SCALING AND UNIVERSAL POWER LAWS IN TIME SERIES OF SEISMIC EVENTS

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It is shown that functions obeying a scale-invariance property can be conveniently represented as linear superpositions of power laws. Seismicity characteristics of earthquake distributions and focal mechanisms are employed to suggest such a power law for the temporal distribution of nearest-neighbours pairs of earthquakes in the limit of short times. Arguments are given for a universal function introduced recently for such pair distribution (A. Corral, Phys. Rev. Lett. **92** 108501 (2004)) and an application is made for Vrancea earthquakes.

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Temporal distribution of earthquakes is of particular importance for assessing seismic risk and hazard. In the absence of more specific knowledge regarding earthquake generating mechanisms, statistical analysis is employed in order to detect possible regular patterns in the spatial, temporal and magnitude distributions of the earthquakes. It is widely agreed that temporal series of earthquakes exhibit two main paterns. First, they may appear as “regular” earthquakes, [1–3] characterized by a mean recurrence time t_r, and secondly each main seismic shock may be accompanied by foreshocks and aftershocks, as described by Omori’s law [4–9]. According to this wisdom, earthquakes are conventionally divided into “regular” earthquakes, quasi-randomly distributed in time by Poisson-like distributions, and accompanying earthquakes governed by Omori’s law. The basic equation in describing earthquakes is the Gutenberg-Richter equation $E = E_0 \exp(bM)$, where E is the energy released in an earthquake, E_0 is a threshod energy and M is the earthquake’s magnitude (usually defined as seimic moment) [10–12]. Parameter b has the value $b = 3.5$ [12].

A model has been put forward recently [13] for accumulating seismic energy in a localized focus, which relates the accumulating time t to energy E

through the relationship $t/t_0 = (E/E_0)^r$, where t_0 is a threshold time. The exponent r is a parameter accounting for the particular geometry of the critical focal zone and the particular mechanism of accumulating energy. It was shown on geometrical grounds that parameter r acquires the value $r = 1/3$ for a uniform mechanism of accumulating energy in a point-like localized focus, or $r = 1/2$ for a fault-planes critical zone with a geometry close to a two-dimensional one. In general, the inverse $1/r$ may be viewed as a fractal dimension [14] in the relationship $E/E_0 \sim (R/R_0)^{1/r}$, where R and R_0 are corresponding scale lengths characterizing the focal zone. The particular values of the parameter r, as depending on the seismic region and the set of data analyzed, can be obtained by statistical fits to data.

Making use of this model, and identifying the accumulation time with the mean recurrence time, the earthquakes distribution in energy rE_0^r/E^{1+r} has been derived, as well as the well-known magnitude distribution $P(M) = \beta\exp(-\beta M)$, where $\beta = br$ (β = 1.17 for $r = 1/3$) [13]. It follows immediately the well-known Gutenberg-Richter distribution law[15–17] $\ln(\Delta N/N_0\Delta M) = \ln\beta - \beta M$, and the Gutenberg-Richter cumulative distribution $\ln(N_{ex}/T) = -\ln t_0 - \beta M$ (recurence law, or excedence rate), where ΔN is the number of earthquakes with magnitude M to $M + \Delta M$ out of a total of N_0 earthquakes occurring in (a long) time T. It turns out that $1/t_0 = N_0/T$ is the seismicity rate. The Gutenberg-Richter laws given above are well documented for a variety of regions, time intervals and magnitudes. For instance, a worldwide analysis for $5.8 < M < 7.3$ indicates $r = 0.39$ (β = 1.38, compares with β = 1.17 corresponding to $r = 1/3$; and $-\ln t_0 = 12.65$ for t_0 measured in years) [12]. Data for Southern California [18] seem to indicate $r = 0.66$ (β = 2.3; and $-\ln t_0 = 17.25$ for t_0 in years). Similarly, a recent analysis for 1999 earthquakes with magnitude $M > 3$ recorded in Vrancea between 1974 and 2004[13] indicates the average values $r = 0.54$ (β = 1.89; and $-\ln t_0 = 9.68$ for t_0 in years; the fit to the Gutenberg-Richter recurrence law gives β = 1.76 and $-\ln t_0 = 8.99$; in general, there are appreciable deviations from these linear laws for extreme values of the magnitude). In addition, as it follows from the distributions given above, the model of accumulation of seismic energy gives a mean recurrence time $t_r = (t_0/\beta\Delta M)\exp(\beta M)$ for earthquakes with magnitude M to $M + \Delta M$, which enter the Poissonian distribution [19] $(1/t_r)\exp(-t/t_r)$ for such earthquakes characterized by a (fixed) mean recurrence time t_r.

