BRANS-DICKE COSMOLOGY WITH DE-BROGLIE-BOHM QUANTUM POTENTIAL

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Brans-Dicke cosmology with two scalar coupled fields is explored using de-Broglie-Bohm quantum potential where the effects of gravity on geometry and the quantum effects on geometry of the space-time are coupled. In the presence of an additional ultra-light quartic potential (ULQP), the universe was found to undergoes at late times an eternal superinflationary regime with positive Brans-Dicke parameter $\omega > 8/3$ and an equation of state parameter equal to -1. The effective cosmological constant depends on the quantum potential, which at late times is found to be the sum of H^2 (H is the Hubble parameter) and induced term $\Lambda_{ind} = -3/4m^2$ where m is the ultra-light masses. In the absence of ULQP, the universe undergoes a phase of power-law inflation unless $\omega < 8/3$. The scalar fields were found to increase slowly linearly with time. Additional features are discussed.

Key words: BD cosmology, quantum potential, scalar fields, superinflationary regime.

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According to the present-day observational evidence, in particular the dynamics of galaxies [1], cluster of galaxies [2] Type Ia supernovae (SNIa) [3] with redshift $z > 0.35$, the CMB [4] and the recent findings of BOOMERANG experiments [5], our universe seems to be dominated by a positive cosmological constant, spatially flat and is today in an accelerated phase. From theoretical point of view, in order to obtain an accelerated regime, a repulsive gravitational effect must be obtained which in general is achieved if the strong energy condition $\rho + 3p \geq 0$ is violated. ρ is the energy density of the perfect exotic fluid called dark energy and p is the pressure which is negative. The nature of the dark energy component is one of the most important challenges in cosmology and particle physics.

There are a large number of theoretical competitive models in literature trying to explain the physical nature of the dark component, including the ΛCDM model [6] consisting a mixture of cosmological constant Λ and cold dark matter (CDM) or WIMPS composed of weakly interacting massive particles which must be relics of a grand unified phase of the Universe, quintessence with self-interacting scalar field and purely exponential and inverse power law potentials is considered [7], K-essence [8], viscous fluid [9], Chaplygin gas [10, 11], Generalized Chaplygin gas model (GCGM) [12, 13], Brans-Dicke (BD) pressureless solutions with non-minimal coupling [14–16], decaying Higgs fields where the dark energy field was assumed to be a decayed scalar component of a supermultiplet field in the early Universe that creates inertial mass through spontaneous symmetry breaking, *e.g.* a Higgs field [17], dual role of the Ricci scalar [18, 19], tachyons as a dark energy source having non-minimal coupling with curvature and self-interacting inverse cubic potential [20], minimally coupled scalar field theories with equation of state (EOS) $p = \tilde{\omega}\rho$, $\tilde{\omega}$ is the EOS parameter, with $\tilde{\omega} \geq -1$ (recent observations and classical tests do not seem to exclude the regime $\tilde{\omega} < -1$ (phantom field) [21–25]), non-minimal coupling with ultra-light masses [26, 27] etc. Most of these theories are accompanied with problems and many difficulties. For example, within the framework of the ΛCDM model, the vacuum energy is assumed to be constant while the matter energy density decreases with cosmic time, their ratio must be set to a specific infinitesimally small value (10^{-120}) in the early Universe so as to nearly coincide today, *i.e.* there exist a huge of discrepancy of about 120 orders of magnitude between the predicted and the observed values of the cosmological constant. This type of specific evolution is called the "cosmic coincidence" problem (CCP). In addition, several constraints and fine tuning of parameters for the potential used to model quintessence are required. This discrepancy may be alleviated using supersymmetric arguments but not totally solved. Moreover, the addressed question: "why the universe starts to accelerate very recently after the completion of formation of galaxies and clusters of galaxies" is not solved even if a positive Λ is present. Because of these conceptual and theoretical problems connected to the cosmological constant, many other alternatives have been proposed in particular the "tracker field" quintessence cosmological model [28] with exponential self-interaction and Gaussian potentials. It's equation of motion is an attractor like in a sense that for a wide range of initial conditions the equation of motion converges to the same solution. As for popular quintessence theory with scalar field ϕ acting as fluctuating dark energy, fine tuning parameters and several constraints are required. In the GCGM, unifying the description of dark energy and dark matter the density perturbations in the theory exhibits large oscillations in the resulting power spectrum which do not appear in the observed spectrum of mass agglomeration [29]. Moreover, within the framework of scalar tensor theories, it was showed that the Brans-Dicke cosmology bring a negligible correction to the matter density component of Friedmann equations where another tiny density parameter Ω_{BD} in introduced by the theory [30–32]. If this parameter is found to be positive, data will favor the BD model on the standard cosmological model with $\Lambda > 0$. Despite some remarkable interesting features of the theory, Ω_{BD} is too small to be detected from observations.

