddot{Q}_m + f_{mn}^r (\partial_0 A^{0n}) Q_r + f_{mn}^r A^{0n} (\partial_0 Q_r) + \partial_i (D^i)_m^r Q_r = 0 \tag{24}$$

We can note that, step by step, the remaining ghost variables can be expressed in terms of the real fields A_μ^m. It is also important to see that the equations from (15) to (18), written for the ghost fields, contain the ghost momenta P_μ^m and P^m only. The other equations mix the fields and their associated momenta.

5. CONCLUDING REMARKS

Let us note that: (*i*) the equations of motion for the real (physical variables) are not affected by the choose of a particular form of the gauge fixing function K. They have the same form before and after the gauge fixing procedure is applied; (*ii*) in the extended phase space there is a symplectic structure given by the generalized Poisson bracket that can be used in order to describe the dynamics of the system seen as being generated both by the real and by the ghost variables; (*iii*) by writing down the equations of motion for the ghost fields, we will obtain some equations which should allow in principle to express the ghost momenta P_μ^m and P^m in terms of the real fields A_μ^m. By keeping walking, these expressions of P_μ^m, used in their own equations of motion, offer the possibility of obtaining also the ghost fields in terms of A_μ^m, and afterwords the real fields $B_m^{\mu\nu}$ again in terms of A_μ^m. By that, the fields A_μ^m prove to be the fundamental fields of the model. This is an important conclusion which will be speculated when a mechanical Freedman-Townsend model will be defined by similarity with the Yang-Mills model [12]. A single set of color factors, $f^{(m)}(t)$, will be enough in this respect. The problem will be tackled out in another occasion.

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REFERENCES

1. E. Witten, *Nucl. Phys*. **B 268**, 2532 (1986).
2. S. P. de Alwis, M. T. Grisaru, L. Mezincescu, *Nucl. Phys*. **B 303**, 57 (1988).
3. M. Henneaux, *Phys. Lett*. **B 368**, 83 (1996).
4. C. Battle, J. Gomis, *Phys. Rev*. **D 38**, 1169 (1988).
5. C. Bizdadea, E. M. Cioroianu, S. O. Saliu, *Annalen Phys*. **11**, 151 (2002).
6. S. O. Saliu, *Phys. Scripta* **68**, 219 (2003).
7. R. Constantinescu, C. Ionescu, *Mod. Phys. Lett*. **A**, Vol. **15**, No 16, 1037 (2000).
8. G. Barnich, R. Constantinescu, Ph. Gregoire, *Phys. Lett*. **B 293**, 353 (1992).
9. M. Henneaux, C. Teitelboim, *Quantization of gauge systems,* Princeton Univ. Press, Princeton, 1992.
10. A. Babalean, R. Constantinescu, C. Ionescu, *J. Phys. A: Math Gen* **32**, 3005 (1999).
11. A. Dresse, J. M. L. Fisch, P. Gregoire, M. Henneaux, *Nucl. Phys*. **B 354**, 191 (1991).
12. S. G. Matincan, G. K. Savvidi, N. G. Ter-Arutyunyan-Savvidi, *Sov. Phys. JETP* 53 (3), 421 (1981).