The accompanying seismic activity of foreshocks and aftershocks enjoyed extensive investigations, in special in connection with the critical-point theory and self-organized criticality [20–22] It was shown recently that such associated earthquakes may appear by a self-replication mechanism with leads to Omori's law [23]. According to Omori's law, the rate of accompanying earthquakes is given by $dn/d\tau \sim 1/|\tau|$, and the rate of released energy goes like $dE/d|\tau| \sim -1/\tau^2$ (or $E \sim 1/|\tau|$), where τ denotes time measured with respect to the occurrence of the main shock ($\tau < 0$ for foreshocks and $\tau > 0$ for aftershocks). Both relationships seem to be supported by empirical evidence [5, 7].

Recently, it becomes increasingly apparent that the picture described above is more complex. It was recognized, at least for small and moderate earthquakes, that their magnitude and occurrence time are distinct, independent statistical variables [24]. In addition, correlations effects of various sorts, including clustering [25, 26] or self-organized seismic criticality, [7, 20, 27, 28] seem to be present in statistical distributions of earthquakes, beside random occurrence. In this context, the temporal pair distribution $D(\tau)$ of nearest-neighbouring earthquakes [29–31] received recently a great deal of attention in statistical studies of earthquakes [32–35]. In general, the probability density of N serial events denoted by i and occurring at time t_i can be written as $\rho(t) = = N^{-1}\sum_i \delta(t_i - t)$. Similarly, the pair distribution of nearest-neighbours separated by time τ is given

$$D(\tau) = \frac{dN}{Nd\tau} = \frac{1}{N}\sum_i \delta(t_{i+1} - t_i - \tau)\ . \tag{1}$$

This function is also known as the recurrence, or waiting time distribution, or next-earthquake distribution. Usually, N represents the number N_{ex} of earthquakes with magnitude M greater than a certain cutoff magnitude M_c. According to the Gutenberg-Richter recurrence law given above it can be written as $N/T = t_0^{-1}e^{-\beta M_c}$. Function $D(\tau)$ must depend on a dimensionless variable $R\tau$, where R^{-1} is a characteristic scale time. Since the only characteristic time in the problem is the inverse of the excedence rate we may take $R = t_0^{-1}e^{-\beta M_c}$. The pair distribution can therefore be written as $D(\tau) = Rf(R\tau)$, where f is a function which remains to be determined. It was recently suggested [33, 34] that the pair distribution of nearest-neighbouring earthquakes can be written in this form, where f is a universal function (in the sense that it does not depend on the cutoff parameter M_c). It can be said that functions $D(\tau)$ which satisfy the relationship $D(\tau) = Rf(R\tau)$, where $f(\tau)$ is a universal function, possess a scale-invariance property. Such a function $D(\tau)$ can also be represented as $R^{-1}D(R^{-1}\tau) = f(\tau)$, and, since $f(\tau)$ does not depend on R, it follows that function $D(R^{-1}\tau)$ is given by a function $F(R^{-1}, R^{-1}\tau) = D(R^{-1}\tau) = Rf(\tau)$ of two variables, which satisfies the equation

$$F + x\partial F/\partial x + u\partial F/\partial u = 0, \tag{2}$$

where $x = R^{-1}$ and $u = R^{-1}\tau$. Equation (2) is obtained by taking the derivative of $R^{-1}F(R^{-1}, R^{-1}\tau) = f(\tau)$ with respect to R^{-1} and making use of the fact that $f(\tau)$ does not depend on the cutoff R. The elementary solutions of this equation are $F(x,u) = x^{-1+\alpha}u^{-\alpha}$, *i.e.* $D(\tau) = R(R\tau)^{-\alpha}$ and universal power laws $f(\tau) = \tau^{-\alpha}$, where α is any (real) exponent. The general solution is a superposition of such power laws.