In the present work, we will try to combine the de-Broglie-Bohm quantum theory to BD scalar theory. Our motivation came in fact from de-Broglie comment that the quantum theory of motion for relativistic spinless particles is very similar to the classical theory of motion in a conformally flat space-time where the conformal factor is related to the Bohm's quantum potential [33]. One can interpret this by stating that "the effects of gravity on geometry is highly coupled to the quantum effects on the geometry of the space-time; moreover, the quantum and the gravitational effects of matter has geometrical nature" [34]. Accordingly, the conformal metric contains both gravity and quantum and the other background metric including only gravity. On the other hand, it was proved that using two scalar fields, one can relax this preassumption and on the equations of motion, the correct form of quantum potential will be achieved. In other words, the presence of the quantum potential is equivalent to a curved space-time with its corresponding metric. Consequently, it seems that there is a dual role of geometry in physics: the space-time geometry is sometimes gravitational and sometimes quantum [35, 36]. Since the equations governing the geometry are non-linear, the curvature due to the quantum potential may have a large influence on the classical contribution to the curvature of the space-time. In order that the above assumptions contribute on the dark energy problem, we will consider a quantum potential-like related to the scalar field χ like $Q = \lambda^2 \Box\chi/\chi = \lambda^2 \ddot{\chi}/\chi$ for a time-dependent scalar field in units $\hbar = c = 1$ where λ is a scale factor assumed to be of the order of H^{-1}, H is the Hubble parameter. The next step is to write the Lagrangian of the theory by including the conformal factor as a scalar field and introducing another scalar field (quantum potential in fact). Physical intuition leads us to that both fields are coupled together. One then expect that a relation between the quantum potential and conformal factor will emerge. The total action of the theory is written as:

$$S^{Total} = \\ = \frac{\sqrt{-g}}{16\pi}\int\left(\varphi\left(R-2\Lambda\right)-\frac{\omega}{\varphi}\nabla^{\mu}\varphi\nabla_{\mu}\varphi-\frac{1}{\varphi}\nabla^{\mu}\chi\nabla_{\mu}\chi+V\left(\varphi,\chi\right)+V\left(\varphi\right)\right)d^4x+S^{matter} \tag{1}$$

where g is the scalar metric, R is the scalar curvature, ω is a dimensionless constant, S^{matter} is the action of ordinary matter treated as fluid and which is covariantly conserved, *i.e.* $\nabla^{\mu}T_{\mu\nu}^{(matter)} = 0$ and in order to bring the quantum effects, we consider the potentials

$$V(\chi,\varphi)+V(\varphi)=\frac{3m^2}{4}\left(e^{\alpha H^{-2}\ddot{\chi}/\chi}-1\right)\varphi+\Lambda\left(2\beta\varphi H^{-2}\frac{\ddot{\chi}}{\chi}-1\right)^b-\frac{3}{4}m^2+\frac{3}{4}m^2w\varphi^4 \quad (2)$$

where terms containing the quantum potential are added. α, b and β are free positive parameters assumed in what follows to be equal to unity for simplicity. The first and second terms in equation (2) are written such that in the classical limit ($Q = 0$), terms on φ will survive. Moreover, we expect that the scalar field χ increases with time as a power-law, a lesson we learn from BD cosmology [37], and consequently at very late times ($t \to \infty$), the two parentheses in equation (2) vanishes and the potential $V(\varphi)$ dominates.