It is reasonable to assume that for long times t the number of nearest-neighbours pairs $D(\tau)$ corresponding to small values of τ is proportional to t, *i.e.* $D(\tau) \sim t$. These events can also be viewed as releasing the energy E accumulated in time t in the short interval of time τ, so we may write, according to Omori's law, $E \sim 1/\tau$. On the other hand, according to the model of accumulating the seismic energy discussed herein, the time t and energy E are related trough $t \sim E^r$. These relationships lead to $D(\tau) \sim t \sim E^r \sim 1/\tau^r$, *i.e.* the power law $D(\tau) \sim 1/(R\tau)^r$ for $\tau \to 0$, with the exponent α equal to the parameter r. Therefore, it is assumed that the exponent α in the power law of the pair distribution for short times τ can tentatively be taken as the parameter r of the model of accumulating seismic energy. Such a power law for the pair distribution in the limit of short times indicates correlations in pair distribution, in agreement with clustering effects [25].

For large values of τ it is natural to expect an exponential behavior $D(\tau) \sim e^{-R\tau/B}$, in agreement with an uncorrelated, quasi-random, Poisson-like distribution of pairs [19, 36]. It is worth noting that such an exponential can be represented as a superposition of power laws. The parameter B in this exponential may originate in the corrections to the simplified relationship $t \sim E^r$ for small values of t and E. Indeed, if the cutoff parameters t_0 and E_0 are introduced explicitly by $t \to t + t_0$ and $E \to E + E_0$, this relationship becomes $1 + t/t_0 = (1 + E/E_0)^r$. It leads to an excedence rate $N_{ex}/T = t_0^{-1}(1 + e^{bM})^{-r}$, which is slightly different from the simplified form $N_{ex}/T = t_0^{-1}e^{-\beta M}$ (where $\beta = br$). The difference between the two relationships can be seen for small values of t, *i.e.* large values of τ (where parameter B is effective), which means vanishing magnitudes. For instance, the relationship given above yields a correction factor of the order ~ 2^r for $M = 0$, which amounts to 1.26 for $r = 1/3$. This correction factor may account for the parameter $B > 1$ in the exponential given above, which means that the cutoff parameter $R = t_0^{-1}(1 + e^{bM_c})^{-r}$ is smaller, in fact, than its simplified value $R = t_0^{-1}e^{-\beta M_c}$.

Taking into account the asymptotic behaviour of $D(\tau)$ given above, both for $\tau \to 0$ and $\tau \to \infty$, we are led to suggest that the pair distribution (1) can be written as

$$D(\tau) = CR \cdot \frac{1}{(R\tau)^{\alpha}} \cdot e^{-R\tau/B} \quad , \tag{3}$$

such that the normalization constant C satisfies $CB^{1-\alpha}\Gamma(1-\alpha) = 1$, where Γ is Euler's gamma function, as it was pointed out recently [32–34]. The exponent α is tentatively identified here with the parameter r, and is viewed as a fitting parameter for the class of functions given by equation (3).

The values $C = 1/2$, $B = 1.58$ and $\alpha = 0.33$ in (3) have been established recently, [32–34] by an extensive analysis of earthquakes recorded in a large variety of world wide regions, spanning various time intervals and magnitude ranges. The universal function in (3) has also been discussed recently in connection with correlation effects [35]. The exponent $\alpha = 0.33$ corresponds to rather moderate values $R\tau < 1$, *i..e.* for time $\tau \le R^{-1}$ comparable with the mean seismicity rate R^{-1}, in agreement with moderate and strong earthquakes statistics, and possibly suggesting a generic model of localized, point-like seismic focus (compare with $r = 1/3$). It is worth noting that the exponent $\alpha = 0.33$ derived by such an analysis of the empirical data agrees with the corresponding value $r = 1/3$ of the parameter r, as suggested above. In the limit $R\tau \to 0$ the exponent α in (3) exhibits a certain variability, acquiring values that are close to the values of the parameter r obtained from the fit of the Gutenberg-Richter recurrence law. For instance, $\alpha = 0.57$ for Vrancea earthquakes in this limit, in agreement with $r = 0.54$ indicated above. It is believed that the pair distribution undergoes a non-stationary criticality [32, 33] in the limit $R\tau \to 0$, where aftershocks tend to increase the value of the exponent α toward Omori's law exponent $\alpha = 1$.