The ultra-light quartic potential $V(\varphi)$ is introduced based on our recent work on non-minimal coupling theories with ultra-light masses where it was proved that it has important cosmological consequences [26, 27]. The parameter $w << 1$ and $m \approx H$ are the ultra-light masses. In fact, these ultra-light masses are part of the dark energy hidden sector and have desirable feature for the description of the accelerated universe [38–41]. Their presence signals that the corresponding potentials are very shallow. In extended supergravity theories ultra light fields necessarily come in a package with too small Λ. The equations of motion look like:

$$\begin{aligned}R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}&=-\frac{1}{\varphi}T_{\mu\nu}+\frac{\omega}{\varphi^2}\left(\nabla_\mu\varphi\nabla_\nu\varphi-\frac{1}{2}g_{\mu\nu}\nabla_\rho\varphi\nabla^\rho\varphi\right)\\&+\frac{1}{\varphi}\left(\nabla_\mu\nabla_\nu\varphi-g_{\mu\nu}\Box\varphi\right)+\frac{1}{\varphi^2}\left(\nabla_\mu\chi\nabla_\nu\chi-\frac{1}{2}g_{\mu\nu}\nabla_\rho\chi\nabla^\rho\chi\right)\end{aligned} \quad (3)$$

$$\begin{aligned}&R+2\omega\frac{\Box\varphi}{\varphi}-\omega\frac{\nabla^\mu\varphi\nabla_\mu\varphi}{\varphi^2}+\frac{\nabla^\mu\chi\nabla_\mu\chi}{\varphi^2}+2\Lambda+\\&+3m^2w\varphi^3+\frac{3m^2}{4}\left(e^{H^{-2}\ddot{\chi}/\chi}-1\right)+2\Lambda H^{-2}\frac{\ddot{\chi}}{\chi}=0\end{aligned} \quad (4)$$

$$\frac{\Box\chi}{\varphi}-\frac{\nabla_\mu\chi\nabla^\mu\varphi}{\varphi^2}-\frac{3m^2\varphi}{4}\frac{H^{-2}\ddot{\chi}}{\chi^2}e^{H^{-2}\ddot{\chi}/\chi}-\frac{2\Lambda\varphi H^{-2}\ddot{\chi}}{\chi^2}=0 \quad (5)$$

where

$$T_{\mu\nu}=-\frac{1}{\sqrt{-g}}\frac{\delta}{\delta g_{\mu\nu}}\int d^4x\sqrt{-g}L_m \quad (6)$$

is the stress-energy momentum tensor. For the Friedmann-Robertson-Walker flat space-time metric

$$ds^2=dt^2-a^2(t)\left[dr^2+r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right)\right] \quad (7)$$

where $a(t)$ is the scale factor of the universe, the resulting equations of motion are:

$$\frac{\dot{a}^2}{a^2}=\frac{\omega}{6}\frac{\dot{\varphi}^2}{\varphi^2}-\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi}+\frac{1}{6}\frac{\dot{\chi}^2}{\varphi^2}+\frac{\Lambda}{3}+\frac{8\pi\rho}{3\varphi}+ \\ +\frac{m^2\left(w\varphi^4-1\right)}{8\varphi}+\frac{m^2}{8}\left(e^{H^{-2}\ddot{\chi}/\chi}-1\right)+\frac{\Lambda}{6\varphi}\left(2\varphi H^{-2}\frac{\ddot{\chi}}{\chi}-1\right) \tag{8}$$