The pair distribution $D(\tau)$ given by (1) has been analyzed for 1999 earthquakes with magnitude $M > M_c = 3$ recorded in Vrancea between 1974 and 2004 [37]. The function $D(\tau)$ *vs* τ is shown in Fig. 1 (on a logarithmic scale) for various cutoff magnitudes M_c. It exhibits a rather large dispersion, especialy for greater cutoff magnitudes ($M_c = 3.8$) and large values of $R\tau$. The data collapse on rescaling with cutoff parameter R, as shown in Fig. 2, except, possibly, for the limit $R\tau \to 0$, where a variability may appear. The fit employing the universal function given by (3) gives $\alpha = 0.25$, $B = 1.17$ and $C = 0.71$ (with 13% error), in fair agreement with the fitting parameters given in Refs. 32 and 33. The poor statistics for Vrancea, especially for earthquakes with higher magnitude, prevents a more reliable analysis.

One of the most interesting applications of the pair distribution is the computation of the next-earthquake probability for earthquakes with magnitude greater than M, which, according to its definition (1), is given by $D(\tau)$ for the cutoff parameter $R = t_0^{-1} e^{-\beta M}$ in equation (3). This probability is shown in Fig. 3 for Vrancea, for $M = 5$ and $\alpha = 0.25$, $B = 1.17$, $C = 0.71$. It corresponds to

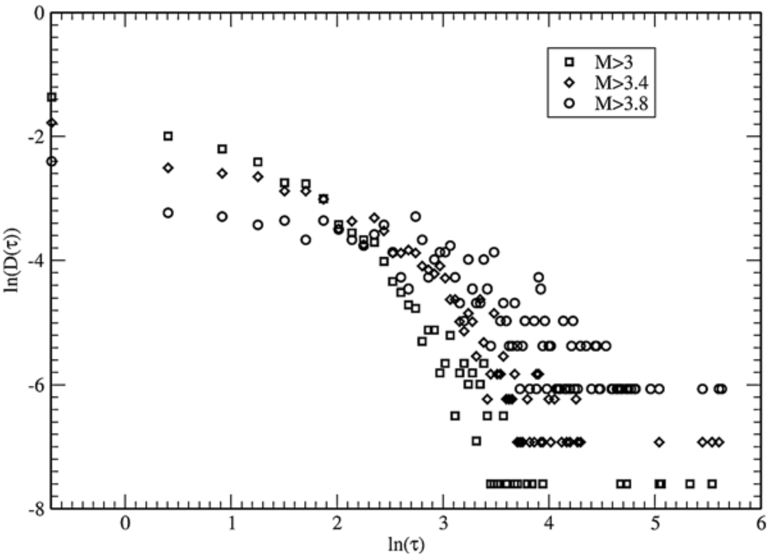

Fig. 1 – Pair distribution *vs.* time (measured in days) for various cutoff magnitudes ($M_c = 3$, 3.4, 3.8) for 1999 ($M_c = 3$) earthquakes recorded in Vrancea between 1974 and 2004 [37].

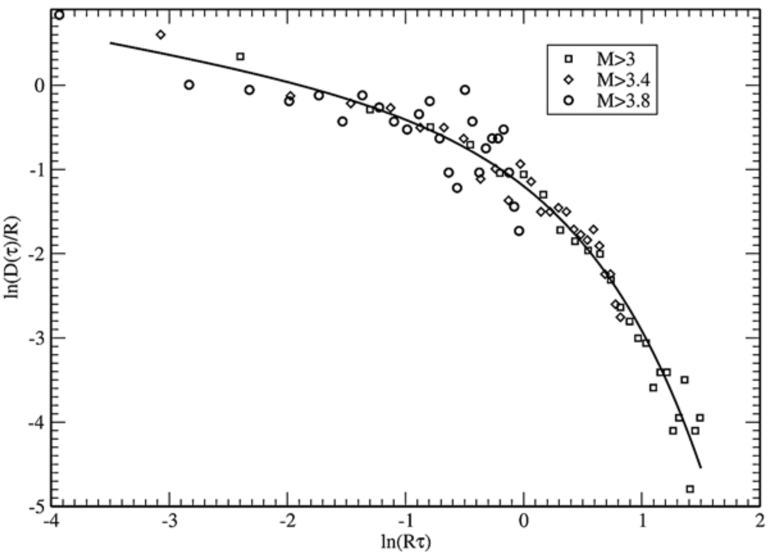

Fig. 2 – Rescaled pair distributions and the fit provided by equation (3) (solid curve, $\alpha = 0.25$, $B = 1.17$, $C = 0.71$) for 1999 earthquakes recorded in Vrancea between 1974 and 2004 ($M > M_c = 3$) [37].

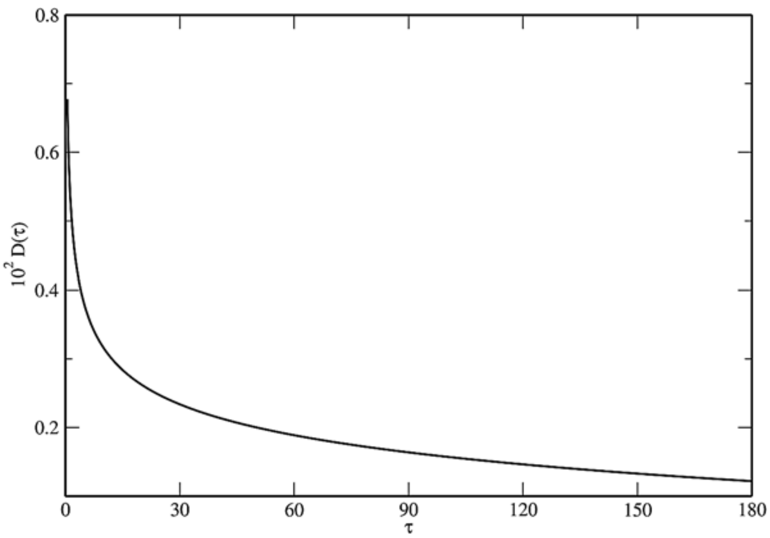

Fig. 3 – Temporal distribution for successive pairs of earthquakes with magnitude $M > 5$ as estimated by equation (3) for Vrancea ($\alpha = 0.25$, $B = 1.17$, $C = 0.71$; time τ on the abscissa is measured in days).

$R = 1.6 \times 10^{-3}$ day^{-1}, *i.e.* 18 earthquakes with $M > 5$ for 30 years, which differs slightly from $R = t_0^{-1} e^{-\beta M} = 3 \times 10^{-3}$ day^{-1} obtained by fitting the Gutenberg-Richter recurrence law to all earthquakes with magnitude $M > M_c = 3$ ($-\ln t_0 = 8.99$ and $\beta = 1.76$). By making use of (3), it may be estimated, for instance, that the probability of having two earthquakes in the same day, in Vrancea, with magnitude greater than $M = 5$ is ~0.8%. It may be slightly greater since the exponent α is increased for such short times, as, for instance, due to possible ocurrence of aftershocks. In general, the probability of having the next-earthquake in time τ is given by $\int_0^{\tau} d\tau' \cdot D(\tau') = CB^{1-r}\gamma(1-\alpha, R\tau/B)$, where γ is Euler's incomplete gamma function.

In conclusion, a temporal pair distribution of nearest-neighbouring earthquakes is analyzed herein by general scale arguments and seismicity parameters, in agreement with the universal function established recently by statistical analysis of extensive empirical data sets [32–34]. The results are applied to Vrancea earthquakes, especially in connection with assessing short-term seismicity.

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