$$\ddot{\varphi}+3\frac{\dot{a}}{a}\dot{\varphi}+\frac{2}{3+2\omega}\frac{\dot{\chi}^2}{\varphi}=\frac{1}{2\omega+3}\left(8\pi(\rho-3p)-\frac{3}{2}m^2-\frac{3}{2}m^2w\varphi^4+\right. \\ \left.+\frac{3m^2}{4}\left(e^{H^{-2}\ddot{\chi}/\chi}-1\right)\varphi-2\Lambda+2\varphi H^{-2}\frac{\ddot{\chi}}{\chi}\right) \tag{9}$$

$$\ddot{\chi}\left(1-\frac{\varphi^2}{H^2\chi^2}\left(\frac{3m^2}{4}e^{H^{-2}\ddot{\chi}/\chi}+\Lambda\right)\right)+3\frac{\dot{a}}{a}\dot{\chi}-\frac{\dot{\varphi}}{\varphi}\dot{\chi}=0 \tag{10}$$

In order to solve equations (8)–(10), we assume for simplicity that the cosmological constant varies with both fields like

$$\Lambda=\frac{H^2\chi^2}{\varphi^2}-\frac{3m^2}{4}e^{H^{-2}\ddot{\chi}/\chi}=\frac{H^2\chi^2}{\varphi^2}-\frac{3m^2}{4}e^{Q} \tag{11}$$

where $H\equiv\dot{a}/a$. This is the effective cosmological constant in the theory which depends on the quantum potential. In addition, we will assume that the fluid density is:

$$\rho=3p+\frac{3}{16\pi}m^2+\frac{3}{16\pi}m^2w\varphi^4-\frac{3m^2}{32\pi}\left(e^{H^{-2}\ddot{\chi}/\chi}-1\right)\varphi-\frac{\varphi H^{-2}}{4\pi}\frac{\ddot{\chi}}{\chi}+\frac{\Lambda}{4\pi} \tag{12}$$

Equation (10) gives $3\dot{a}/a=\dot{\varphi}/\varphi$ for $\dot{\chi}\neq 0$ which after inserting in equation (9) gives:

$$\ddot{\varphi}+\frac{\dot{\varphi}^2}{\varphi}+\frac{2}{3+2\omega}\frac{\dot{\chi}^2}{\varphi}=0 \tag{13}$$

while equation (9) gives:

$$\left(4-\frac{3\omega}{2}\right)\frac{\dot{a}^2}{a^2}=\frac{1}{6}\frac{\dot{\chi}^2}{\varphi^2}+\frac{\Lambda}{3}+\frac{8\pi\rho}{3\varphi}+\frac{m^2\left(w\varphi^4-1\right)}{8\varphi}+ \\ +\frac{m^2}{8}\left(e^{H^{-2}\ddot{\chi}/\chi}-1\right)+\frac{\Lambda}{6\varphi}\left(2\varphi H^{-2}\frac{\ddot{\chi}}{\chi}-1\right) \tag{14}$$

Assuming a power-law behavior for both scalar fields as $\varphi\propto\varphi_0\left(t/t_0\right)^r$ and $\chi\propto\chi_0\left(t/t_0\right)^n$ where r and n are dimensionless constants, with φ_0 and χ_0 are the values of the scalar field at the origin of time t_0 assumed all three equal to one for simplicity without any loss of generality in our subsequent calculations. It is easy to check that equation (13) gives $r = n = (3+2\omega)/4(2+\omega)$. For $\omega > -3/2$, $r > 0$ and the scalar fields increase with time. If in contrast, $\omega < -3/2$, $r < 0$ and the scalar fields decrease with time. The cosmological constant behaves like:

$$\Lambda = H^2 - \frac{3m^2}{4} e^{r(r-1)/H^2 t^2} \tag{15}$$

which at late times tends at late times $(t \to \infty)$ to $H^2 - (3m^2/4)$ which is positive as long as $m^2 < 4H^2/3$. As a result, the model may explain why the cosmological constant is too small. This effective cosmological constant can be viewed as the sum of the standard Einstein's lambda and a negative induced time-dependent one given by:

$$\Lambda_{induced} = -\frac{3m^2}{4} e^{r(r-1)/H^2 t^2} \tag{16}$$

which tends to a constant negative value at larger times. Notes that at the critical time

$$t_c = H^{-1} \sqrt{\frac{r(r-1)}{\ln \frac{4H^2}{3m^2}}} = H^{-1} \sqrt{\frac{(5+2\omega)(3+2\omega)}{(8+4\omega)^2 \ln \frac{3m^2}{4H^2}}} \tag{17}$$

the cosmological constant vanishes. Notice that the following constraints $\omega > -3/2$ ($r > 0$) or $\omega < -5/2$ ($r > 1$) are required. The gravitational coupling which is given in BD theory [41] by $G = (4+2\omega)/(4+3\omega)\varphi$ may take in our scenario a positive value, in contrast to other models where the big price to pay is $G < 0$ [9, 42, 43]. Before investigating about the scale factor behavior, it is worthmentioning that the fluid density given by equation (12) behaves like:

$$\rho \approx 3p + \frac{3m^2 + 4\Lambda}{16\pi} + \frac{3}{16\pi} m^2 w t^{4r} - \frac{1}{4\pi} \frac{r(r-1)}{H^2 t^{2-r}} \tag{18}$$

If $r < 0$, then the fluid density tends to $\rho \approx 3p + (H^2/4\pi)$ for $t \to \infty$. We are more interested on the opposite case $r > 0$ in order to have a decaying fluid density as required by most cosmological theories. Therefore, a decaying ultra-light masse just like $m \propto t^{-1}$ is required and additionally $r < 1/2$. As a result, at very larger times, the fluid density tends to $\rho \approx 3p + (H^2/4\pi)$, a solution which is satisfied for $r < 1/2$ and for every ω. From equation (14), choosing $\rho = H^2/16\pi$, one obtains $p = -H^2/16\pi$ and therefore $p = -\rho$ and therefore we are dealing with a dark energy problem or quintessence.

Further, from equation (14) and in particular at late times, the dominant factor is $-m^2/8\varphi$ which consequently gives for $r < 1/2$, a scale factor behaving like $a \propto \exp(\delta t^{3-r})$ which corresponds to a superaccelerated regime. Here $\delta = 1/(2(3-r)\sqrt{(3\omega-8)})$ augmented by the constraint $\omega > 8/3$.

We summarize:

The scalar fields increases lowly with time as $\varphi,\ \chi \propto t^{1/2}$, the pressure and the density behaves as $p = -\rho$, the universe undergoes a superaccelerated regime with $\omega > 8/3$. In the absence of the potential $V(\varphi)$, the dominant term in equation (14) is the first term on the RHS and consequently, it is easy to show that we have a power-law inflation as $a \propto t^{\nu}$ where $\nu = r/\sqrt{24-9\omega}$ with the constraints $\sqrt{24-9\omega} < r < 1/2$ in order to have $\nu > 1$ (accelerated expansion) and $\omega < 8/3$. This shows the role of the potential $V(\varphi)$ in the theory. Its presence yields a superinflationary regime and power-law inflation if the ultra-light masses are set equal to zero or if $V(\chi, \varphi)$ dominates over $V(\varphi)$.

The cosmological approach developed in this paper is based on de-Broglie-Bohm quantum theory of gravity, in particular the quantum potential. Many interesting features and consequences on the description of the Brans-Dicke scalar cosmology are explored. The role of the quantum potential and de-Broglie-Bohm quantum theory of gravity must of course be supplemented by a fundamental treatment and investigations. Further consequences and details are in progress.

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